Inverzní matice

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}, \quad |A| = -3, \quad A^{-1} = -\frac{1}{3} \begin{pmatrix} 3 & -2 \\ -3 & 1 \end{pmatrix}$$

adjungovaná matice

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ -1 & 1 & 3 \end{pmatrix} \qquad A^{-1} = -\frac{1}{4} \begin{pmatrix} \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} \\ -\begin{vmatrix} 0 & 2 \\ -1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} \\ \begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} \end{pmatrix} = -\frac{1}{4} \begin{pmatrix} 1 & 7 & 5 \\ -2 & 2 & -2 \\ 1 & -3 & 1 \end{pmatrix}$$

$$|A| = 3-4-1-2 = -4$$

---

Jauška, př. 2.2.1

$$f: A_2 \to A_3, \quad [1,1] \to [4,4,0], \quad [-1,1] \to [2,8,0], \quad [-1,-1] \to [-2,2,-2]$$

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\begin{aligned} a_{11}\cdot 1 + a_{12}\cdot 1 + b_1 &= 4 \\ a_{21}\, 1 + a_{22}\, 1 + b_2 &= 4 \\ a_{31}\, 1 + a_{32}\, 1 + b_3 &= 0 \end{aligned} \qquad \begin{aligned} a_{11}(-1) + a\ \ 1 + b_1 &= 2 \\ a_{21}(-1) + a_{22}\, 1 + b_2 &= 8 \\ a_{31}(-1) + a_{32}\, 1 + b_3 &= 0 \end{aligned} \qquad \begin{aligned} a_{11}(-1) + a_{12}(-1) + b_1 &= -2 \\ a_{21}(-1) + a_{22}(-1) + b_2 &= 2 \\ a_{31}(-1) + a_{32}\, 1 + b_3 &= -2 \end{aligned}$$

---

$$\begin{aligned} a_{11} + a_{12} + b_1 &= 4 \\ -a_{11} + a_{12} + b_1 &= 2 \\ -a_{11} + a_{12} + b_1 &= -2 \end{aligned} \qquad \begin{aligned} a_{21} + a_{22} + b_2 &= 4 \\ -a_{21} + a_{22} + b_2 &= 8 \\ -a_{21} - a_{22} + b_2 &= 2 \end{aligned} \qquad \begin{aligned} a_{31} + a_{32} + b_3 &= 0 \\ -a_{31} + a_{32} + b_3 &= 0 \\ -a_{31} + a_{32} + b_3 &= -2 \end{aligned}$$

$$\left(\begin{array}{ccc|c|c|c} 1 & 1 & 1 & 4 & 4 & 0 \\ -1 & 1 & 1 & 2 & 8 & 0 \\ -1 & -1 & 1 & -2 & 2 & -2 \end{array}\right) \sim \cdots \sim \left(\begin{array}{ccc|c|c|c} 1 & 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & 2 & 3 & 1 \\ 0 & 0 & 1 & 1 & 3 & -1 \end{array}\right)$$

$$\begin{aligned} a_{11} &= 1 \\ a_{12} &= 2 \\ b_1 &= 1 \end{aligned} \qquad \begin{aligned} a_{21} &= -2 \\ a_{22} &= 3 \\ b_2 &= 3 \end{aligned} \qquad \begin{aligned} a_{31} &= 0 \\ a_{32} &= 1 \\ b_3 &= -1 \end{aligned}$$

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -2 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

$$\begin{aligned} x_1' &= x_1 + 2x_2 + 1 \\ x_2' &= -2x_1 + 3x_2 + 3 \\ x_3' &= x_2 - 1 \end{aligned}$$

Př. Určete rovnice af. zobrazení

$$[1,1] \to [-1,-1,-1] \quad , \quad [1,2] \to [0,-3,-1] \quad , \quad [2,2] \to [1,-2,1]$$

$$\left(\begin{array}{ccc|c|c|c} 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & 2 & 1 & 0 & -3 & -1 \\ 2 & 2 & 1 & 1 & -2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & -1 & -1 & -1 \\ 0 & 1 & 0 & 1 & -2 & 0 \\ 0 & 0 & -1 & 3 & 0 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 2 & -1 & 2 \\ 0 & 1 & 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & -3 & 0 & -3 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|c|c|c} 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & -3 & 0 & -3 \end{array}\right) \longrightarrow \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} -3 \\ 0 \\ -3 \end{pmatrix}$$

$$x_1' = x_1 + x_2 - 3$$
$$x_2' = x_1 - 2x_2 + 0$$
$$x_3' = 2x_1 \quad - 3$$

---

$A_2 \to A_1$ $\quad [2,1] \to [2] \quad , \quad [3,2] \to [0] \quad , \quad [0,1] \to [10]$

$$(x_1') = (a_{11}\ a_{12}) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + b_1$$

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 2 \\ 3 & 2 & 1 & 0 \\ 0 & 1 & 1 & 10 \end{array}\right) \sim \left(\begin{array}{ccc|c} 6 & 3 & 3 & 6 \\ 6 & 4 & 2 & 0 \\ 0 & 1 & 1 & 10 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 1 & 1 & 2 \\ 0 & 1 & -1 & -6 \\ 0 & 1 & 1 & 10 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 1 & 1 & 2 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & 2 & 16 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|c} 2 & 1 & 0 & -6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & 0 & -8 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 8 \end{array}\right)$$

$A = (-4, 2)$

$B = (8)$

$$\boxed{x_1' = -4x_1 + 2x_2 + 8}$$

$$z = -4x + 2y + 8$$

$f: A_2 \to A_3$ $\qquad \vec{v}_1 = (-2,0) \longrightarrow v_1' = (-2,4,0)$

$R = [1,1] \to R' = [4,4,0]$ $\qquad \vec{v}_2 = (-2,-2) \longrightarrow v_2' = (-6,-2,-2)$

I

$$\begin{pmatrix} u_1' \\ u_2' \\ u_3' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \qquad \left(\begin{array}{cc|c|c|c} -2 & 0 & -2 & 4 & 0 \\ -2 & -2 & -6 & -2 & -2 \end{array}\right) \sim \left(\begin{array}{cc|ccc} 1 & 0 & 1 & -2 & 0 \\ 0 & 1 & 2 & 3 & 1 \end{array}\right)$$

$$\underline{\underline{A = \begin{pmatrix} 1 & 2 \\ -2 & 3 \\ 0 & 1 \end{pmatrix}}} \qquad \varphi_f: \begin{pmatrix} u_1' \\ u_2' \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -2 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -2 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \qquad \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -2 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\begin{aligned} 4 &= 1 + 2 + b_1 \\ 4 &= -2 + 3 + b_2 \\ 0 &= 0 + 1 + b_3 \end{aligned} \quad \to \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x_1' \\ x_2' \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -2 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

III

$R = [1,1] \qquad Q = R + \vec{v}_1 = [-1,1] \qquad S = R + \vec{v}_2 = [-1,1]$

$R' = [4,4,0] \qquad Q' = R' + \vec{v}_1' = [2,8,0] \qquad S' = R' + \vec{v}_2' = [-2,2,-2]$

II Sloupce matice A jsou souřadnic obrazů $\vec{e}_1, \vec{e}_2, \vec{e}_3, \dots$

$$\varphi(\vec{v}_1) = \varphi(-2\vec{e}_1 + 0\vec{e}_2) = -2\varphi(\vec{e}_1) + 0 \cdot \varphi(\vec{e}_2) = -2\vec{e}_1 + 4\vec{e}_2 + 0\vec{e}_3$$

$$\varphi(\vec{v}_2) = \varphi(-2\vec{e}_1 - 2\vec{e}_2) = -2\varphi(\vec{e}_1) - 2\varphi(\vec{e}_2) = -6\vec{e}_1 - 2\vec{e}_2 - 2\vec{e}_3$$

$$-2\varphi(\vec{e}_1) = -2\vec{e}_1 + 4\vec{e}_2 \longrightarrow \varphi(\vec{e}_1) = \vec{e}_1 - 2\vec{e}_2 + 0\vec{e}_3$$

$$-2\vec{e}_1 + 4\vec{e}_2 - 2\varphi(\vec{e}_2) = -6\vec{e}_1 - 2\vec{e}_2 - 2\vec{e}_3$$

$$\vdots$$

$$\varphi(\vec{e}_2) = 2\vec{e}_1 + 3\vec{e}_2 + 1\vec{e}_3$$

$f: A_2 \to A_2$

$B\,[-1,0] \longrightarrow B'\,[0,3]$

$\vec{u}_1 = (2,-1) \longrightarrow \vec{u}_1{}' = (3,-2)$

$\vec{u}_2 = (1,2) \longrightarrow \vec{u}_2{}' = (4,-1)$

$$\left(\begin{array}{cc|c|c} 2 & -1 & 3 & -2 \\ 1 & 2 & 4 & -1 \end{array}\right) \sim \left(\begin{array}{cc|c|c} 1 & 2 & 4 & -1 \\ 2 & -1 & 3 & -2 \end{array}\right) \sim \left(\begin{array}{cc|c|c} 1 & 2 & 4 & -1 \\ 0 & -5 & -5 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c|c} 1 & 2 & 4 & -1 \\ 0 & 1 & 1 & 0 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cc|c|c} 1 & 0 & 2 & -1 \\ 0 & 1 & 1 & 0 \end{array}\right) \qquad A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$2 \cdot (-1) + 1 \cdot 0 + b_1 = 0$

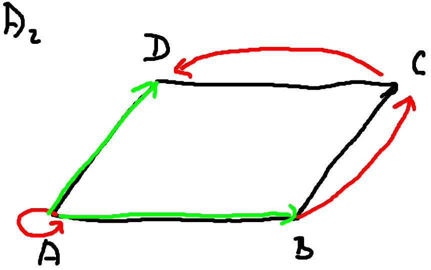
$-1\,(-1) + 0 \cdot 0 + b_2 = 3$

$b_1 = 2 \qquad b_2 = 2$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\boxed{\begin{aligned} x_1' &= 2x_1 + x_2 + 2 \\ x_2' &= -x_1 + 2 \end{aligned}}$$

$f: A_2 \to A_2$




$A \to A$

$B \to C$

$C \to D$

$R = \langle A, \vec{AB}, \vec{AD} \rangle$

$A = [0,0]$, $B = [1,0]$, $C = [1,1]$, $D = [0,1]$

$A' = [0,0]$ $\quad B' = [1,1]$ $\quad C' = [0,1]$

$$\left(\begin{array}{ccc|c|c} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c|c} 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c|c} 1 & 1 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right) \qquad \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$x_1' = x_1 - x_2$

$x_2' = x_1$

Urči samodružné body zobrazení

$$f([x_1,x_2]) = [x_1,x_2]$$

a) $\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$x_1' = 2x_1 - x_2 + 1$ $\qquad x_1 = 2x_1 - x_2 + 1$ $\qquad x_1 - x_2 + 1 = 0$

$x_2' = x_1 + 2x_2 + 3$ $\qquad x_2 = x_1 + 2x_2 + 3$ $\qquad x_1 + x_2 + 3 = 0$

$x_1 - x_2 = -1$

$x_1 + x_2 = -3$

$$\left(\begin{array}{cc|c} 1 & -1 & -1 \\ 1 & 1 & -3 \end{array}\right) \longrightarrow \begin{array}{l} x_1 = -2 \\ x_2 = -1 \end{array} \qquad [-2,-1]$$

b) f: $x_1' = x_1$ $\qquad x_1 = x_1$

$x_2' = x_1 + 2x_3 + 2$ $\qquad x_2 = x_1 + 2x_3 + 2$

$x_3' = 2x_1 - x_2 + 2x_3 + 1$ $\qquad x_3 = 2x_1 - x_2 + 2x_3 + 1$

$0 = 0 \cdot x_1$

$x_1 - x_2 + 2x_3 + 2 = 0$

$2x_1 - x_2 + x_3 + 1 = 0$

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & -1 & 2 & -2 \\ 2 & -1 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & -2 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 = t$ $\qquad t - x_2 + 2x_3 = -2$ $\qquad x_3 = t - 1$

$\qquad x_2 - 3x_3 = 3$ $\qquad x_2 = 3t$

$[t, 3t, -1+t]$

$x_1 = t$

$x_2 = 3t$

$x_3 = -1 + t$

---

DÚ

1) Urči souřadnicové vyjádření af. zobrazení, pokud znáte

$A = [-1,0,1] \longrightarrow A' = [-1,-1,0]$

$B = [0,-1,1] \longrightarrow B' = [0,0,1]$

$C = [-1,1,0] \longrightarrow C' = [-1,0,1]$

$D = [2,2,2] \longrightarrow D' = [2,7,13]$

Řeš: $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$

$B = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

2) Určete souřadnicové vyjádření af. zobrazení, znáte-li

$R = [0,-1,3] \longrightarrow R' = [2,-3,-1]$

$\vec{u}_1 = (1,1,0) \longrightarrow \vec{u}_1' = (1,2,0)$

$\vec{u}_2 = (-1,0,2) \rightarrow \vec{u}_2' = (1,1,-4)$

$\vec{u}_3 = (2,1,-1) \rightarrow \vec{u}_3' = (1,1,3)$

Řeš.: $A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 3 & 0 \\ 2 & -2 & -1 \end{pmatrix}$

$B = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$

3) V $A_2$ je dán trojúhelník $\triangle ABC$, vzhledem ke vhodné bázi napište rovnice af. zobrazení

$A \to B$, $B \to C$, $C \to A$

Řeš. např. $A = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

4) V $A_3$ je dán čtyřstěn $ABCD$, ... napište rovnice af. zobrazení

$A \to C$, $B \to D$, $C \to A$, $D \to B$

Řeš. $A = \begin{pmatrix} 0 & 0 & 1 \\ -1 & -1 & -1 \\ 1 & 0 & 0 \end{pmatrix}$

$B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

5) Určete samodružné body zobrazení

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

Řeš. $[-2,0,-1]$