

$$|BCS\rangle = \prod_k (u_k + v_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger) |vac\rangle$$

členy s různými indexy

$$\{c_\alpha, c_\beta\} = 0 \quad \{c_\alpha^\dagger, c_\beta^\dagger\} = 0 \quad \{c_\alpha, c_\beta^\dagger\} = \delta_{\alpha\beta}$$

$$c_\alpha c_\beta = -c_\beta c_\alpha$$

komutují: $c_{k\uparrow} c_{-k\downarrow} c_{k'\uparrow} c_{-k'\downarrow} = c_{k'\uparrow} c_{-k'\downarrow} c_{k\uparrow} c_{-k\downarrow}$!

$$\langle BCS | BCS \rangle = \langle \prod_{k'} (u_{k'}^* + v_{k'}^* c_{-k'\downarrow}^\dagger c_{k'\uparrow}^\dagger) \prod_{k''} (u_{k''} + v_{k''} c_{k''\uparrow} c_{-k''\downarrow}) | \rangle = \left| \begin{array}{l} \text{Přeromání} \\ \text{členů} \\ \text{v součinu} \end{array} \right| = \left| \prod_{k'} (A_{k'} + B_{k'}) \prod_{k''} (C_{k''} + D_{k''}) \right| = \prod_{k'} (A_{k'} + B_{k'}) (C_{k'} + D_{k'})$$

Prežijí pouze členy, které nezmění stav | \rangle nebo se do něj vrátí

$$= \prod_{k'} (|u_{k'}|^2 + |v_{k'}|^2) = 1 \quad \leftarrow \text{normační podmínka}$$

$$\Downarrow$$

$$|u_{k'}|^2 + |v_{k'}|^2 = 1$$

$$\langle \hat{N} \rangle = \langle BCS | \hat{N} | BCS \rangle = ?$$

$$\hat{N} = \sum_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} = \sum_k c_{k\uparrow}^\dagger c_{k\uparrow} + \sum_k c_{k\downarrow}^\dagger c_{k\downarrow}$$

$$\langle \hat{N} \rangle = \sum_k \langle c_{k\uparrow}^\dagger c_{k\uparrow} \rangle + \sum_k \langle c_{k\downarrow}^\dagger c_{k\downarrow} \rangle$$

$$\langle c_{k\uparrow}^\dagger c_{k\uparrow} \rangle = ?$$

$$\langle c_{k\uparrow}^\dagger c_{k\uparrow} \rangle = \langle BCS | c_{k\uparrow}^\dagger c_{k\uparrow} | BCS \rangle = \langle \prod_{k'} (u_{k'}^* + v_{k'}^* c_{-k'\downarrow}^\dagger c_{k'\uparrow}^\dagger) c_{k\uparrow}^\dagger c_{k\uparrow} \prod_{k''} (u_{k''} + v_{k''} c_{k''\uparrow} c_{-k''\downarrow}) | \rangle =$$

$$= \langle \prod_{k' \neq k} (u_{k'}^* + v_{k'}^* c_{-k'\downarrow}^\dagger c_{k'\uparrow}^\dagger) (u_k^* + v_k^* c_{-k\downarrow}^\dagger c_{k\uparrow}^\dagger) c_{k\uparrow}^\dagger c_{k\uparrow} (u_k + v_k c_{k\uparrow} c_{-k\downarrow}) \prod_{k'' \neq k} (u_{k''} + v_{k''} c_{k''\uparrow} c_{-k''\downarrow}) | \rangle =$$

$$= \langle \prod_{k' \neq k} (u_{k'}^* + v_{k'}^* c_{-k'\downarrow}^\dagger c_{k'\uparrow}^\dagger) (u_{k'} + v_{k'} c_{k'\uparrow} c_{-k'\downarrow}) (u_k^* + v_k^* c_{-k\downarrow}^\dagger c_{k\uparrow}^\dagger) c_{k\uparrow}^\dagger c_{k\uparrow} (u_k + v_k c_{k\uparrow} c_{-k\downarrow}) | \rangle =$$

$$\prod_{k' \neq k} (|u_{k'}|^2 + |v_{k'}|^2)$$

Prežije pouze člen $|v_k|^2 c_{-k\downarrow} c_{k\uparrow} c_{k\uparrow}^\dagger c_{k\uparrow} c_{k\uparrow}^\dagger c_{k\uparrow} c_{-k\downarrow} = |v_k|^2$

$$\langle c_{k\uparrow}^\dagger c_{k\uparrow} \rangle = |v_k|^2$$

$\langle C_{kb}^+ C_{kb} \rangle = ?$ $\langle C_{kb}^+ C_{kb} \rangle = \langle BCS | C_{kb}^+ C_{kb} | BCS \rangle =$

$= \langle | \prod_{k' \neq k} (u_{k'}^* + v_{k'}^* C_{-k'\uparrow}^+ C_{k'\uparrow}) (u_{-k'} + v_{-k'} C_{k'\downarrow}^+ C_{-k'\downarrow}) C_{kb}^+ C_{kb} (u_{-k} + v_{-k} C_{-k\uparrow}^+ C_{k\downarrow}^+) \prod_{k'' \neq k} (u_{k''} + v_{k''} C_{-k''\uparrow}^+ C_{k''\downarrow}^+) | \rangle =$

$=$ | obdobně jako pro $\langle C_{k\uparrow}^+ C_{k\uparrow} \rangle$ vyjde: $= |v_{-k}|^2$

$\langle C_{kb}^+ C_{kb} \rangle = |v_{-k}|^2$ (with some crossed-out terms)

$\langle \hat{N} \rangle = \sum_k \langle C_{k\uparrow}^+ C_{k\uparrow} \rangle + \sum_k \langle C_{kb}^+ C_{kb} \rangle = 2 \sum_k |v_k|^2$

$\langle N^2 \rangle = \langle \sum_{k\sigma} C_{k\sigma}^+ C_{k\sigma} \sum_{l\sigma} C_{l\sigma}^+ C_{l\sigma} \rangle = \langle \sum_{k\sigma l\sigma} C_{k\sigma}^+ C_{k\sigma} C_{l\sigma}^+ C_{l\sigma} \rangle = ?$
 $\langle C_{k\sigma}^+ C_{k\sigma} C_{l\sigma}^+ C_{l\sigma} \rangle = ?$

Pro případ $k \neq l$: $\langle C_{k\sigma}^+ C_{k\sigma} C_{l\sigma}^+ C_{l\sigma} \rangle = \langle C_{k\sigma}^+ C_{k\sigma} C_{l\sigma}^+ C_{l\sigma} \rangle$

$= \langle | \prod_{k' \neq k, l} (u_{k'}^* + v_{k'}^* C_{-k'\uparrow}^+ C_{k'\uparrow}) (u_{-k'} + v_{-k'} C_{k'\downarrow}^+ C_{-k'\downarrow}) (u_{k'}^* + v_{k'}^* C_{-k'\uparrow}^+ C_{k'\uparrow}) C_{k\sigma}^+ C_{k\sigma} (u_{-k} + v_{-k} C_{-k\uparrow}^+ C_{k\downarrow}^+) (u_{k'}^* + v_{k'}^* C_{-k'\uparrow}^+ C_{k'\uparrow}) C_{l\sigma}^+ C_{l\sigma} (u_{-l} + v_{-l} C_{-l\uparrow}^+ C_{l\downarrow}^+) C_{l\sigma}^+ C_{l\sigma} (u_{-l} + v_{-l} C_{-l\uparrow}^+ C_{l\downarrow}^+) | \rangle$

obdobně jako při výpočtu $\langle C_{k\uparrow}^+ C_{k\uparrow} \rangle$ $|v_k|^2$

$|v_k|^2$

$k \neq l$: $\langle C_{k\sigma}^+ C_{k\sigma} C_{l\sigma}^+ C_{l\sigma} \rangle = |v_k|^2 \cdot |v_l|^2$

Pro případ $k=l$: $\langle C_{k\sigma}^+ C_{k\sigma} C_{k\sigma}^+ C_{k\sigma} \rangle = \langle C_{k\uparrow}^+ C_{k\uparrow} C_{k\uparrow}^+ C_{k\uparrow} \rangle =$

$= \langle | \prod_{k' \neq k} (u_{k'}^* + v_{k'}^* C_{-k'\uparrow}^+ C_{k'\uparrow}) (u_{-k'} + v_{-k'} C_{k'\downarrow}^+ C_{-k'\downarrow}) (u_{k'}^* + v_{k'}^* C_{-k'\uparrow}^+ C_{k'\uparrow}) C_{k\sigma}^+ C_{k\sigma} (u_{-k} + v_{-k} C_{-k\uparrow}^+ C_{k\downarrow}^+) (u_{k'}^* + v_{k'}^* C_{-k'\uparrow}^+ C_{k'\uparrow}) C_{k\sigma}^+ C_{k\sigma} (u_{-k} + v_{-k} C_{-k\uparrow}^+ C_{k\downarrow}^+) | \rangle$

Prežijte pouze člen $|v_k|^2 \cdot C_{-k\uparrow}^+ C_{k\uparrow}^+ C_{k\uparrow}^+ C_{k\uparrow}^+ C_{k\uparrow}^+ C_{k\uparrow}^+ C_{k\uparrow}^+ C_{-k\downarrow}^+$

$|v_k|^2$

pro $k=l$ platí:

$\langle C_{k\sigma}^+ C_{k\sigma} C_{k\sigma}^+ C_{k\sigma} \rangle = |v_k|^2$

$$\langle N^2 \rangle = \left\langle \sum_{k \neq m} c_{k\uparrow}^\dagger c_{k\uparrow} c_{m\uparrow}^\dagger c_{m\uparrow} \right\rangle = \left\langle \sum_{k \neq l} c_{k\uparrow}^\dagger c_{k\uparrow} c_{l\uparrow}^\dagger c_{l\uparrow} \right\rangle + \left\langle \sum_{k \neq m} c_{k\downarrow}^\dagger c_{k\downarrow} c_{m\downarrow}^\dagger c_{m\downarrow} \right\rangle =$$

= $\left| \begin{array}{l} \text{V každé sémě jsou 4} \\ \text{príspevky od různých spinů} \end{array} \right| = 4 \cdot \sum_k |v_k|^2 + 4 \cdot \sum_k \sum_{l \neq k} |v_k|^2 |v_l|^2$

$$\langle (N - \langle N \rangle)^2 \rangle = \langle N^2 \rangle - \langle N \rangle^2 = 4 \sum_k |v_k|^2 + 4 \sum_k \sum_{l \neq k} |v_k|^2 |v_l|^2 - 4 \cdot \sum_{k,l} |v_k|^2 |v_l|^2 =$$

$$= 4 \sum_k |v_k|^2 - 4 \sum_{\substack{k,l \\ k=l}} |v_k|^2 |v_l|^2 = 4 \cdot \sum_k |v_k|^2 (1 - |v_k|^2) = 4 \sum_k |v_k|^2 \cdot |u_k|^2$$

$$\langle c_{-k\downarrow} c_{k\uparrow} \rangle = \left\langle \prod_{k'=k} (u_{k'}^* + v_{k'}^* c_{-k'\downarrow}^\dagger c_{k'\uparrow}) (u_k + v_k c_{k\uparrow}^\dagger c_{-k\downarrow}) (u_k^* + v_k^* c_{-k\downarrow}^\dagger c_{k\uparrow}) c_{-k\downarrow} c_{k\uparrow} (u_k + v_k c_{k\uparrow}^\dagger c_{-k\downarrow}) \right\rangle$$

1

Přešijte pouze člen

$$u_k^* v_k c_{-k\downarrow} c_{k\uparrow} c_{k\downarrow}^\dagger c_{k\uparrow}^\dagger$$

$$\langle c_{-k\downarrow} c_{k\uparrow} \rangle = u_k^* v_k$$

$$4) \langle C_{k\uparrow}^+ C_{-k\downarrow}^+ C_{-l\downarrow} C_{l\uparrow} \rangle =$$

• Pro $k \neq l$

$$= \langle | \tilde{1} \rangle \underbrace{\left(\sum_{k' \neq k, l} (u_{k'}^* + v_{k'}^* C_{-k'\downarrow}^+ C_{k'\uparrow}) \right) (u_{k'} + v_{k'} C_{k'\uparrow} C_{-k'\downarrow})}_{\text{Pr\u00e9zije ponze}} \underbrace{\left(\sum_{k' \neq k, l} (u_{k'}^* + v_{k'}^* C_{-k'\downarrow}^+ C_{k'\uparrow}) C_{k'\uparrow} C_{-k'\downarrow} \right)}_{\text{Pr\u00e9zije ponze}} \underbrace{\left(\sum_{l' \neq l} (u_{l'}^* + v_{l'}^* C_{-l'\downarrow}^+ C_{l'\uparrow}) C_{-l'\downarrow} C_{l'\uparrow} \right) (u_{l'} + v_{l'} C_{l'\uparrow} C_{-l'\downarrow})}_{\text{Pr\u00e9zije ponze}} | \tilde{1} \rangle$$

$$= u_k v_k^* C_{-k\downarrow}^+ C_{k\uparrow}^+ C_{k\uparrow} C_{-k\downarrow} + u_l^* v_l C_{-l\downarrow} C_{l\uparrow} C_{l\uparrow}^+ C_{-l\downarrow}^+$$

$$= u_k^* u_l v_k^* v_l$$

• Pro $k = l$

$$\langle C_{k\uparrow}^+ C_{-k\downarrow}^+ C_{-k\downarrow} C_{k\uparrow} \rangle =$$

$$= \langle | \tilde{1} \rangle \underbrace{\left(\sum_{k' \neq k} (u_{k'}^* + v_{k'}^* C_{-k'\downarrow}^+ C_{k'\uparrow}) \right) (u_{k'} + v_{k'} C_{k'\uparrow} C_{-k'\downarrow})}_{\text{Pr\u00e9zije ponze}} \underbrace{\left(\sum_{k' \neq k} (u_{k'}^* + v_{k'}^* C_{-k'\downarrow}^+ C_{k'\uparrow}) C_{k'\uparrow} C_{-k'\downarrow} \right)}_{\text{Pr\u00e9zije ponze}} \underbrace{\left(\sum_{k' \neq k} (u_{k'} + v_{k'} C_{k'\uparrow} C_{-k'\downarrow}) \right) (u_{k'}^* + v_{k'}^* C_{-k'\downarrow}^+ C_{k'\uparrow})}_{\text{Pr\u00e9zije ponze}} | \tilde{1} \rangle$$

$$= |v_k|^2 C_{-k\downarrow}^+ C_{k\uparrow}^+ C_{k\uparrow} C_{-k\downarrow} + |v_k|^2 C_{-k\downarrow} C_{k\uparrow} C_{k\uparrow}^+ C_{-k\downarrow}^+$$

$$= |v_k|^2$$

$$\langle C_{k\uparrow}^+ C_{-k\downarrow}^+ C_{-l\downarrow} C_{l\uparrow} \rangle = \begin{cases} u_k^* u_l v_k^* v_l & k \neq l \\ |v_k|^2 & k = l \end{cases}$$