$$|\Psi_A\rangle \approx \underset{\substack{\uparrow \\ \text{bogoliubovské} \\ \text{vakuum}}}{|\Psi_0\rangle} - \sum_n \frac{\langle n|H_{int}|\Psi_0\rangle}{\underbrace{E_n - E_0}_{\text{exc. energie}}} \underset{\nwarrow \text{ exc. stavy BCS}}{|n\rangle}$$

(1)

$$H_{BCS} = \sum_k \sum_\sigma E_k b^+_{k\sigma} b_{k\sigma}$$

$$c_{k\uparrow} = u^*_k b_{k\uparrow} + v_k b^+_{-k\downarrow}$$

$$c^+_{-k\downarrow} = -v^*_k b_{k\uparrow} + u_k b^+_{-k\downarrow}$$

$$v_k = v_{-k} \in \mathbb{R}$$

$$u_k = u_{-k} \in \mathbb{R}$$

$$H_{int} = \frac{e\hbar}{2m} \sum_{k\sigma} \left(\vec{k} + \frac{\vec{q}}{2}\right) \vec{a}_q \, c^+_{k+q\sigma} c_{k\sigma}$$

$$\sum_\sigma c^+_{k+q} c_{k\sigma} = c^+_{k+q\uparrow} c_{k\uparrow} + c^+_{k+q\downarrow} c_{k\downarrow}$$

$$= \left(u_{k+q} b^+_{k+q\uparrow} + v_{k+q} b_{-k-q\downarrow}\right)\left(u_k b_{k\uparrow} + v_k b^+_{-k\downarrow}\right) +$$

$$+ \left(-v_{k+q\uparrow} b_{-k-q\uparrow} + u_{k+q} b^+_{k+q\downarrow}\right)\left(-v_{+k} b^+_{-k\uparrow} + u_{+k} b_{k\downarrow}\right)$$

$$\sum_\sigma c^+_{k+q} c_{k\sigma}|\Psi_0\rangle = \left(u_{k+q} v_k b^+_{k+q\uparrow} b^+_{-k\downarrow} - u_{k+q} v_k b^+_{k+q\downarrow} b^+_{-k\uparrow}\right)|\Psi_0\rangle$$

$$+ \left(b b^+\right)|\Psi_0\rangle$$

$$\langle \underset{\substack{\uparrow \\ >0}}{n}| b b^+ |\Psi_0\rangle = 0$$

$$\langle n| \sum_{k\sigma} c^+_{k+q\sigma} c_{k\sigma} |\Psi_0\rangle = \sum_k u_{k+q} v_k \langle n| \left(b^+_{k+q\uparrow} b^+_{-k\downarrow} - b^+_{k+q\downarrow} b^+_{-k\uparrow}\right)|\Psi_0\rangle$$

$$\langle n|(b^{+}_{k+q\uparrow}b^{+}_{-k\downarrow}-b^{+}_{k+q\downarrow}b^{+}_{-k\uparrow}|\Psi_0\rangle \neq 0 \iff$$ ②

$$|n\rangle = b^{+}_{k+q\uparrow}b^{+}_{-k\downarrow}|\Psi_0\rangle \ \vee \ |n\rangle = b^{+}_{k+q\downarrow}b^{+}_{-k\uparrow}|\Psi_0\rangle$$

$$\Rightarrow \sum_n \langle n|\sum_{k\sigma} c^{+}_{k+q\sigma}c_{k\sigma}|\Psi_0\rangle|n\rangle =$$

$$= \sum_k u_{k+q}v_k\,(b^{+}_{k+q}b^{+}_{-k\downarrow}-b^{+}_{k+q\downarrow}b^{+}_{-k\uparrow})|\Psi_0\rangle$$

$$\Rightarrow \sum_n \frac{\langle n|H_{int}|\Psi_0\rangle}{E_n-E_0}|n\rangle = \frac{e\hbar}{2m}\sum_k(\bar{k}+\frac{\bar{q}}{2})\bar{a}_q\frac{u_{k+q}v_k}{E_{k+q}+E_k}(b^{+}_{k+q}b^{+}_{-k\downarrow}+$$

$$-b^{+}_{k+q\downarrow}b^{+}_{-k\uparrow})|\Psi_0\rangle = |\delta\Psi\rangle$$

$$\langle\Psi_A|\bar{j}_p+\bar{j}_d|\Psi_A\rangle \approx \langle\Psi_0|\bar{j}_d|\Psi_0\rangle+\langle\Psi_0|\bar{j}_p|\Psi_0\rangle-\langle\Psi_0|\bar{j}_p|\delta\Psi\rangle+$$

$$-\langle\delta\Psi|\bar{j}_p|\Psi_0\rangle$$

- $$\bar{j}_d(\bar{q}) = -\frac{e^2}{m}\frac{1}{V}\bar{a}_q\sum_{k\sigma}c^{+}_{k\sigma}c_{k\sigma}$$

$$\langle\Psi_0|\bar{j}_d(\bar{q})|\Psi_0\rangle = -\frac{e^2}{m}\frac{1}{V}\bar{a}_q\overbrace{\langle\Psi_0|\sum_{k\sigma}c^{+}_{k\sigma}c_{k\sigma}|\Psi_0\rangle}^{N} = -\frac{e^2}{m}n\,\vec{a}_q$$

- $$\bar{j}_p(\bar{q}) = \frac{e\hbar}{m}\frac{1}{V}\sum_{k\sigma}(\vec{k}-\frac{\bar{q}}{2})c^{+}_{k-q\sigma}c_{k\sigma}$$

$$\sum_\sigma c^{+}_{k-q\sigma}c_{k\sigma} = c^{+}_{k-q\uparrow}c_{k\uparrow}+c^{+}_{k-q\downarrow}c_{k\downarrow} =$$

$$= (u_{k-q}b^{+}_{k-q\uparrow}+v_{q-k}b_{q-k\downarrow})(u^{*}_{k}b_{k\uparrow}+v_k b^{+}_{-k\downarrow})+$$

$$+(-v_{k-q}b_{q-k\uparrow}+u_{k-q}b_{k-q\downarrow})(-v_k b^{+}_{-k\uparrow}+u_k b_{k\downarrow})$$

③

• $\langle \Psi_0 | \hat{j}_r(\bar{q}) | \Psi_0 \rangle$

$$\langle \Psi_0 | \sum_\sigma c^+_{k-q} c_{k\sigma} | \Psi_0 \rangle = \langle \Psi_0 | v_{q-k} v_k \, b_{q-k\downarrow} b^+_{-k\downarrow} | \Psi_0 \rangle +$$

$$+ \langle \Psi_0 | v_{q-k} b_{q-k\uparrow} v_k b^+_{-k\uparrow} | \Psi_0 \rangle =$$

$$= 2 v_k^2 \delta_q^0$$

$$\langle \Psi_0 | \hat{j}_r | \Psi_0 \rangle = \frac{e\hbar}{m} \frac{1}{V} \sum_k 2 v_k^2 \cdot k = \left| v_k = v_{-k} \right| = 0$$

• • $\langle \Psi_0 | \hat{j}_r(\bar{q}) | \delta \Psi \rangle$

irelevantní

$$\sum_\sigma c^+_{k-q\sigma} c_{k\sigma} = v_{q-k} u_k b_{q-k\downarrow} b_{k\uparrow} - v_{q-k} u_k b_{q-k\uparrow} b_{k\downarrow} + \dots$$

$$\dots \langle \Psi_0 | \sum_k \sum_{k'} \overbrace{v_{k-q} u_k}^{A_k} \left[b_{q-k\downarrow} b_{k\uparrow} - b_{q-k\uparrow} b_{k\downarrow} \right] \left[b^+_{k'+q\uparrow} b^+_{-k'\downarrow} - b^+_{k'+q\downarrow} b^+_{-k'\uparrow} \right] | \Psi_0 \rangle$$

$$= \langle \Psi_0 | \sum_{kk'} A_k \left[b_{q-k\downarrow} b_{k\uparrow} b^+_{k'+q\uparrow} b^+_{-k'\downarrow} - b_{q-k\downarrow} b_{k\uparrow} b^+_{k'+q\downarrow} b^+_{-k'\uparrow} + \right.$$

$$\left. + b_{q-k\uparrow} b_{k\downarrow} b^+_{k'+q\downarrow} b^+_{-k'\downarrow} - b_{q-k\uparrow} b_{k\downarrow} b^+_{k'+q\uparrow} b^+_{-k'\downarrow} \right] | \Psi_0 \rangle =$$

$$= \sum_{kk'} A_k \left[\delta_k^{k'+q} \delta_{-k'}^{q-k} + \delta_k^{-k'} \delta_{q-k}^{k'+q} + \delta_{-k}^{q-k} \delta_k^{k'+q} + \delta_{q-k}^{k'+q} \delta_k^{-k'} \right]$$

$$\langle \varphi_0 | \bar{j}_p | \delta\varphi \rangle = \sum_k \bar{k}\, \vec{a}_q \frac{u_{k+q} v_k}{E_{k+q} + E_k} \Big[\left(k + \tfrac{1}{2} q\right) A_{k+q} + \left(-k - \tfrac{q}{2}\right) A_{-k}$$

$$+ \left(k + \frac{q}{2}\right) A_{k+q} + \left(-k - \frac{q}{2}\right) A_{-k} \Big] =$$

$$= \left| \begin{array}{l} A_{k+q} = v_k u_{k+q} \\ A_{-k} = v_{k+q} u_k \end{array} \right| = \sum_k \bar{k}\, \vec{a}_q \left(k + \frac{q}{2}\right) \Big[2 v_k u_{k+q} - 2 v_{k+q} u_k \Big]$$

$$\underset{q \to 0}{=} \underline{0}\ \checkmark$$