1) Určete rovnice souměrnosti podle přímky p: $\begin{aligned} x &= 1 + t \\ y &= 2 - t \\ z &= 0 + 3t \end{aligned}$

$[1,2,0] \rightarrow [1,2,0]$

$\vec{s} = (1,-1,3) \rightarrow (1,-1,3)$

$\vec{v}_1 \perp p \qquad (1,1,0) \rightarrow (-1,-1,0)$

$\vec{v}_2 \perp p \qquad (0,3,1) \rightarrow (0,-3,-1)$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & -1 & 3 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 0 & 3 & 1 & 0 & -3 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & -1 & 3 \\ 0 & 2 & -3 & -2 & 0 & -3 \\ 0 & 3 & 1 & 0 & -3 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & -1 & 3 \\ 0 & 1 & -\frac{3}{2} & -1 & 0 & -\frac{3}{2} \\ 0 & 0 & \frac{11}{2} & 3 & -3 & \frac{7}{2} \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & -\frac{7}{11} & \frac{7}{11} & \frac{12}{11} \\ 0 & 1 & 0 & -\frac{2}{11} & -\frac{9}{11} & -\frac{6}{11} \\ 0 & 0 & 1 & \frac{6}{11} & -\frac{6}{11} & \frac{7}{11} \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{9}{11} & -\frac{2}{11} & \frac{6}{11} \\ 0 & 1 & 0 & -\frac{2}{11} & -\frac{9}{11} & -\frac{6}{11} \\ 0 & 0 & 1 & \frac{6}{11} & -\frac{6}{11} & \frac{7}{11} \end{array}\right)$$

$$\boxed{\begin{aligned} x' &= -\frac{9}{11}x - \frac{2}{11}y + \frac{6}{11}z + b_1 \\ y' &= -\frac{2}{11}x - \frac{9}{11}y - \frac{6}{11}z + b_2 \\ z' &= \frac{6}{11}x - \frac{6}{11}y + \frac{7}{11}z + b_3 \end{aligned}}$$

$1 = -\frac{9}{11}1 - \frac{2}{11}2 + \frac{6}{11}\cdot 0 + b_1 \qquad b_1 = \frac{24}{11}$

$2 = -\frac{2}{11}1 - \frac{9}{11}2 - \frac{6}{11}\cdot 0 + b_2 \qquad b_2 = \frac{42}{11}$

$0 = \frac{6}{11}1 - \frac{6}{11}2 + \frac{7}{11}\cdot 0 + b_3 \qquad b_3 = \frac{6}{11}$

2) Určete $p, q, r \in R$ tak, aby zobrazení byla podobnost

$x' = x - 2y + 2z + 4$

$y' = px + 2y + z - 2$

$z' = qx + ry + 2z - 2$

$$\begin{pmatrix} 1 & p & q \\ -2 & 2 & r \\ 2 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & 2 \\ p & 2 & 1 \\ q & r & 2 \end{pmatrix} = \begin{pmatrix} 1+p^2+q^2 & -2+2p+qr & 2+p+2q \\ -2+2p+qr & 8+r^2 & -2+2r \\ 2+p+2q & -2+2r & 9 \end{pmatrix}$$

$1 + p^2 + q^2 = 9 \qquad p^2 + q^2 = 8$

$8 + r^2 = 9$ ✓

$-2 + 2p + qr = 0 \qquad -2 + 2p + q = 0$

$2 + p + 2q = 0 \qquad 2 + p + 2q = 0 \longrightarrow p = 2, \; q = -2$

$-2 + 2r = 0 \longrightarrow r = 1$

$$\boxed{\begin{aligned} x' &= x - 2y + 2z + 4 \\ y' &= 2x + 2y + z - 2 \\ z' &= -2x + y + 2z - 2 \end{aligned}}$$

3) Určete $p \in R$ tak, aby zobrazení f byla podobnost

$A = [1,2] \to A' = [3,-1]$, $B = [0,1] \to B' = [4,2]$, $C = [1,1] \to C' = [p,1]$.

$\overrightarrow{AB} = (-1,-1)$, $\overrightarrow{A'B'} = (1,3)$ $|\overrightarrow{AB}| = \sqrt{2}$, $|\overrightarrow{A'B'}| = \sqrt{10}$ $k = \sqrt{5}$

$\overrightarrow{AC} = (0,-1)$, $\overrightarrow{A'C'} = (p-3, 2)$ $|\overrightarrow{AC}| = 1$, $|\overrightarrow{A'C'}| = \sqrt{(p-3)^2+4} = \sqrt{p^2-6p+13}$

$\overrightarrow{BC} = (1,0)$, $\overrightarrow{B'C'} = (p-4,-1)$, $|\overrightarrow{BC}| = 1$, $|\overrightarrow{B'C'}| = \sqrt{(p-4)^2+1} = \sqrt{p^2-8p+17}$

$p^2-6p+13 = 5$
$p^2-8p+17 = 5$ $\quad 2p = 4$, $p = 2$

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4) Rozhodněte, zda f je podobnost v E_2, pokud ano rozložte ji na stejnolehlost a shodnost.

$f: x' = -3y + 9$
$y' = 3x + 3$

$A = \begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix}$, $A^T = \begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix} = 9\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $k = 3$

stejn. h: $x' = 3x$, $y' = 3y$

shodnost s_1: $x' = -y + 3$, $y' = x + 1$

$f = h \circ s_1$ $\quad x' = 3(-y+3) = -3y+9$, $y' = 3(x+1) = 3x+3$

shodnost s_2: $x' = -y+9$, $y' = x+3$

$f = s_2 \circ h$ $\quad x' = -3y+9$, $y' = 3x+3$

Sam. bod f

$x = -3y + 9$ $\quad x + 3y - 9 = 0$ $\quad S\ [0,3]$
$y = 3x + 3$ $\quad 3x - y + 3 = 0$

$\triangle ABC$

$A = [0,0] \to [9,3]$

$B = [1,0] \to [9,6]$

$C = [0,1] \to [6,3]$

5) V E_3 je dáno zobrazení f. Rozhodněte, zda je to podobnost, určete její koeficient. Rozložte na stejnolehlost a shodnost.

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 9 & 2 & 6 \\ 2 & 9 & -6 \\ 6 & -6 & -7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} -10 \\ -10 \\ -20 \end{pmatrix} \qquad A^T = \begin{pmatrix} 9 & 2 & 6 \\ 2 & 9 & 6 \\ 6 & -6 & -7 \end{pmatrix}$$

$$\begin{pmatrix} 9 & 2 & 6 \\ 2 & 9 & 6 \\ 6 & -6 & -7 \end{pmatrix} \cdot \begin{pmatrix} 9 & 2 & 6 \\ 2 & 9 & 6 \\ 6 & -6 & -7 \end{pmatrix} = \begin{pmatrix} 121 & 0 & 0 \\ 0 & 121 & 0 \\ 0 & 0 & 121 \end{pmatrix} = 11^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad k = 11$$

stej. h: $x' = 11x$, $y' = 11y$, $z' = 11z$

f: $x' = 9x + 2y + 6z - 10$, $y' = 2x + 9y - 6z - 10$, $z' = 6x - 6y - 7z - 20$

$S = [2, 0, -1]$

shodnost s_1:

$$x' = \frac{9}{11}x + \frac{2}{11}y + \frac{6}{11}z - 10$$
$$y' = \frac{2}{11}x + \frac{9}{11}y - \frac{6}{11}z - 10$$
$$z' = \frac{6}{11}x - \frac{6}{11}y - \frac{7}{11}z - 20$$

$f: s_1 \circ h$

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6) f: $x' = 3x + 4y + 1$, $y' = 4x - 3y - 1$ $\qquad$ g: $x' = 3x - 4y + 6$, $y' = 4x + 3y + 8$

Určete koeficienty f, g.
Určete koeficienty $f \circ g$.

$$\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix} \cdot \begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix} = \begin{pmatrix} 25 & 0 \\ 0 & 25 \end{pmatrix} = 5^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad k_f = 5$$

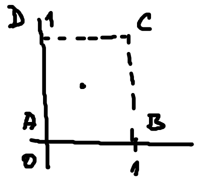
$$\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \cdot \begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 25 & 0 \\ 0 & 25 \end{pmatrix} = 5^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad k_g = 5$$

$f \circ g$:
$$x' = 3(3x - 4y + 6) + 4(4x + 3y + 8) + 1$$
$$y' = 4(3x - 4y + 6) - 3(4x + 3y + 8) - 1$$

$$x' = 25x + 51$$
$$y' = -25y - 1$$

$$\begin{pmatrix} 25 & 0 \\ 0 & -25 \end{pmatrix} \cdot \begin{pmatrix} 25 & 0 \\ 0 & -25 \end{pmatrix} = \begin{pmatrix} 25^2 & 0 \\ 0 & 25^2 \end{pmatrix} = 25^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad k_{f \circ g} = 5 \cdot 5 = 25$$

4) Je dán čtverec $ABCD$, se středem S. Určete rovnice podobnosti, která zobrazí $A \to B$, $B \to D$, $S \to C$



$A = [0,0] \to [1,0]$

$B = [1,0] \to [0,1]$

$S = [\frac{1}{2}, \frac{1}{2}] \to [1,1]$

$$X' = AX + B$$

$$\left(\begin{array}{ccc|cc} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|cc} 1 & 1 & 2 & 2 & 2 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|cc} 1 & 1 & 2 & 2 & 2 \\ 0 & -1 & -1 & -2 & -1 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|cc} 1 & 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|cc} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right)$$

$$A = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\boxed{\begin{aligned} x' &= -x + y + 1 \\ y' &= x + y \end{aligned}}$$

Příklady na procvičení:

1) Rozhodněte, zda jde o podobnost a určete samodružný bod

$$x' = -2x + y + 3$$
$$y' = -x - 2y + 4$$

$[\frac{13}{10}, \frac{9}{10}]$

2) Určete číslo $p \in \mathbb{R}$ tak, aby zobrazení byla podobnost

$f: \; [0,0] \to [0,2], \; [1,1] \to [0,0], \; [2,0] \to [2,p]$

$[p = 0]$

3) Určete, zda je zobrazení podobnost, pokud ano, rozložte ho na stejnolehlost a shodnost

$$x' = 2x - 2y + z + 1$$
$$y' = -2x - y + 2z + 3$$
$$z' = x + 2y + 2z - 125$$

$x' = 3x$
$y' = 3y$
$z' = 3z$

$x' = \frac{2}{3}x - \frac{2}{3}y + \frac{1}{3}z + \frac{1}{3}$

$y' = -\frac{2}{3}x - \frac{1}{3}y + \frac{2}{3}z + 1$

$z' = \frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}z - \frac{125}{3}$