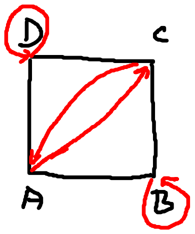
1)



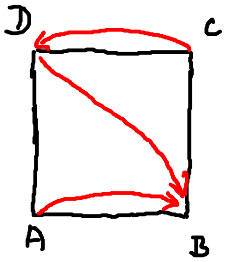
$A \to C$
$B \to B$
$C \to A$
$D \to D$

$|AB| = 1 = |CB|$ $\quad$ $|BC| = 1 = |BA|$

$|AC| = \sqrt{2} = |CA|$ $\quad$ $|BD| = \sqrt{2} = |BD|$

$|AD = 1 = |CD|$ $\quad$ $|CD| = 1 = |AD|$

✓

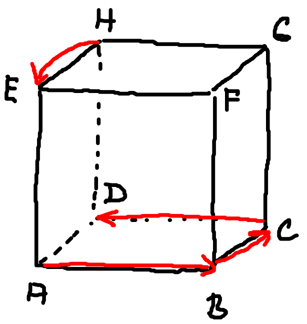


$A \to B$
$C \to D$
$D \to B$

$|AC| = \sqrt{2} = |BD|$

$|AD| = 1 \neq |BB|$

NE



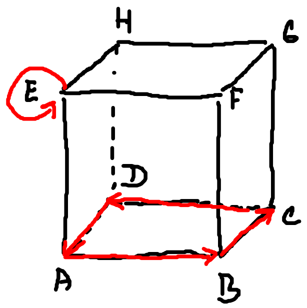
$A \to B$
$B \to C$
$C \to D$
$H \to E$

$|AB| = 1 = |BC|$ $\quad$ $|BC| = 1 = |CD|$

$|AC| = \sqrt{2} = |BD|$ $\quad$ $|BH| = \sqrt{3} = |CE|$

$|AH| = \sqrt{2} = |BE$ $\quad$ $|CH| = \sqrt{2} = |DE|$

✓



$A \to B$
$B \to C$
$C \to D$
$D \to A$
$E \to E$

$|AE| = 1 \neq |BE| = \sqrt{2}$

NE

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2) a Rozhodněte, zda jde o shodné zobrazení $E_2 \to E_3$

$x' = \frac{2}{3}x - \frac{2}{3}y + 1$

$y' = \frac{2}{3}x + \frac{1}{3}y + 3$

$z' = \frac{1}{3}x + \frac{2}{3}y + 0$

$$A = \begin{pmatrix} \frac{2}{3} & -\frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}, \quad A^T = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

b) $x' = \frac{3}{5}x - \frac{1}{5}y + \frac{9}{5}$

$y' = \frac{1}{5}x + \frac{3}{5}y - \frac{7}{5}$

je podobnost ?

$$A = \begin{pmatrix} \frac{3}{5} & -\frac{1}{5} \\ \frac{1}{5} & \frac{3}{5} \end{pmatrix}, \quad A^T = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{pmatrix}$$

$$A^T A = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} \frac{3}{5} & -\frac{1}{5} \\ \frac{1}{5} & \frac{3}{5} \end{pmatrix} = \begin{pmatrix} \frac{10}{25} & 0 \\ 0 & \frac{10}{25} \end{pmatrix} = \frac{10}{25} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\frac{10}{25} = k^2$$

3) Určete parametry $p, q, r \in \mathbb{R}$ tak, aby rovnice vyjadřovaly shodnost

$$x' = \frac{3}{4}x + qy + 1$$
$$y' = px + ry - 1$$

$$A = \begin{pmatrix} \frac{3}{4} & q \\ p & r \end{pmatrix}, \quad A^T = \begin{pmatrix} \frac{3}{4} & p \\ q & r \end{pmatrix} \qquad A^T A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{4} & p \\ q & r \end{pmatrix} \begin{pmatrix} \frac{3}{4} & q \\ p & r \end{pmatrix} = \begin{pmatrix} \frac{9}{16} + p^2 & \frac{3}{4}q + pr \\ \frac{3}{4}q + pr & q^2 + r^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\frac{9}{16} + p^2 = 1 \qquad p^2 = \frac{7}{16}, \quad p = \pm\frac{\sqrt{7}}{4}$$

$$\frac{3}{4}q + pr = 0 \qquad \frac{3}{4}q \pm \frac{\sqrt{7}}{4}r = 0 \quad /\cdot 4$$

$$q^2 + r^2 = 1 \qquad 3q \pm \sqrt{7}r = 0 \longrightarrow q = \mp\frac{\sqrt{7}}{3}r$$

$$\frac{7}{9}r^2 + r^2 = 1 \rightarrow \frac{16}{9}r^2 = 1 \rightarrow r = \pm\frac{3}{4}$$

(p, q, r)

$\left(\frac{\sqrt{7}}{4}, -\frac{\sqrt{7}}{4}, \frac{3}{4}\right)$

$\left(-\frac{\sqrt{7}}{4}, \frac{\sqrt{7}}{4}, \frac{3}{4}\right)$

$\left(\frac{\sqrt{7}}{4}, \frac{\sqrt{7}}{4}, -\frac{3}{4}\right)$

$\left(-\frac{\sqrt{7}}{4}, -\frac{\sqrt{7}}{4}, -\frac{3}{4}\right)$

4) Je dán rovnostranný $\triangle ABC$. Jaké shodné zobrazení vznikne složením středových symetrií $\varphi_1(A)$, $\varphi_2(B)$, tedy $\varphi_2 \circ \varphi_1$.

posunutí o $2\overrightarrow{AB}$

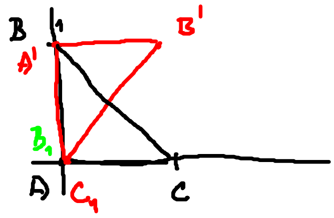
5) Je dána shodnost v E_2. Rozlož ji na osové souměrnosti

$$f: \quad x' = y, \qquad y' = -x + 1$$

$A = [0,0] \rightarrow A' = [0,1]$

$B = [0,1] \rightarrow B' = [1,1]$

$C = [1,0] \rightarrow C' = [0,0]$



f_1: $A \rightarrow A'$, o_1 ... osa úsečky AA', $y = \frac{1}{2}$, $y - \frac{1}{2} = 0$

$$x' = x - \frac{2 \cdot 0}{1}\left(y - \tfrac{1}{2}\right)$$
$$y' = y - \frac{2 \cdot 1}{1}\left(y - \tfrac{1}{2}\right)$$

$$\begin{aligned} x' &= x \\ y' &= y - 2y + 1 \end{aligned}$$

$$\boxed{f_1: \quad x' = x, \qquad y' = -y + 1}$$

$A \rightarrow A'$

$B \rightarrow B_1 = [0,0]$

$C \rightarrow C_1 = [1,1]$

f_2: musí B_1 přejít B', osa úsečky B_1B', tedy o_2: $y = -x + 1$, $x + y - 1 = 0$,

$$x' = x - \frac{2 \cdot 1}{2}(x + y - 1)$$
$$y' = y - \frac{2 \cdot 1}{2}(x + y - 1)$$

$$\boxed{\begin{aligned} x' &= -y + 1 \\ y' &= -x + 1 \end{aligned}} \quad f_2$$

$A \rightarrow A'$

$B \rightarrow B_1 \rightarrow B'$

$C \rightarrow C_1 \rightarrow C'$ ✓

$f_2 \circ f_1$:

$$\begin{aligned} x' &= -(-y+1) + 1 = y \\ y' &= -x + 1 \qquad\quad = -x + 1 \end{aligned} \qquad \underline{\underline{f}}$$

6) Rozložte f v E_3 na rovinné souměrnosti

$A=[0,0,3] \to A'=[1,5,1]$, $B=[3,0,0] \to B'=[1,2,-2]$

$C=[1,1,0] \to C'=[0,2,0]$, $D=[0,1,1] \to D'=[0,3,1]$

f_1: $\vec{AA'} = \vec{n}_1 = (1,5,-2)$, $S_1 = \left[\frac{1}{2}, \frac{5}{2}, 2\right]$

$x+5y-2z+d=0$

$\underline{x+5y-2z-9=0}$

$a=1,\ b=5,\ c=-2$

$a^2+b^2+c^2=30$

$$x' = x - \frac{2\cdot 1}{30}(x+5y-2z-9)$$
$$y' = y - \frac{2\cdot 5}{30}(x+5y-2z-9)$$
$$z' = z - \frac{2\cdot(-2)}{30}(x+5y-2z-9)$$

f_1:
$$x' = \frac{14}{15}x - \frac{1}{3}y + \frac{2}{15}z + \frac{3}{5}$$
$$y' = -\frac{1}{3}x - \frac{2}{3}y + \frac{2}{3}z + 3$$
$$z' = \frac{2}{15}x + \frac{2}{3}y + \frac{11}{15}z - \frac{18}{15}$$

$A \to A'$,

$B \to B_1 = \left[\frac{17}{5}, 2, -\frac{4}{5}\right]$, $C \to C_1 = \left[\frac{6}{5}, 2, -\frac{2}{5}\right]$, $D \to D_1 = \left[\frac{2}{5}, 3, \frac{1}{5}\right]$

f_2: $B_1 \to B'$

$\vec{B_1B'} = \vec{n}_2 = \left(-\frac{12}{5}, 0, -\frac{6}{5}\right)$, $S_1 = \left[\frac{11}{5}, 2, -\frac{7}{5}\right]$

$-\frac{12}{5}x - \frac{6}{5}z + d = 0$

$2x+z-3=0$

$a=2$
$b=0$
$c=1$
$a^2+b^2+c^2=5$

$$x' = x - \frac{2\cdot 2}{5}(2x+z-3)$$
$$y' = y - \frac{2\cdot 0}{5}(2x+z-3)$$
$$z' = z - \frac{2\cdot 1}{5}(2x+z-3)$$

f_2:
$$x' = -\frac{3}{5}x - \frac{4}{5}z + \frac{12}{5}$$
$$y' = y$$
$$z' = -\frac{4}{5}x + \frac{3}{5}z + \frac{6}{5}$$

$A \to A'$

$B \to B_1 \to B'$

$C \to C_1 \to C_2 = [2,2,0]$, $D \to D_1 \to D_2 = [2,3,1]$

f_3: volím tak, aby $C_2 \to C'$, osa úsečky C_2C' , $\vec{C_2C'} = \vec{n}_3 = (-2,0,0)$, $S_3 = [1,2,0]$

$-2x+d=0$ $\quad -2x+2=0$

$\boxed{x-1=0}$

$a=1,\ b=0,\ c=0$

$$x' = x - \frac{2\cdot 1}{1}(x-1)$$
$$y' = y - \frac{2\cdot 0}{1}(x-1)$$
$$z' = z - \frac{2\cdot 0}{1}(x-1)$$

f_3:
$$x' = -x+2$$
$$y' = y$$
$$z' = z$$

$A \to A'$, $B \to B_1 \to B'$, $C \to C_1 \to C_2 \to C'$, $D \to D_1 \to D_2 \to D'$

$f = f_3 \circ f_2 \circ f_1$

7) Rovnice otočení v E_2 o úhel $\alpha \neq 0$ kolem počátku, resp. jiného bodu

$$x' = x\cos\alpha - y\sin\alpha \qquad\qquad x' = x\cos\alpha - y\sin\alpha + p$$
$$y' = x\sin\alpha + y\cos\alpha \quad , \quad y' = x\sin\alpha + y\cos\alpha + q$$

Urči rovnice otočení o $\alpha = 60°$, kolem středu $S = [3,-2]$

$\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$ $\qquad x' = \frac{1}{2}x - \frac{\sqrt{3}}{2}y + p$ $\qquad S \to S$

$\qquad y' = \frac{\sqrt{3}}{2}x + \frac{1}{2}y + q$

$3 = \frac{1}{2}\cdot 3 - \frac{\sqrt{3}}{2}(-2) + p$

$-2 = \frac{\sqrt{3}}{2}\cdot 3 + \frac{1}{2}(-2) + q$ $\qquad p = \frac{3}{2} - \sqrt{3}$, $q = -1 - \frac{3\sqrt{3}}{3}$

$$\boxed{\begin{aligned} x' &= \tfrac{1}{2}x - \tfrac{\sqrt{3}}{2}y + \tfrac{3}{2} - \sqrt{3} \\ y' &= \tfrac{\sqrt{3}}{2}x + \tfrac{1}{2}y - 1 - \tfrac{3\sqrt{3}}{3} \end{aligned}}$$

8) Je dáno otočení $\quad x' = \frac{3}{5}x - \frac{4}{5}y + 1$, urči střed a úhel otočení

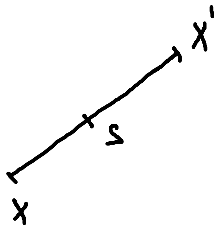
$\qquad y' = \frac{4}{5}x + \frac{3}{5}y - 2$

$\qquad\qquad \sin\alpha = \frac{4}{5}$

$x = \frac{3}{5}x - \frac{4}{5}y + 1$ $\quad \dots \quad y = 0$, $x = \frac{5}{2}$ $\qquad \underline{S = [\frac{5}{2}, 0]}$ $\qquad \underline{\alpha = \arcsin\frac{4}{5}}$

$y = \frac{4}{5}x + \frac{3}{5}y - 2$

9) Urči rovnice středové symetrie v E_3 se středem $S = [1, 0, -2]$.



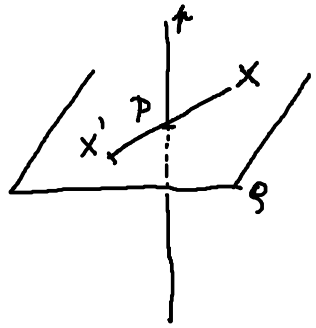
$\overrightarrow{SX'} = -\overrightarrow{SX}$

$X' - S = -(X - S)$

$\boxed{X' = -X + 2S}$

$$\boxed{\begin{aligned} x' &= -x + 2 \\ y' &= -y \\ z' &= -z - 4 \end{aligned}}$$

10) Určete v E_3 rovnici souměrnosti podle přímky $p: \begin{array}{l} x = 1+t \\ y = 2-t \\ z = 0+3t \end{array}$ $\quad \vec{s}_p = (1,-1,3)$



$X = [x_1, y_1, z_1] \to X' = [x_1', y_1', z_1']$

$\rho: \ x - y + 3z + d = 0 \qquad d = -x_1 + y_1 - 3z_1$

$x_1 - y_1 + 3z_1 + d = 0$

$x - y + 3z - x_1 + y_1 - 3z_1 = 0$

$1 + t - (2-t) + 3\cdot 3t - x_1 + y_1 - 3z_1 = 0 \ \to \ t = \frac{1}{11}(x_1 - y_1 + 3z_1 + 1)$

$P = \left[\frac{12}{11} + \frac{1}{11}(x_1 - y_1 + 3z_1),\ \frac{21}{11} - \frac{1}{11}(x_1 - y_1 + 3z_1),\ \frac{3}{11} + \frac{3}{11}(x - y_1 + 3z_1)\right]$

$$\begin{aligned} x' &= -x + \frac{24}{11} + \frac{2}{11}x - \frac{2}{11}y + \frac{6}{11}z \\ y' &= -y + \frac{42}{11} - \frac{2}{11}x + \frac{2}{11}y - \frac{6}{11}z \\ z' &= -z + \frac{6}{11} + \frac{6}{11}x - \frac{6}{11}y + \frac{18}{11}z \end{aligned} \quad \to \quad \boxed{\begin{aligned} x' &= -\frac{9}{11}x - \frac{2}{11}y + \frac{6}{11}z + \frac{24}{11} \\ y' &= -\frac{2}{11}x - \frac{9}{11}y - \frac{6}{11}z + \frac{42}{11} \\ z' &= \frac{6}{11}x - \frac{6}{11}y + \frac{7}{11}z + \frac{6}{11} \end{aligned}}$$

Příklady na procvičení.

1) Rozhodněte, zda jde o shodné zobrazení $E_3 \to E_3$

$$\begin{aligned} x' &= -\tfrac{1}{2}y \\ y' &= \tfrac{\sqrt{3}}{2}y + 3 \\ z' &= x - 2 \end{aligned}$$

[ano]

2) Určete $s \in \mathbb{R}$ tak, aby šlo o shodnost

$A = [0,0] \to A' = [-8,-6]$, $B = [s,0] \to B' = [0,0]$ $\qquad$ [$s = \pm 10$]

3) Jsou dány body $A = [-4,-1]$, $B = [4,3]$, $C = [2,7]$. Určete rovnice osových souměrností $f_1: A \to B$, $f_2: B \to C$. Určete a klasifikujte zobrazení

$h = f_1 \circ f_2$, $g = f_2 \circ f_1$

$h: \begin{array}{l} x' = -x + 2 \\ y' = -y + 6 \end{array}$ (stř. souměrnost $S = [-1,3]$)

$f_1: \begin{array}{l} x' = -\frac{3}{5}x - \frac{4}{5}y + \frac{4}{5} \\ y' = -\frac{4}{5}x + \frac{3}{5}y + \frac{2}{5} \end{array}$ $\qquad f_2: \begin{array}{l} x' = \frac{3}{5}x + \frac{4}{5}y - \frac{14}{5} \\ y' = \frac{4}{5}x - \frac{3}{5}y + \frac{28}{5} \end{array}$

$g: \begin{array}{l} x' = -x + 2 \\ y' = -y + 6 \end{array}$ $\qquad h = g$

4) Určete rovnice otočení kolem osy z o úhel 30°.

$$x' = \frac{\sqrt{3}}{2}x - \frac{1}{2}y$$
$$y' = \frac{1}{2}x + \frac{\sqrt{3}}{2}y$$
$$z' = z$$

5) Určete rovnice souměrnosti v E_3 podle přímky p: $\begin{aligned} x &= 1 + t \\ y &= 0 - 2t \\ z &= 3 + 3t \end{aligned}$

$$x' = -\frac{6}{7}x - \frac{2}{7}y + \frac{3}{7}z + \frac{4}{7}$$
$$y' = -\frac{2}{7}x - \frac{3}{7}y - \frac{6}{7}z + \frac{20}{7}$$
$$z' = \frac{3}{7}x - \frac{6}{7}y + \frac{2}{7}z + \frac{12}{7}$$