$$A = \begin{pmatrix} \frac{1}{5} & 0 & \frac{2}{5} \\ 0 & 1 & 0 \\ \frac{2}{5} & 0 & \frac{4}{5} \end{pmatrix}$$

$$\begin{vmatrix} \frac{1}{5}-\lambda & 0 & \frac{2}{5} \\ 0 & 1-\lambda & 0 \\ \frac{2}{5} & 0 & \frac{4}{5}-\lambda \end{vmatrix} = (1-\lambda) \begin{vmatrix} \frac{1}{5}-\lambda & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5}-\lambda \end{vmatrix} = (1-\lambda) \left[\left(\frac{1}{5}-\lambda\right)\left(\frac{4}{5}-\lambda\right) - \frac{4}{25} \right] =$$

$$= \cdots = (1-\lambda)(\lambda-1)\lambda \qquad \lambda_1 = 0 \qquad \lambda_{2,3} = 1$$

$\lambda_1 = 0$

$$\frac{1}{5}x_1 + \frac{2}{5}x_3 = 0$$
$$x_2 = 0$$
$$\frac{2}{5}x_1 + \frac{4}{5}x_3 = 0$$

$x_2 = 0$

$$\frac{1}{5}x_1 + \frac{2}{5}x_3 = 0$$
$$x_1 + 2x_3 = 0$$

$x_3 = t$

$x_1 = -2t$

$$(-2t, 0, t) \sim (-2, 0, 1) \quad \vec{v}_1$$

$\lambda_{2,3} = 1$

$$-\frac{4}{5}x_1 + \frac{2}{5}x_3 = 0$$
$$\frac{2}{5}x_1 - \frac{1}{5}x_3 = 0$$

$x_2 = t$

$$2x_1 - x_3 = 0$$

$x_3 = s$ $\qquad x_1 = \frac{1}{2}s$

$$\left(\frac{1}{2}s, t, s\right) \to \left(\frac{1}{2}, 0, 1\right) \ \vec{v}_2, \quad (0, 1, 0) \ \vec{v}_3$$

$$\vec{v}_2 \cdot \vec{v}_3 = 0$$

$$\vec{v}_1 \cdot \vec{v}_2 = 0 \qquad \vec{v}_1 \cdot \vec{v}_3 = 0$$

$$\begin{pmatrix} \frac{1}{5} & 0 & \frac{2}{5} \\ 0 & 1 & 0 \\ \frac{2}{5} & 0 & \frac{4}{5} \end{pmatrix} \sim \begin{pmatrix} \frac{1}{5} & 0 & \frac{2}{5} \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad h(A)$$

$$A = \begin{pmatrix} 3 & -6 & 0 \\ 1 & -2 & 0 \\ -1 & 2 & 0 \end{pmatrix}$$

$$\begin{vmatrix} 3-\lambda & -6 & 0 \\ 1 & -2-\lambda & 0 \\ -1 & 2 & -\lambda \end{vmatrix} = -\lambda \begin{vmatrix} 3-\lambda & -6 \\ 1 & -2-\lambda \end{vmatrix} = -\lambda \left[(3-\lambda)(-2-\lambda) + 6 \right] =$$

$$= -\lambda^2(\lambda - 1) \qquad \lambda_{1,2} = 0 \qquad \lambda_3 = 1$$

$\lambda_{1,2} = 0$

$$3x_1 - 6x_2 = 0$$
$$x_1 - 2x_2 = 0$$
$$-1 + 2x_2 = 0$$

$$x_1 - 2x_2 = 0$$

$x_3 = t$ $\qquad x_2 = s$ $\qquad x_1 = 2s$

$$(2s, s, t) \to (2, 1, 0), \quad (0, 0, 1)$$

$\lambda_3 = 1$

$$2x_1 - 6x_2 = 0$$
$$x_1 - 3x_2 = 0$$
$$-x_1 + 2x_2 - x_3 = 0$$

$$x_1 - 3x_2 = 0$$
$$-x_1 + 2x_2 - x_3 = 0$$

$$\begin{pmatrix} 1 & -3 & 0 \\ -1 & 2 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -3 & 0 \\ 0 & +1 & +1 \end{pmatrix}$$

$x_1 - 3x_2 = 0$

$x_2 + x_3 = 0$

$x_3 = t$

$x_2 = -t$

$x_1 = -3t$

$$(-3t, -t, t) \sim (-3, -1, 1)$$

$$h(A) = 1$$

$-5+5i-i-1=-6+4i$

$$\left.\begin{aligned}(1-i)x_1 - x_2 &= 0\\ 6x_1+(-5-i)x_2+2x_3&=0\\ 8x_1-6x_2+(3-i)x_3&=0\end{aligned}\right\}$$

$x_2=(1-i)x_1$

$6x_1+(-5-i)(1-i)x_1+2x_3=0$

$8x_1-6(1-i)x_1+(3-i)x_3=0$

$4i\,x_1+2x_3=0 \qquad x_3=-2i\,x_1$

$(2+6i)x_1+(3-i)x_3=0$

$(2+6i)x_1+(3-i)(-2i)x_1=0$

$0x_1=0$

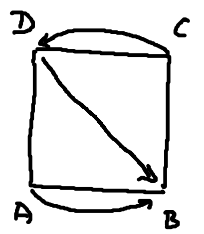
$x_1=t$

$x_2=(1-i)t$

$x_3=-2i\,t$

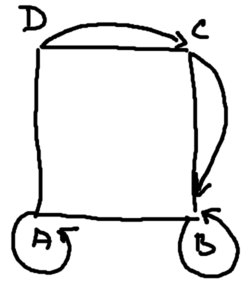
$(1,1-i,-2i)$

$f: A_2 \to A_2$



$A \to B$
$C \to D$
$D \to B$

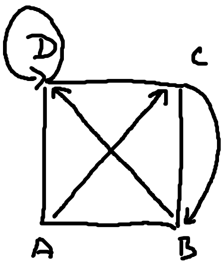
ANO



$\vec{AB} \parallel DC$

$\vec{A'B'} = \vec{AB} \nparallel \vec{D'C'} = \vec{CB}$

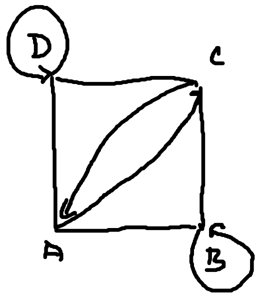
NE



$\vec{AB} \parallel \vec{DC}$

$\vec{A'B'} = \vec{CD} \nparallel \vec{D'C'} = \vec{DB}$

NE

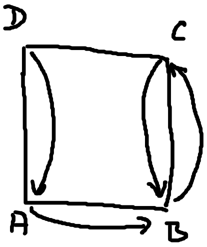


$\vec{AB} \parallel \vec{DC}$
$\downarrow$
$\vec{CB} \parallel \vec{DA}$ ✓

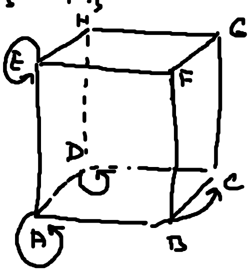
$\vec{BC} \parallel AD$
$\vec{BA} \parallel CD$ ✓

ANO

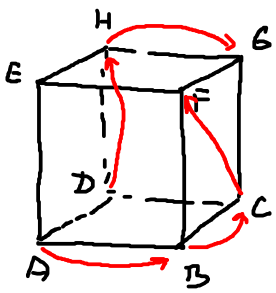
Dú



$A_3 \to A_3$



ANO



$\vec{AB} \parallel \vec{CD}$
$\downarrow$
$\vec{BC} \nparallel \vec{FH}$

NE

Určete $p, q, r \in \mathbb{R}$ tak, aby zobrazení bodů bylo afinní

$A = [1,1,1] \to A' = [-9,-9,21]$ $\quad \vec{AB} = (0,0,1)$

$B = [1,1,2] \to B' = [-9,-7,18]$ $\quad \vec{AC} = (0,0,3)$ $\quad \vec{AC} = 3\vec{AB}$

$C = [1,1,4] \to C' = [p,q,r]$

$\vec{A'B'} = (0,+2,-3) \qquad \vec{A'C'} = (p+9,\ q+9,\ r-21)$

$p+9 = 3 \cdot 0$
$q+9 = 3 \cdot 2$
$r-21 = 3 \cdot (-3)$

$$\boxed{\begin{aligned} p &= -9 \\ q &= -3 \\ r &= 12 \end{aligned}}$$

Dú $\quad A = [3,1,3] \to A' = [1,1,-9]$ $\quad C = [11,5,-5]$ $\quad p = 5$

$B = [5,2,1] \to B' = [2,3,1]$ $\quad C' = [p,q,r]$ $\quad q = 5$

$r = 31$

Rozhodněte zda $f: A_2 \to A_2$ je afinní

$$f([x,y]) = [3x-10, 3y+2]$$

$A = [a_1, a_2]$, $B = [b_1, b_2]$, $C = [c_1, c_2]$

$A' = [3a_1-10, 3a_2+2]$, $B' = [3b_1-10, 3b_2+2]$, $C' = [3c_1-10, 3c_2+2]$

$\vec{AC} = \lambda \vec{BC}$ $\quad \vec{AC} = (c_1-a_1, c_2-a_2)$, $\vec{BC} = (c_1-b_1, c_2-b_2)$

$\vec{AC} = \lambda \vec{BC}$ $\quad (c_1-a_1, c_2-a_2) = \lambda(c_1-b_1, c_2-b_2)$

$$\vec{A'C'} = (3c_1-10-(3a_1-10), 3c_2+2-(3a_2+2)) = (3(c_1-a_1), 3(c_2-a_2)) =$$

$$= (3\lambda(c_1-b_1), 3\lambda(c_2-b_2)) = \lambda(3(c_1-b_1), 3(c_2-b_2))$$

$$\vec{B'C'} = (3c_1-10-(3b_1-10), 3c_2+2-(3b_2+2)) = (3(c_1-b_1), 3(c_2-b_2))$$

$$\vec{A'C'} = \lambda \vec{B'C'}$$

$f: A_2 \to A_2$ $\quad f([x,y]) = [2x+1, xy]$

⋮

$A = [0,0] \to [1,0]$	$\vec{AC} = (2,0)$	$\vec{A'C'} = (4,0)$	$\vec{A'C'} = 2\vec{A'B'}$
$B = [1,0] \to [3,0]$	$\vec{AB} = (1,0)$	$\vec{A'B'} = (2,0)$	
$C = [2,0] \to [5,0]$	$\vec{AC} = 2\vec{AB}$		

$A = [0,0] \to [1,0]$	$\vec{AC} = (2,2)$	$A'C' = (4,4)$	$\vec{A'C'} \neq 2\vec{A'B'}$
$B = [1,1] \to [3,1]$	$\vec{AB} = (1,1)$	$A'B' = (2,1)$	NENÍ
$C = [2,2] \to [5,4]$	$\vec{AC} = 2\vec{AB}$		