

# Homework problems #1

1. Helmholtz free energy of liquids can be approximated by

$$F(T, V, N) = -kT \ln \left\{ \left( \frac{h^2}{2\pi mkT} \right)^{-3N/2} \left[ \frac{4}{3}\pi \left( \left( \sqrt{2} \frac{V}{N} \right)^{1/3} - \sigma \right)^3 \right]^N \right\},$$

where  $h$ ,  $m$ ,  $k$  and  $\sigma$  are constants. Determine the equation of state of this gas  $p = p(T, V)$ , the difference of specific heats  $c_p - c_v$ , and show that both formulae correspond to ideal gas for  $V/N \gg \sigma$ .

2. *Computer problem:* The number of microstates of the system of  $N$  classical noninteracting particles with energy lower than  $E$  in a volume  $V$  is given by

$$\Omega(E) = \left( \frac{2\pi mE}{h^2} \right)^{\frac{3}{2}N} \frac{V^N}{N! \Gamma(\frac{3}{2}N + 1)}.$$

Let us study two such systems in a thermal interaction with total energy  $2E$ . We shall study fluctuations in these systems, within which one of the system has energy  $E + \Delta E$  and the second one  $E - \Delta E$ . Plot the number of microstates of combined system as a function of  $\Delta E$  for  $N = 10, 10^2, 10^4$ . (It is advisable to express the energy in units of  $\varepsilon = h^2/(2\pi m)$  an number of states in units of  $\gamma = V^N/(N! \Gamma(\frac{3}{2}N + 1))$ .) Let us assume that  $\Delta E$  changes discontinuously by the value of  $\varepsilon/100$  and determine an error introduced by assuming that the entropy is calculated only from the states corresponding to the equilibrium. (calculate for  $N = 10, 10^2, 10^4, 10^5$ ).

The solution should be submitted not later than on March 16th.