1) Vypočítejte $\int_C (2x-y)\,ds$, kde C je úsečka AB, $A=[1,1]$, $B=[2,3]$

$\vec{s} = \vec{AB} = B - A = (1,2)$, $\quad C: \begin{array}{l} x = 1+t \\ y = 1+2t \end{array}$, $t \in \langle 0,1 \rangle$ $\quad \begin{array}{l} x' = 1 \\ y' = 2 \end{array}$

$$\int_C (2x-y)\,ds = \int_0^1 [2(1+t) - (1+2t)]\sqrt{1^2+2^2}\,dt = \sqrt{5}\int_0^1 (1-0t)\,dt =$$

$$= \sqrt{5}\,[t]_0^1 = \sqrt{5}$$

---

2) Vypočítejte $\int_C (x+y)\,ds$, kde C je úsečka $x-y+1=0$ a $x \in \langle 1,3 \rangle$

$C: \; y = x+1 \quad \begin{array}{l} x = t \\ y = t+1 \end{array}$ $\quad t \in \langle 1,3 \rangle$ $\quad \begin{array}{l} x' = 1 \\ y' = 1 \end{array}$

$$\int_C (x+y)\,ds = \int_1^3 [t+(t+1)]\sqrt{1^2+1^2}\,dt = \sqrt{2}\int_1^3 (2t+1)\,dt = \sqrt{2}\,[t^2+t]_1^3 =$$

$$= \sqrt{2}\,[(9+3)-(1+1)] = 10\sqrt{2}$$

---

3) Vypočítejte $\int_C (x-z)\,ds$, kde C je úsečka KL, $K=[1,0,2]$, $L=[3,1,-1]$

$\vec{s} = \vec{KL} = L - K = (2,1,-3)$, $\quad C: \begin{array}{l} x = 1+2t \\ y = 0+t \\ z = 2-3t \end{array}$ $\quad t \in \langle 0,1 \rangle$ $\quad \begin{array}{l} x' = 2 \\ y' = 1 \\ z' = -3 \end{array}$

$$\int_C (x-z)\,ds = \int_0^1 [(1+2t)-(2-3t)]\sqrt{2^2+1^2+(-3)^2}\,dt =$$

$$= \sqrt{14}\int_0^1 (-1+5t)\,dt = \sqrt{14}\left[-t+\frac{5}{2}t^2\right]_0^1 = \sqrt{14}\left(-1+\frac{5}{2}\right) =$$

$$= \sqrt{14}\cdot\frac{3}{2}$$

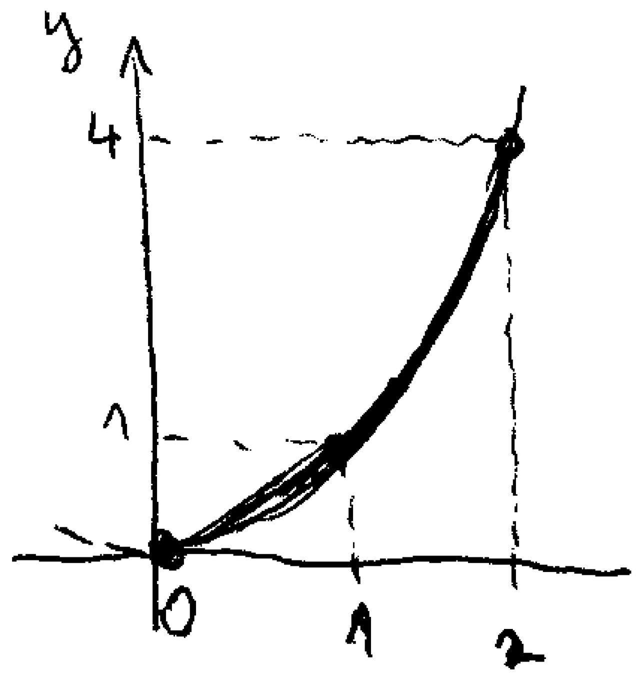
4) Vypočtěte délku úsečky AB, $A=[1,1,2]$, $B=[3,-1,2]$

$$\vec{s}=\overrightarrow{AB}=B-A=(2,-2,0), \quad \begin{aligned} x&=1+2t\\ y&=1-2t\\ z&=2+0t \end{aligned} \quad t\in\langle 0,1\rangle \quad \begin{aligned} x'&=2\\ y'&=-2\\ z'&=0 \end{aligned}$$

$$l=\int_C 1\,ds=\int_0^1 1\cdot\sqrt{2^2+(-2)^2+0^2}\,dt=\int_0^1\sqrt{8}\,dt=\sqrt{8}\,[t]_0^1=\sqrt{8}=2\sqrt{2}$$

---

5) Vypočtěte $\int_C 3x\,ds$, kde C je část paraboly $y=x^2$ mezi body $[0,0]$ a $[2,4]$.



$$\begin{aligned} x&=t\\ y&=t^2 \end{aligned} \quad t\in\langle 0,2\rangle \quad \begin{aligned} x'&=1\\ y'&=2t \end{aligned}$$

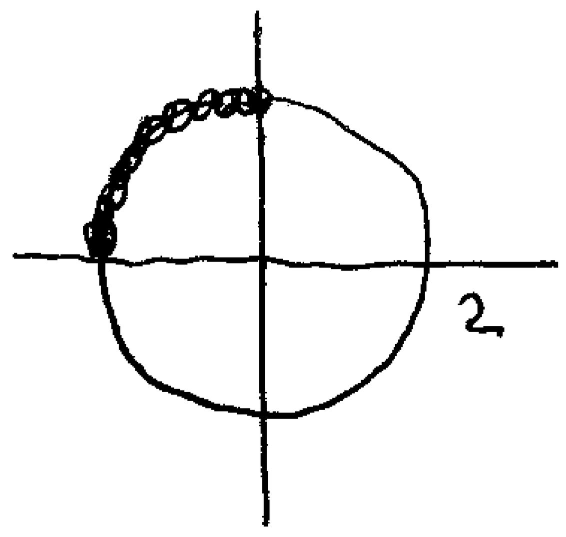
$$\int_C 3x\,ds=\int_0^2 3t\sqrt{1^2+(2t)^2}\,dt=3\int_0^2 t\sqrt{1+4t^2}\,dt=\left|\begin{aligned} 1+4t^2&=z\\ 8t\,dt&=dz\\ t\,dt&=\tfrac{1}{8}dz\\ t=0&\to z=1\\ t=2&\to z=17 \end{aligned}\right|=$$

$$=\frac{3}{8}\int_1^{17}\sqrt{z}\,dz=\frac{3}{8}\cdot\left[\frac{z^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^{17}=\frac{3}{8}\cdot\frac{2}{3}\left[\sqrt{z^3}\right]_1^{17}=$$

$$=\frac{1}{4}\left(\sqrt{17^3}-1\right)$$

---

6) Vypočtěte $\int_C (x+y)\,ds$, kde C je část kružnice $x^2+y^2=4$ a platí $x\le 0$ a $y\ge 0$.



$$\begin{aligned} x&=2\cos t\\ y&=2\sin t \end{aligned} \quad t\in\left\langle \frac{\pi}{2},\pi\right\rangle \quad \begin{aligned} x'&=-2\sin t\\ y'&=2\cos t \end{aligned}$$

$$\int_C (x+y)\,ds=\int_{\frac{\pi}{2}}^{\pi}(2\cos t+2\sin t)\sqrt{(-2\sin t)^2+(2\cos t)^2}\,dt=2\int_{\frac{\pi}{2}}^{\pi}(\cos t+\sin t)\sqrt{4}\,dt$$

$$=4\left[\sin t-\cos t\right]_{\frac{\pi}{2}}^{\pi}=4\left[(\sin\pi-\cos\pi)-\left(\sin\frac{\pi}{2}-\cos\frac{\pi}{2}\right)\right]=$$

$$=4\left[(0-(-1))-(1-0)\right]=0$$