Př 1) Rozhodněte, zda f je shodnost v E_2, pokud ano, určete její rovnice.

$A = [10,0] \to A' = [0,0]$, $B = [25,20] \to B' = [0,25]$ viz Janyška 3.2.1

$|AB| = \sqrt{15^2 + 20^2} = 25$, $|A'B'| = \sqrt{0^2 + 25^2} = 25$

$x' = a_1 x + b_1 y + c_1$
$y' = a_2 x + b_2 y + c_2$

$0 = 10a_1 + 0b_1 + c_1$
$0 = 10a_2 + 0b_2 + c_2$

$0 = 25a_1 + 20b_1 + c_1$
$25 = 25a_2 + 20b_2 + c_2$

$10a_1 + c_1 = 0$
$25a_1 + 20b_1 + c_1 = 0$

$10a_2 + c_2 = 0$
$25a_2 + 20b_2 + c_2 = 25$

$c_1 = 10t$
$a_1 = -t$
$-25t + 20b_1 + 10t = 0$
$b_1 = \frac{3}{4}t$

$c_2 = 10s$
$a_2 = -s$
$-25s + 20b_2 + 10s = 25$
$b_2 = \frac{5}{4} + \frac{3}{4}s$

$$\begin{cases} x' = -tx + \frac{3}{4}ty + 10t \\ y' = -sx + \left(\frac{5}{4} + \frac{3}{4}s\right)y + 10s \end{cases}$$

$$A = \begin{pmatrix} -t & \frac{3}{4}t \\ -s & \frac{5}{4} + \frac{3}{4}s \end{pmatrix}$$

$A^T \cdot A = E_2$

$$\begin{pmatrix} -t & -s \\ \frac{3}{4}t & \frac{5}{4} + \frac{3}{4}s \end{pmatrix} \begin{pmatrix} -t & \frac{3}{4}t \\ -s & \frac{5}{4} + \frac{3}{4}s \end{pmatrix} = \begin{pmatrix} t^2 + s^2 & -\frac{3}{4}t^2 - \frac{5}{4}s - \frac{3}{4}s^2 \\ -\frac{3}{4}t^2 - \frac{5}{4}s - \frac{3}{4}s^2 & \frac{9}{16}t^2 + \left(\frac{5}{4} + \frac{3}{4}s\right)^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$t^2 + s^2 = 1$
$-\frac{3}{4}t^2 - \frac{5}{4}s - \frac{3}{4}s^2 = 0$
$\frac{9}{16}t^2 + \left(\frac{5}{4} + \frac{3}{4}s\right)^2 = 1$

$t^2 = 1 - s^2$
$-\frac{3}{4}(1 - s^2) - \frac{5}{4}s - \frac{3}{4}s^2 = 0$

$s = -\frac{3}{5}$, $t \begin{cases} \frac{4}{5} \\ -\frac{4}{5} \end{cases}$

$t_1 = \frac{4}{5}$, $s = -\frac{3}{5}$
$t_2 = -\frac{4}{5}$, $s = -\frac{3}{5}$

$$\boxed{\begin{aligned} x' &= -\frac{4}{5}x + \frac{3}{5}y + 8 \\ y' &= \frac{3}{5}x + \frac{4}{5}y - 6 \end{aligned}}$$

$$\begin{vmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{vmatrix} = -1$$

nepřímá sh.

$$\boxed{\begin{aligned} x' &= \frac{4}{5}x + \frac{3}{5}y - 8 \\ y' &= \frac{3}{5}x + \frac{4}{5}y - 6 \end{aligned}}$$

$$\begin{vmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{vmatrix} = 1$$

přímá sh.

Př. 2 Je dáno zobrazení f: $B=[0,0,0] \to B'=[0,0,0]$

$C=[1,1,1] \to C'=[1,1,1]$

$A=[1,-1,0] \to A'$, kde A' leží v rovině $\underline{x=0}$.

Určete souřadnice bodu A' tak, aby f bylo shodné zobrazení.

$A'=[0,y,z]$ $\quad |AB|=|A'B'|$ $\quad |AB|=\sqrt{2}$, $|A'B'|=\sqrt{y^2+z^2}$ $\quad \sqrt{2}=\sqrt{y^2+z^2}$

$(-1,1,0)$ $\quad (0,y,z)$

$|AC|=|A'C'|$ $\quad |AC|=\sqrt{5}$ $\quad |A'C'| = \sqrt{1^2+(1-y)^2+(1-z)^2}$

$(0,2,1)$ $\quad (1,1-y,1-z)$

$\sqrt{2}=\sqrt{y^2+z^2}$

$\sqrt{5}=\sqrt{1^2+(1-y)^2+(1-z)^2}$

$y^2+z^2=2$

$(1-y)^2+(1-z)^2=4$

$y_1=1,\ z_1=-1$

$y_2=-1,\ z_2=1$

$A'_1=[0,1,-1]$

$A'_2=[0,-1,1]$

Př. 3 Rozhodněte, zda jde o shodnost, najděte samodružné body a samodružné směry.

$x'=y+3$

$y'=x-3$

$z'=z$

$A=\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}$

$|A|=-1$

$A^T\cdot A=\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

$\begin{vmatrix}-\lambda&1&0\\1&-\lambda&0\\0&0&1-\lambda\end{vmatrix}=(1-\lambda)\cdot\begin{vmatrix}-\lambda&1\\1&-\lambda\end{vmatrix}=(1-\lambda)\cdot(\lambda^2-1)$

$=(1-\lambda)(\lambda-1)(\lambda+1)=0$

$x=y+3$

$y=x-3$

$z=z$

$\boxed{x-y-3=0}$

$\lambda_{1,2}=1$, $\lambda_3=-1$

Pro $\lambda_{1,2}=1$

$-x+y=0$

$x-y=0$

$0+0+0=0$

$x-y=0$

$z=t,\ y=s,\ x=s$

(s,s,t), např. $(1,1,0)$

$(0,0,1)$

Pro $\lambda_3=-1$

$x+y=0$

$x+y=0$

$2z=0$

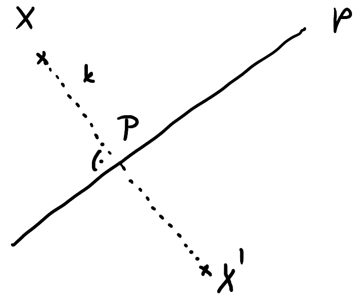
$x=-t$

$y=t$

$z=0$

$(-t,t,0)$, např. $(1,-1,0)$

Př. 4 Odvoďte rovnice souměrnosti v E_2 podle přímky $ax+by+c=0$



$X=[x_1,y_1] \longrightarrow X'=[x_1',y_1']$ $\qquad \vec{n}_p=(a,b)$

$k:\ x = x_1 + at$

$\quad y = y_1 + bt$

$a(x_1+at)+b(y_1+bt)+c=0$

$t = -\dfrac{ax_1+by_1+c}{a^2+b^2}$ pro P

$\overrightarrow{XX'} = 2\overrightarrow{XP}$

$X'-X = 2(P-X)$

$X' = X + 2P - 2X$

$\underline{X' = -X + 2P}$

$P = \left[x_1 - \dfrac{ax_1+by_1+c}{a^2+b^2}a,\ y_1 - \dfrac{a_1x_1+by_1+c}{a^2+b^2}b\right]$

$x_1' = -x_1 + 2\left(x_1 - \dfrac{ax_1+by_1+c}{a^2+b^2}a\right) = x_1 - \dfrac{2a}{a^2+b^2}(ax_1+by_1+c)$

$y_1' = -y_1 + 2\left(y_1 - \dfrac{ax_1+by_1+c}{a^2+b^2}b\right) = y_1 - \dfrac{2b}{a^2+b^2}(ax_1+by_1+c)$

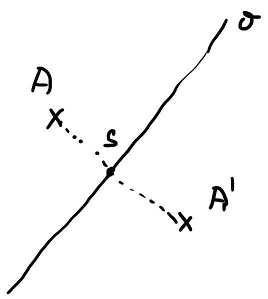
$$\boxed{\begin{aligned} x' &= x - \frac{2a}{a^2+b^2}(ax+by+c) \\ y' &= y - \frac{2b}{a^2+b^2}(ax+by+c) \end{aligned}}$$

Př. 5 Napište rovnice souměrnosti podle přímky $3x-4y+1=0$

$a=3,\ b=-4,\ c=1$

$x' = x - \dfrac{2\cdot 3}{3^2+(-4)^2}(3x-4y+1) = \dfrac{7}{25}x + \dfrac{24}{25}y - \dfrac{6}{25}$

$y' = y - \dfrac{2\cdot(-4)}{3^2+(-4)^2}(3x-4y+1) = \dfrac{24}{25}x - \dfrac{7}{25}y + \dfrac{8}{25}$



Př 6) Určete rovnici souměrnosti podle nadroviny sam. bodů, jestliže bod $A=[1,0,5]$ zobrazí na $A'=[0,5,1]$.

$S=[\frac{1}{2},\frac{5}{2},3]$ $\qquad \vec{AA'}=(-1,5,-4) \sim (1,-5,4)=\vec{n}_\varrho$

$\varrho: \; x-5y+4z+d=0$

$\frac{1}{2}-5\frac{5}{2}+4\cdot 3+d=0 \quad \rightarrow d=0$

$\varrho: \; x-5y+4z=0$

$$x' = x-\frac{2\cdot 1}{1^2+(-5)^2+4^2}(x-5y+4z)$$

$$y' = y-\frac{2\cdot(-5)}{42}(x-5y+4z)$$

$$z' = z-\frac{2\cdot 4}{42}(x-5y+4z)$$

$$x' = \frac{20}{21}x+\frac{5}{21}y-\frac{4}{21}z$$

$$y' = \frac{5}{21}x-\frac{4}{21}y+\frac{20}{21}z$$

$$z' = -\frac{4}{21}x+\frac{20}{21}y+\frac{5}{21}z$$

Na procvičení:

1) Rozhodněte, zda jde o shodnost, pokud ano sestavte rovnice

$A[-1,-1] \rightarrow A'[2,-2]$, $B=[3,3] \rightarrow B'=[6,2]$, $C=[2,1] \rightarrow [4,1]$

$x' = y+3$

$y' = x-1$

2) Určete samodružné body a směry shodnosti

$x' = \frac{3}{5}x-\frac{4}{5}y+1$ $\qquad S[\frac{5}{2},0]$, žádný sam. směr

$y' = \frac{4}{5}x+\frac{3}{5}y-2$

3) Určete osu souměrnosti v E_2, která $A=[2,-3] \rightarrow A'=[-2,5]$ $\qquad x-2y+2=0$

4) Určete rovnice osové souměrnosti s osou $o: \; 2x+3y+1=0$

$$x' = \frac{5}{13}x-\frac{12}{13}y-\frac{4}{13}$$

$$y' = -\frac{12}{13}x-\frac{5}{13}y-\frac{6}{13}$$

5) Určete rovnice souměrnosti podle nadroviny v E_3, jestliže bod

$A = [1, 0, -2] \longrightarrow A' = [3, 2, 0]$

$$x' = \frac{2}{3}x - \frac{1}{3}y - \frac{1}{3}z + \frac{2}{3}$$

$$y' = -\frac{2}{3}x + \frac{1}{3}y - \frac{2}{3}z + \frac{4}{3}$$

$$z' = -\frac{2}{3}x - \frac{2}{3}y + \frac{1}{3}z + \frac{4}{3}$$