

DÜ 7

1. X	$P(X=x \theta=D)$	$P(X=x \theta=S)$	θ	$P(\theta=\theta)$
0	0,7	0,5	D	0,75
1	0,2	0,3	S	0,25
2	0,1	0,2		

$$\bullet EX_D = 0 \cdot 0,7 + 1 \cdot 0,2 + 2 \cdot 0,1 = 0,4 = \mu(D)$$

$$EX_S = 0 \cdot 0,5 + 1 \cdot 0,3 + 2 \cdot 0,2 = 0,7 = \mu(S)$$

$$\mu = 0,4 \cdot 0,75 + 0,7 \cdot 0,25 = \underline{\underline{0,475}}$$

$$\bullet V(D) = 0^2 \cdot 0,7 + 1^2 \cdot 0,2 + 2^2 \cdot 0,1 - (\mu(D))^2 = 0,6 - (0,4)^2 = 0,44$$

$$V(S) = 0^2 \cdot 0,5 + 1^2 \cdot 0,3 + 2^2 \cdot 0,2 - (\mu(S))^2 = 1,1 - (0,7)^2 = 0,61$$

$$V = 0,75 \cdot 0,44 + 0,25 \cdot 0,61 = \underline{\underline{0,4825}}$$

$$\bullet \text{Var}(\mu(\theta)) = E(\mu(\theta)^2) - [E(\mu(\theta))]^2 = E(\mu(\theta)^2) - \mu^2 = \\ = 0,4^2 \cdot 0,75 + 0,7^2 \cdot 0,25 - 0,475^2 = \underline{\underline{0,016875}} = \eta$$

$$\bullet k = \frac{V}{\eta} = \frac{0,4825}{0,016875} \doteq 28,5926$$

$$z = \frac{n}{n+k} = \frac{2}{2+28,5926} \doteq 0,0654$$

$$\bar{X} = \frac{0+1}{2} = \frac{1}{2} \quad (x_1=0 \text{ \& } x_2=1)$$

$$\Rightarrow C_B(\underbrace{x_1, \dots, x_n}_{x_1, x_2}) = 0,0654 \cdot \frac{1}{2} + (1-0,0654) \cdot 0,475 = \underline{\underline{0,4766}}$$

$$2. X \sim \text{Po}(\theta), \quad \pi(\theta) = 3 \cdot \theta^{-4}, \quad \theta > 1$$

$$n=2, \quad \bar{X} = 10 = \frac{20}{2}$$

$$\bullet \mu(\theta) = \theta$$

$$\mu = \int_1^{\infty} \theta \cdot 3 \cdot \theta^{-4} d\theta = \left[-\frac{3}{2\theta^2} \right]_1^{\infty} = \underline{\underline{\frac{3}{2}}}$$

$$\bullet V(\theta) = \text{Var}(X_j | \theta = \theta) = \theta$$

$$V = \int_1^{\infty} \theta \cdot 3 \cdot \theta^{-4} d\theta = \underline{\underline{\frac{3}{2}}}$$

$$\bullet \eta = \text{Var}(\mu(\theta)) = E(\mu(\theta)^2) - [E(\mu(\theta))]^2 =$$

$$= \int_1^{\infty} \theta^2 \cdot 3 \cdot \theta^{-4} d\theta - \left(\frac{3}{2}\right)^2 = \left[-\frac{3}{\theta} \right]_1^{\infty} - \frac{9}{4} = \underline{\underline{\frac{3}{4}}}$$

$$\bullet k = \frac{V}{\eta} = \frac{3/2}{3/4} = 2$$

$$z = \frac{n}{n+k} = \frac{2}{2+2} = \frac{1}{2}$$

$$\bar{X} = 10$$

$$\Rightarrow C_B(x_1, x_2) = \frac{1}{2} \cdot 10 + \frac{1}{2} \cdot \frac{3}{2} = \frac{10 \cdot 2}{2 \cdot 2} + \frac{3}{4} = \underline{\underline{\frac{23}{4}}}$$