DÚ2

1. • $N \sim Po(\lambda)$, $\lambda > 0$

$$\frac{p_N(k)}{p_N(k-1)} = \frac{e^{-\lambda} \cdot \frac{\lambda^k}{k!}}{e^{-\lambda} \cdot \frac{\lambda^{k-1}}{(k-1)!}} = \frac{\lambda}{k} = 0 + \frac{\lambda}{k}$$

$\Rightarrow a = 0,\ b = \lambda$

• $N \sim Bi(n,p)$, $p \in (0,1)$, $n \in \mathbb{N}$

$$\frac{p_N(k)}{p_N(k-1)} = \frac{\binom{n}{k}(1-p)^{n-k} \cdot p^k}{\binom{n}{k-1}(1-p)^{n-k+1} p^{k-1}} = (1-p)^{-1} p \cdot \frac{n-k+1}{k} =$$

$$= \frac{p}{1-p}(-1) + \frac{p}{1-p}\frac{(n+1)}{k}$$

$\Rightarrow a = -\frac{p}{1-p}$, $b = (n+1) \cdot \frac{p}{1-p}$

• $N \sim NeBi(m,p)$, $m > 0$, $p \in (0,1)$

$$\frac{p_N(k)}{p_N(k-1)} = \frac{\binom{k+m-1}{k} p^m (1-p)^k}{\binom{k+m-2}{k-1} p^m (1-p)^{k-1}} = (1-p) \cdot \frac{k+m-1}{k} =$$

$$= (1-p) + \frac{(1-p)(m-1)}{k}$$

$\Rightarrow a = 1-p$, $b = (1-p)(m-1)$

2. $M \sim Po(3)$, $p_M^T(k) = \frac{p_M(k)}{1 - p_M(0)}$, $k = 1, 2, \dots$

$p_M^T(0) = 0$

$$p_M^T(1) = \frac{p_M(1)}{1-p_M(0)} = \frac{e^{-\lambda} \cdot \frac{\lambda^1}{1!}}{1 - e^{-\lambda} \cdot \frac{\lambda^0}{0!}} = \frac{\lambda \cdot e^{-\lambda}}{1 - e^{-\lambda}} = \frac{3 \cdot e^{-3}}{1 - e^{-3}} \doteq \underline{\underline{0{,}157}}$$

$$p_M^T(2) = \frac{p_M(2)}{1-p_M(0)} = \frac{e^{-\lambda} \cdot \frac{\lambda^2}{2!}}{1 - e^{-\lambda}} = \frac{\frac{9}{2} e^{-3}}{1 - e^{-3}} \doteq \underline{\underline{0{,}236}}$$

$$p_M^T(3) = \frac{p_M(3)}{1-p_M(0)} = \frac{e^{-\lambda} \cdot \frac{\lambda^3}{3!}}{1 - e^{-\lambda}} = \frac{\frac{27}{6} \cdot e^{-3}}{1 - e^{-3}} = \frac{\frac{9}{2} \cdot e^{-3}}{1 - e^{-3}} \doteq \underline{\underline{0{,}236}}$$

3. $M \sim Bi(2; 0{,}2)$, $P(M=0) = 0{,}9$, $p_M^M(k) = \frac{1 - p_M^M(0)}{1 - p_M(0)} \cdot p_M(k)$

$p_M^M(0) = 0{,}9$

$$p_M^M(1) = \frac{1 - 0{,}9}{1 - p_M(0)} \cdot p_M(1) = \frac{0{,}1}{1 - (1-p)^2} \cdot \binom{2}{1} \cdot 0{,}2^1 \cdot 0{,}8^1 =$$

$$= \frac{0{,}1}{1 - 0{,}8^2} \cdot 2 \cdot 0{,}2 \cdot 0{,}8 = \underline{\underline{0{,}089}}$$

$$p_M^M(2) = \frac{0{,}1}{1 - 0{,}8^2} \cdot p_M(2) = \frac{0{,}1}{1 - 0{,}8^2} \cdot \binom{2}{2} \cdot 0{,}2^2 \cdot 0{,}8^0 = \underline{\underline{0{,}011}}$$

4. $N \sim Po(\lambda)$, $\lambda > 0$

$$G_N(s) = \sum_{k=0}^{\infty} e^{-\lambda} \cdot \frac{\lambda^k}{k!} \cdot s^k = e^{-\lambda} \cdot \sum_{k=0}^{\infty} \frac{(\lambda s)^k}{k!} = e^{-\lambda} \cdot e^{\lambda s} = e^{\lambda(s-1)}$$

$$G'_N(1) = EN$$

$$= \lambda \cdot e^{\lambda(s-1)} \Big|_{s=1} = \underline{\underline{\lambda}}$$

$$G''_N(1) = E(N(N-1))$$

$$= \lambda^2 \cdot e^{\lambda(s-1)} \Big|_{s=1} = \underline{\underline{\lambda^2}}$$

$$\operatorname{Var} N = EN^2 - (EN)^2 = G''_N(1) + G'_N(1) - (G'_N(1))^2 =$$

$$= \lambda^2 + \lambda - (\lambda)^2 = \underline{\underline{\lambda}}$$

5. Věta: $N_1, \dots, N_n$ nez. $N_i \sim Po(\lambda_i)$

$\Rightarrow N = \sum_{i=1}^n N_i \sim Po(\sum_{i=1}^n \lambda_i)$

$$G_{N_i}(s) = e^{\lambda_i(s-1)}$$

$$G_{N_1+\dots+N_n}(s) = \prod_{i=1}^n G_{N_i}(s) = \prod_{i=1}^n e^{\lambda_i(s-1)} = e^{(s-1)\cdot\sum \lambda_i} = e^{\lambda(s-1)}$$

kde $\lambda = \sum_{i=1}^n \lambda_i$.

$N \sim Po(\lambda) \Leftarrow$ Poissonova generující fce