Introduction

What are the main physical processes which determine the structure of stars ?

- Stars are held together by gravitation – attraction exerted on each part of the star by all other parts
- Collapse is resisted by internal thermal pressure.
- These two forces play the principal role in determining stellar structure – they must be (at least almost) in balance
- Thermal properties of stars – continually radiating into space. If thermal properties are constant, continual energy source must exist
- Theory must describe - origin of energy and transport to surface

We make two fundamental assumptions :

1) Neglect the rate of change of properties – assume constant with time
2) All stars are spherical and symmetric about their centres

We will start with these assumptions and later reconsider their validity

For our stars – which are isolated, static, and spherically symmetric – there are four basic equations to describe structure. All physical quantities depend on the distance from the centre of the star alone

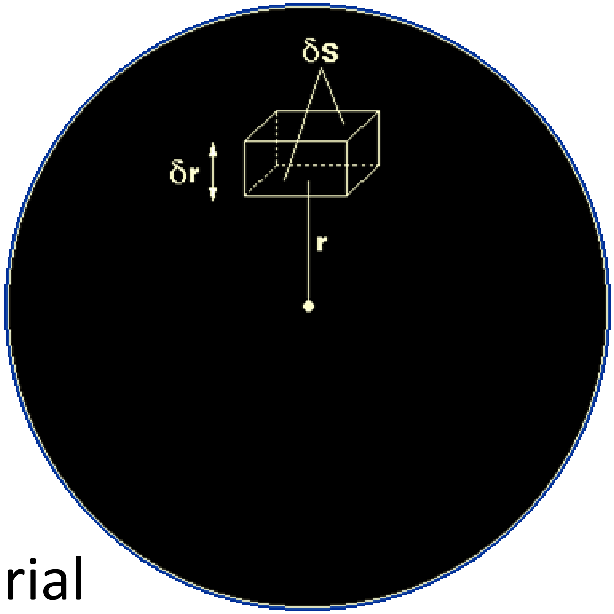
1) **Equation of hydrostatic equilibrium:** at each radius, forces due to pressure differences balance gravity
2) **Conservation of mass**
3) **Conservation of energy:** at each radius, the change in the energy flux = local rate of energy release
4) **Equation of energy transport:** relation between the energy flux and the local gradient of temperature

These basic equations supplemented with

- Equation of state: pressure of a gas as a function of its density and temperature
- Opacity: how opaque the gas is to the radiation field
- Core nuclear energy generation rate

Equation of hydrostatic support

Balance between gravity and internal pressure is known as ***hydrostatic equilibrium***



Mass of element

$\delta m = \rho(r)\delta s\delta m$ where $\rho(r)$ is density at r

Consider forces acting in radial direction

1. Outward force: pressure exerted by stellar material on the lower face: $P(r)\delta s$
2. Inward force: pressure exerted by stellar material on the upper face, and gravitational attraction of all stellar material lying within *r*

$$P(r+\delta r)\delta s + \frac{GM(r)}{r^2}\delta m = P(r+\delta r)\delta s + \frac{GM(r)}{r^2}\rho(r)\delta s\delta r$$

In hydrostatic equilibrium:

$$P(r)\delta s = P(r+\delta r)\delta s + \frac{GM(r)}{r^2}\delta m$$

$$\Rightarrow P(r+\delta r) - P(r) = -\frac{GM(r)}{r^2}\rho(r)\delta r$$

If we consider an infinitesimal element, we write

$$\frac{P(r+\delta r)\ - P(r)}{\delta r} = \frac{dP(r)}{dr} \qquad \text{for } \delta r \ \to 0$$

Hence rearranging above we get

$$\frac{dP(r)}{dr} = -\frac{GM(r)(r)}{r^2}$$

The **equation of hydrostatic support**

Accuracy of hydrostatic assumption

We have assumed that the gravity and pressure forces are balanced - how valid is that?

Consider the case where the outward and inward forces are not equal, there will be a resultant force acting on the element which will give rise to an acceleration a

$$P(r+\delta r)\delta s + \frac{GM(r)}{r^2}\delta s\delta r - P(r)\delta = \rho(r)\delta s\delta$$

$$\Rightarrow \frac{dP(r)}{dr} + \frac{GM(r)}{r^2}\rho(r) = \rho(r)a$$

Now acceleration due to gravity is $g = \frac{GM(r)}{r^2}$

$$\Rightarrow \frac{dP(r)}{dr} + g\rho = \rho(r)a$$

Which is the **generalised form** of the **equation of hydrostatic support**

Accuracy of hydrostatic assumption

Now suppose there is a resultant force on the element (LHS $\neq$ 0). Suppose their sum is small fraction of gravitational term (β)

$$\beta\rho(r)g = \rho(r)a$$

Hence there is an inward acceleration of $a = \beta g$

Assuming it begins at rest, the spatial displacement d after time t is $d = \frac{1}{2}at^2 = \frac{1}{2}\beta g t^2$

If we allowed the star to collapse i.e. set $d = r$ and substitute

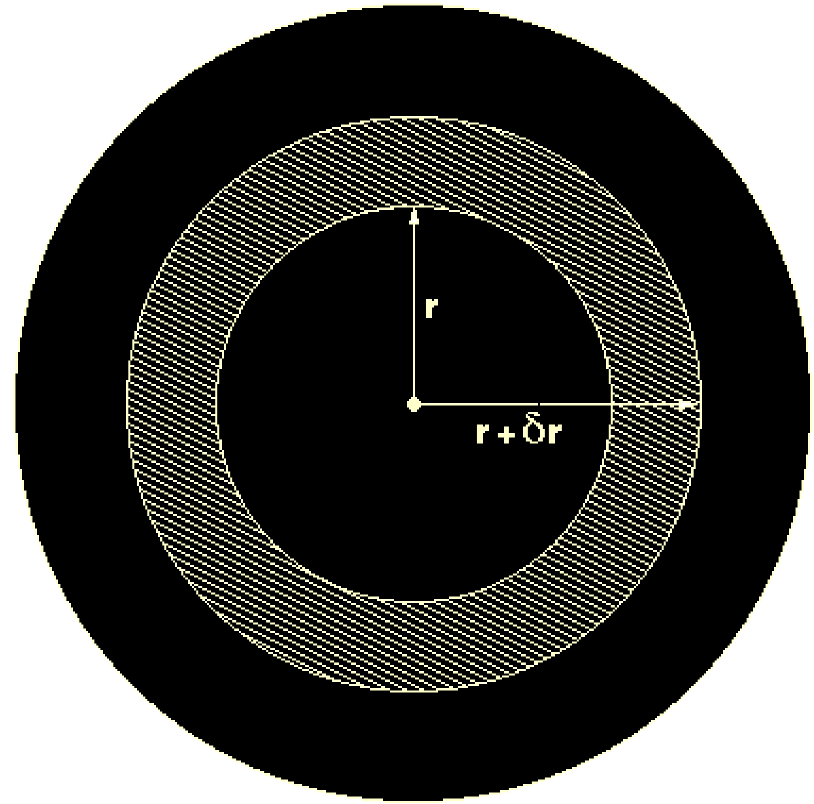
$$g = \frac{GM}{r^2} \Rightarrow t = \frac{1}{\sqrt{\beta}}\left(\frac{2r^3}{GM}\right)^{\frac{1}{2}} \text{ assumming } \beta \approx 1 \Rightarrow t_d = \left(\frac{2r^3}{GM}\right)^{\frac{1}{2}}$$

t_d is known as the ***dynamical time***

Equation of mass conservation

Mass $M(r)$ contained within a star of radius *r* is determined by the density of the gas $\rho(r)$

Consider a thin shell inside the star with radius r and outer radius $r + \delta r$



$$\delta V = 4\pi r^2 \, \delta r$$

$$\delta M = \delta V \rho(r) = 4\pi r^2 \delta r \rho(r)$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

In the limit where $\delta r \rightarrow 0$

This is the **equation of mass conservation**

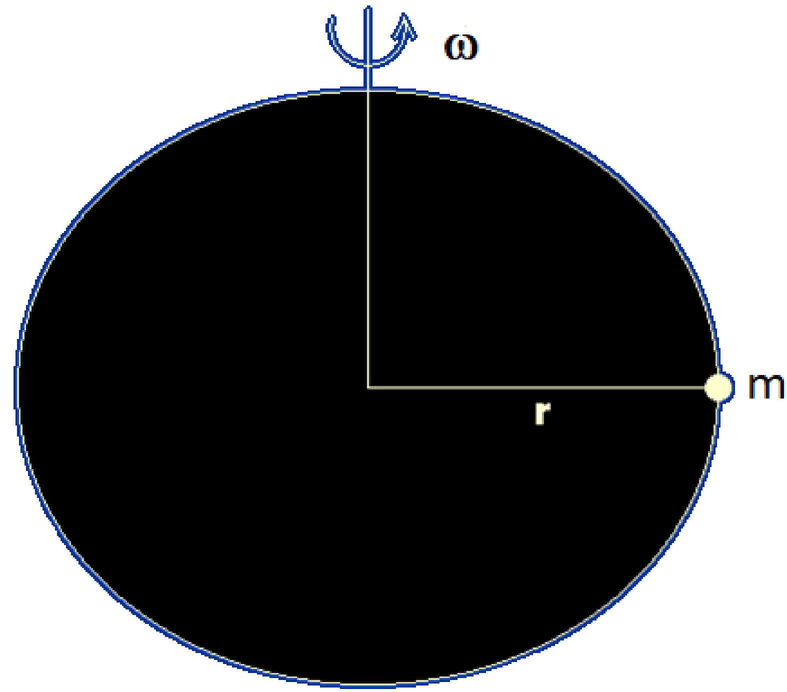
Accuracy of spherical symmetry assumption

Stars are rotating gaseous bodies – to what extent are they flattened at the poles ?

If so, departures from spherical symmetry must be accounted for

Consider mass δm near the surface of star of mass M and radius r

Element will be acted on by additional inwardly acting force to provide circular motion



Centripetal force is given as $m\omega^2 r$ where ω is the ***angular velocity*** of the star. There will be no departure from spherical symmetry provided that

$$\frac{m\omega^2 r}{GMm/r^2} \ll 1 \text{ or } \omega^2 \ll \frac{GM}{r^3}$$

Accuracy of spherical symmetry assumption

Note the RHS of this equation is similar to t_d

$$t_d = \left(\frac{2r^3}{GM}\right)^{\frac{1}{2}} \text{ or } \frac{GM}{r^3} = \frac{2}{t_d^2} \Rightarrow \omega^2 \ll \frac{2}{t_d^2}$$

and $\omega = \frac{2\pi}{P}$ where P is the ***rotation period***

Spherical symmetry is met if $P \gg t_d$

For example t_d(Sun) $\sim$ 2000s and $P \sim 1$ month

$\Rightarrow$ For the majority of stars, departures from spherical symmetry can be ignored

Some stars do rotate rapidly and rotational effects must be included in the structure equations - can change the output of models