Komplexní čísla

1. Upravte a výsledek zapište v algebraickém tvaru: $\frac{2+i}{2-i} - \frac{i-1}{i} =$

2. Vypočítejte: $\overline{\overline{(1+i)}.\overline{(3+2i)}} =$

3. Vypočítejte: $\left|\frac{|4-3i|+i}{3-2i}\right| =$

4. Nakresli do Gaussovy roviny obrazy všech komplexních čísel, pro která platí:

   $|z - 1 + 2i| \leq 5$

5. Řešte rovnici v **C**: $2.z + 3.\bar{z} = 5 + i$

6. Řešte kvadratickou rovnici v **C**: $z^2 - (3 + 2i).z + 5 + i = 0$

7. Řešte binomickou rovnici v **C** a kořeny zobrazte v Gaussově rovině: $z^4 = -6 - 6\sqrt{3}\,i$

Řeš.:

1. $\frac{2+i}{2-i}\cdot\frac{2+i}{2+i} - \frac{i-1}{i}\cdot\frac{i}{i} = \frac{4+4i+i^2}{5} - \frac{i^2-i}{i^2} = \frac{3+4i}{5} + (-1) - i = \frac{3+4i-5-5i}{5} = \frac{-2-i}{5} = -\frac{2}{5} - \frac{i}{5}$ 2

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2. $\overline{\overline{(1+i)}.\overline{(3+2i)}} = \overline{(1-i).(3-2i)} = \overline{3+2i^2-5i} = \overline{1-5i} = 1+5i$ 2

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3. $\left|\frac{|4-3i|+i}{3-2i}\right| = \left|\frac{\sqrt{4^2+(-3)^2}+i}{3-2i}\right| = \left|\frac{5+i}{3-2i}\cdot\frac{3+2i}{3+2i}\right| = \left|\frac{15+13.i+2i^2}{13}\right| = \left|\frac{13+13.i}{13}\right| = |1+i| = \sqrt{2}$ 2

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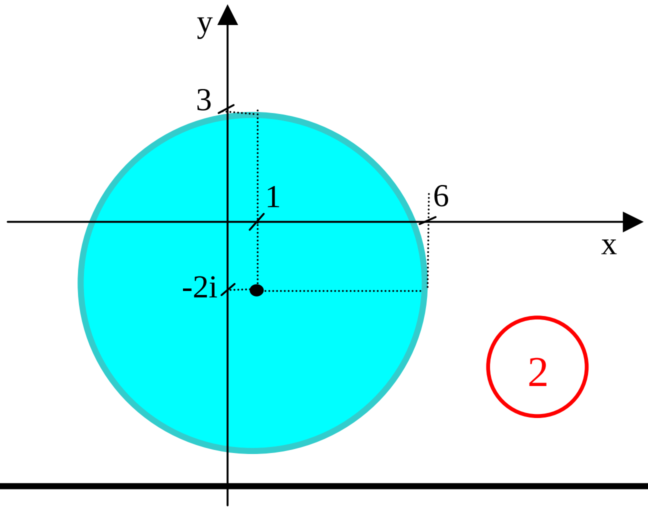
4. $|z - 1 + 2i| \leq 5$

   $|z - (1 - 2i)| \leq 5$



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5. $2.z + 3.\bar{z} = 5 + i$

   $z = x + yi;$ $\bar{z} = x - yi$

   $2.(x + yi) + 3.(x - yi) = 5 + i$

   $2x + 2yi + 3x - 3yi = 5 + i$

   $5x - yi = 5 + i$

   $5x = 5;$ $-y = 1$

   $x = 1;$ $y = -1$

   K = {1 - i} 2

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6. $z^2 - (3 + 2i).z + 5 + i = 0$

$$z_{1,2} = \frac{3+2i \pm \sqrt{(3+2i)^2 - 4.(5+i)}}{2} = \frac{3+2i \pm \sqrt{9+12i-4-20-4i}}{2} = \frac{3+2i \pm \sqrt{-15+8i}}{2}$$

$\sqrt{-15+8i} = x + yi, kde\ x, y \in R$ $\qquad y = \frac{4}{x} \rightarrow x^2 - \frac{16}{x^2} = -15 \qquad x = \pm 1 \rightarrow y = \pm 4$

$-15+8i = x^2 - y^2 + 2xyi \qquad x^4 + 15x^2 - 16 = 0 \qquad \sqrt{-15+8i} = \{\pm 1 \pm 4i\}$

$x^2 - y^2 = -15; \quad 2xy = 8 \qquad (x^2+16).(x^2-1) = 0$

2

$$z_{1,2} = \frac{3+2i \pm (1+4i)}{2} = \begin{cases} 2 + 3i \\ 1 - i \end{cases}$$

K = {2 + 3i; 1 - i} 2

4

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7. $z^4 = -6 - 6\sqrt{3}\,i$

Pomocný výpočet: $a = -6 - 6\sqrt{3}\,i$ ….. do goniometrického tvaru

$|a| = \sqrt{(-6)^2 + (-6\sqrt{3})^2} = \sqrt{144} = 12;$ $\quad \cos x = -\frac{-6}{12} = -\frac{1}{2};$ $\quad \sin x = \frac{-6\sqrt{3}}{12} = -\frac{\sqrt{3}}{2}$

$x = \frac{4\pi}{3}$ 2

Hledám $z = |z|.(\cos x + i.\sin x)$ tak, aby $z^4 = a$.

Tedy: $|z|^4.(\cos 4x + i.\sin 4x) = 12.\left(\cos\frac{4\pi}{3} + i.\sin\frac{4\pi}{3}\right)$

$|z|^4 = 12 \qquad \wedge \qquad 4x = \frac{4\pi}{3} + 2k\pi\ ,\ k \in \{0,1,2,3\}$

$|z| = \sqrt[4]{12} \qquad \wedge \qquad x = \frac{\pi}{3} + \frac{k\pi}{2}\ ,\ k \in \{0,1,2,3\}$

Závěr: $z_k = \sqrt[4]{12}.\left(\cos\left(\frac{\pi}{3} + \frac{k\pi}{2}\right) + i.\sin\left(\frac{\pi}{3} + \frac{k\pi}{2}\right)\right),\ k \in \{0,1,2,3\}$

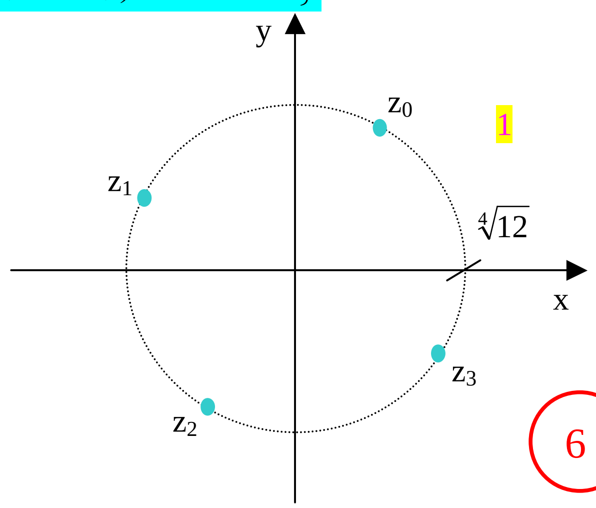
$K = \bigcup_k \left\{\sqrt[4]{12}.\left(\cos\left(\frac{\pi}{3} + \frac{k\pi}{2}\right) + i.\sin\left(\frac{\pi}{3} + \frac{k\pi}{2}\right)\right), k \in \{0,1,2,3\}\right\}$

$z_0 = \sqrt[4]{12}.\left(\cos\frac{\pi}{3} + i.\sin\frac{\pi}{3}\right);$

$z_1 = \sqrt[4]{12}.\left(\cos\frac{5\pi}{6} + i.\sin\frac{5\pi}{6}\right);$

$z_2 = \sqrt[4]{12}.\left(\cos\frac{4\pi}{3} + i.\sin\frac{4\pi}{3}\right);$

$z_1 = \sqrt[4]{12}.\left(\cos\frac{11\pi}{6} + i.\sin\frac{11\pi}{6}\right)$ 3



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Známka:	20 – 18 bodů …… 1
	17 – 14 bodů …... 2
	13 – 8 bodů ……. 3
	7 – 4 body ……. 4