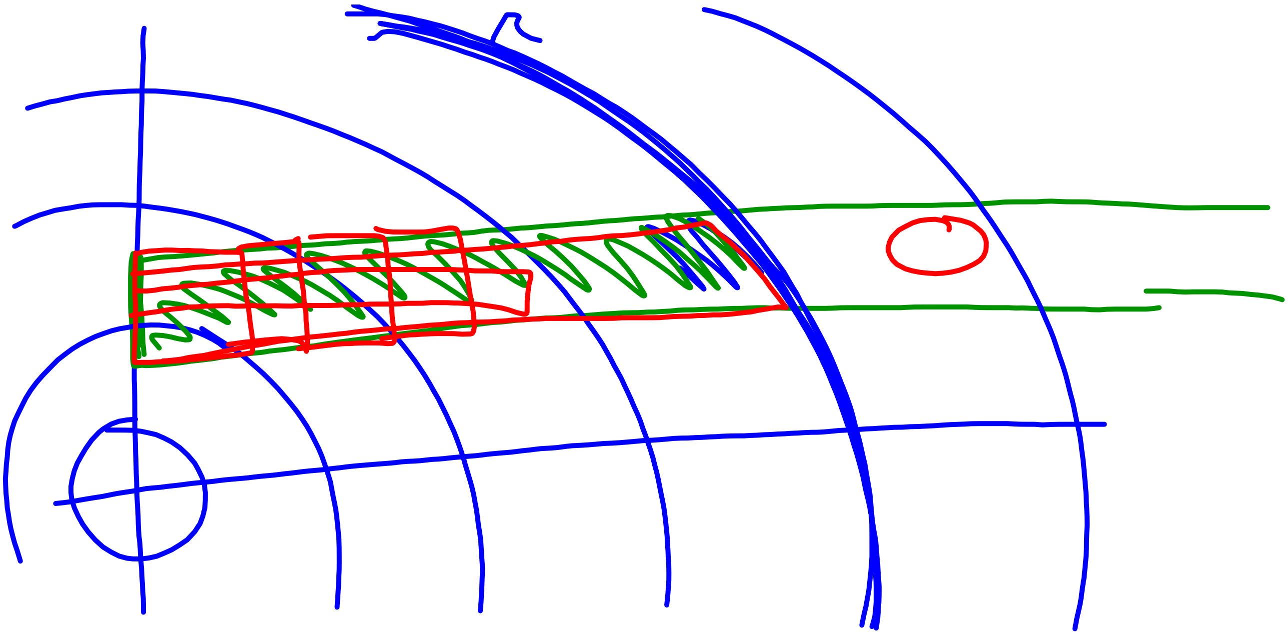
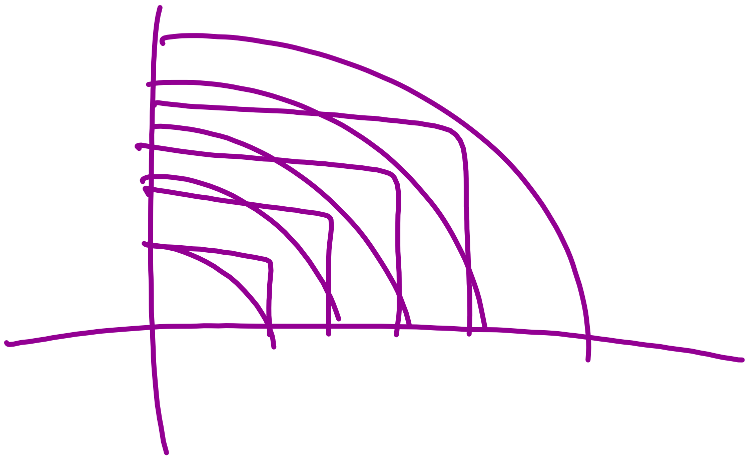
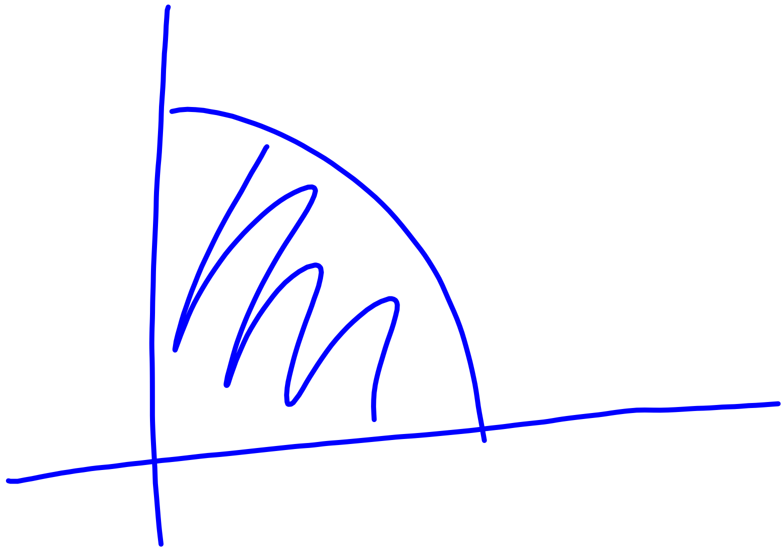
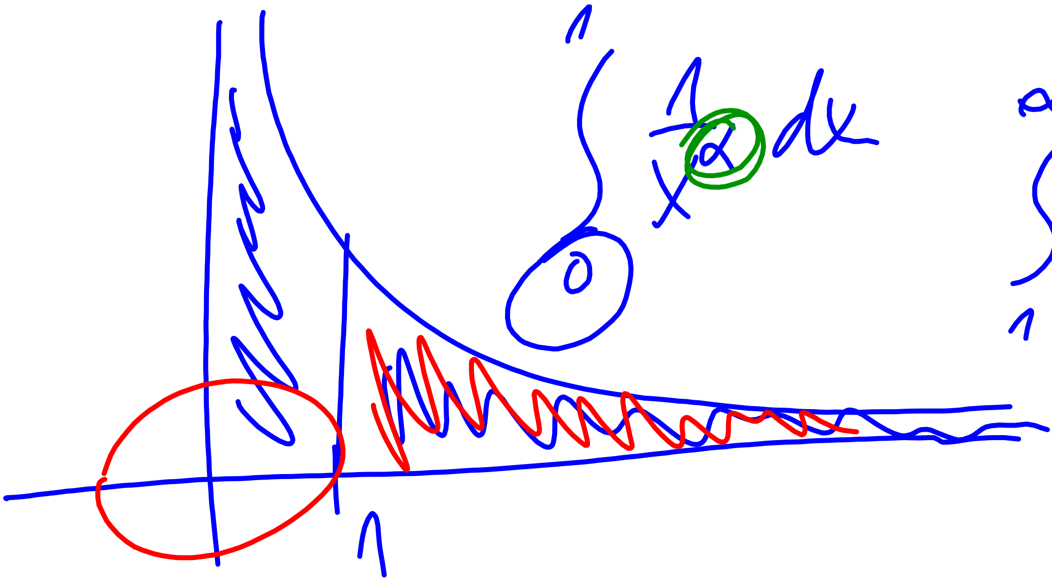






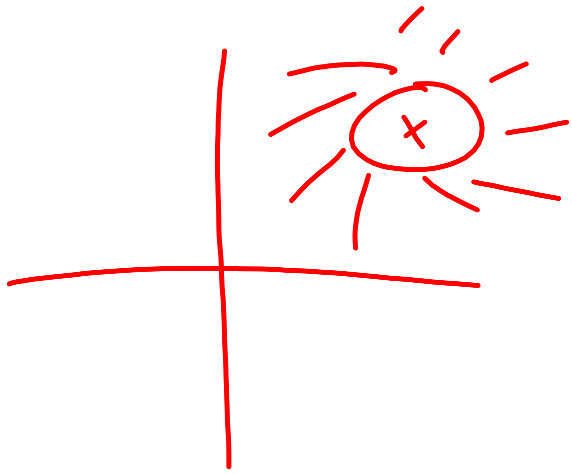
$$\iint \sin x^2 \cdot \cos y^2 \, dx \, dy$$

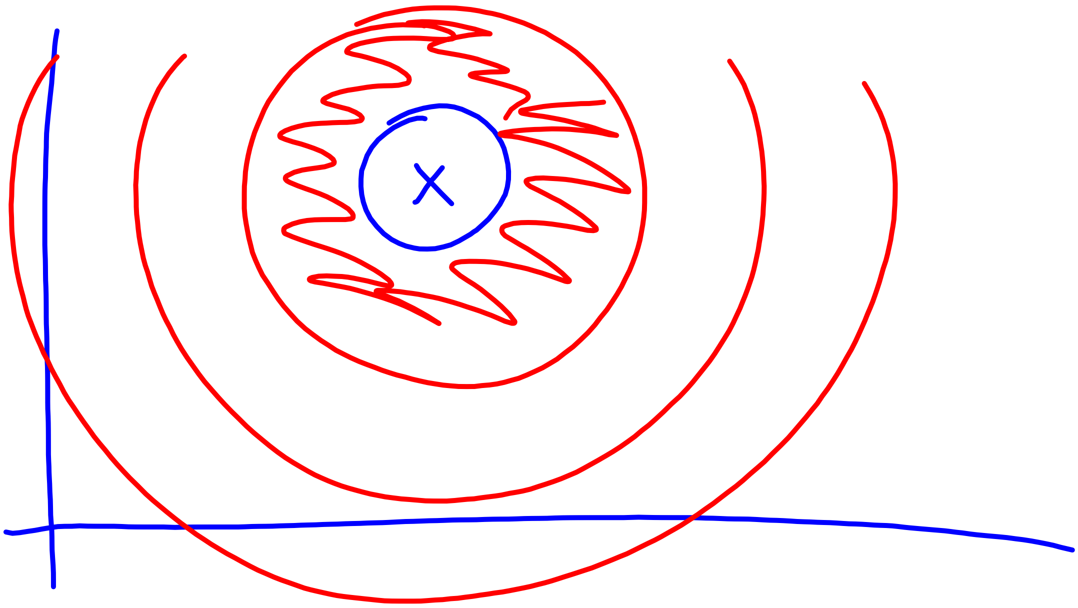
$$\int \cos y^2 \boxed{\int \sin x^2 \, dx} \, dy$$



$$\int_0^1 \frac{1}{x^{\alpha}}\,dx$$

$$\int_1^{\infty} \frac{1}{x^{\alpha}}\,dx$$



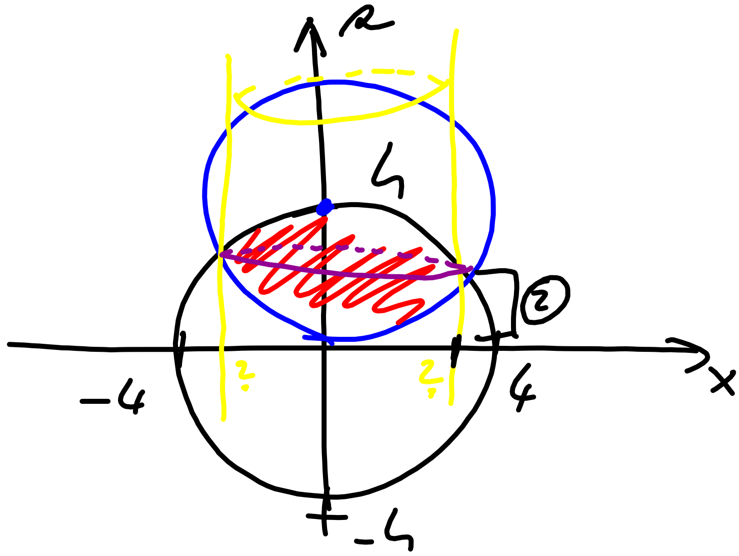


$V = ?$

$$\left.\begin{aligned} x^2+y^2+z^2&=16 \\ x^2+y^2+z^2&=8z \end{aligned}\right\} \circledast$$

$x^2+y^2+z^2-8z=0$

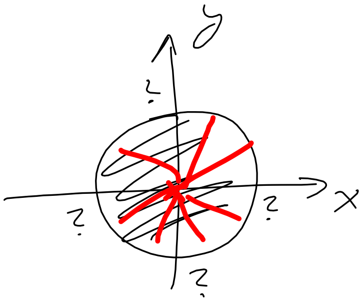
$x^2+y^2+(z-4)^2=16$



a) VÁLCOVÉ

$\varphi \in [0, 2\pi]$

$\rho \in [0, \sqrt{12}]$



$8a = 16$

$a = 2$

$x^2 + y^2 + 4 = 16$

$x^2 + y^2 = 12$

$$z \in \left[4-\sqrt{16-\rho^2},\ \sqrt{16-\rho^2}\right]$$

<u>Dolní</u>: $(z-4)^2 = 16 - x^2 - y^2$

$$z-4 = \pm\sqrt{16-x^2-y^2}$$

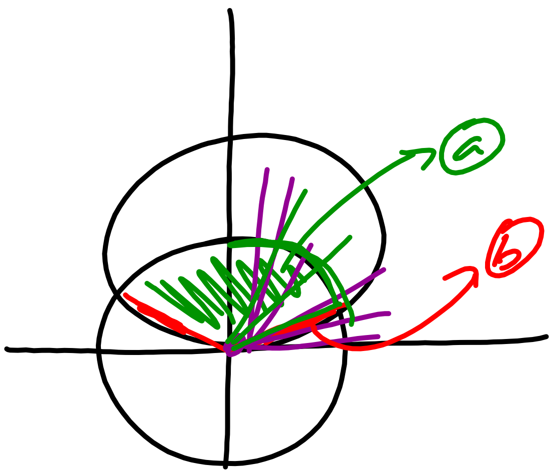
$$z = \pm\sqrt{16-x^2-y^2} + 4$$

<u>Horní</u>: $z^2 = 16 - x^2 - y^2$

$$z = \pm\sqrt{16-x^2-y^2}$$

$$\int_0^{\sqrt{12}} \int_0^{2\pi} \int_{4-\sqrt{16-\varrho^2}}^{\sqrt{16-\varrho^2}} \varrho \cdot 1 \, dz \, d\varphi \, d\varrho = \ldots$$

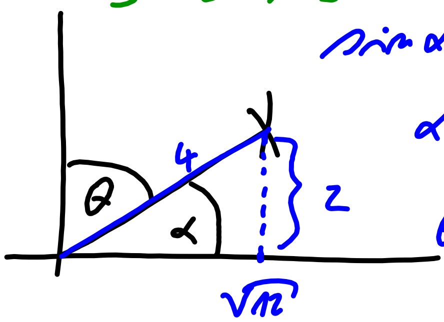
$$\ldots = \frac{80}{3}\pi$$



a $\varphi \in [0, 2\pi]$

$\theta \in [0, \frac{\pi}{3}]$

$\rho \in [0, 4]$



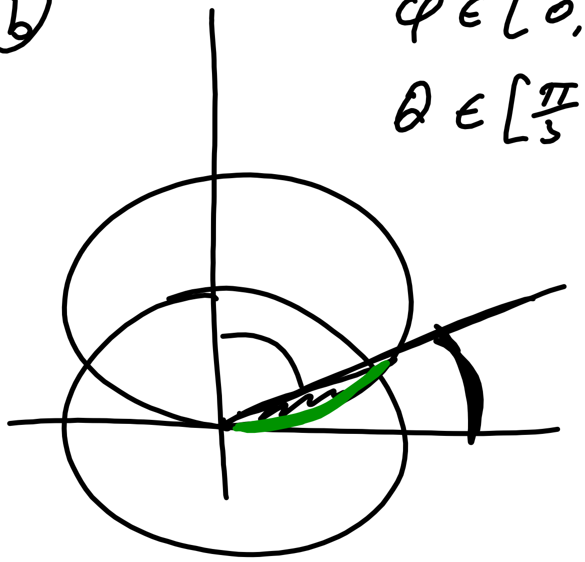
$\sin\alpha = \frac{2}{4} = \frac{1}{2}$

$\alpha = \frac{\pi}{6}$

$\theta = \frac{\pi}{2} - \frac{\pi}{6}$

$$(a) = \int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{4} r^2 \cdot \sin\theta \, d\rho \, d\theta \, d\varphi = \ldots = \frac{64}{3}\pi$$

(b)



$\varphi \in [0, 2\pi]$

$\theta \in [\frac{\pi}{3}, \frac{\pi}{2}]$

$\rho \in [0, 8 \cdot \cos\theta]$

$\rho^2 = 8z$

$\rho^{\cancel{2}} = 8 \cdot \cancel{\rho} \cos\theta$

$$(b) = \int_0^{2\pi} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_0^{8\cdot\cos\theta} \rho^2 \cdot \sin\theta \, d\rho \, d\theta \, d\varphi =$$

$$= \ldots = \frac{16}{3}\pi$$

$$\boxed{(a) + (b) = \frac{80}{3}\pi}$$

$$\iint\limits_A f(x,y)\,dA, \quad A: x^2+y^2 \le 15+2x$$

$$x^2+y^2 \le 10x+8y-25$$

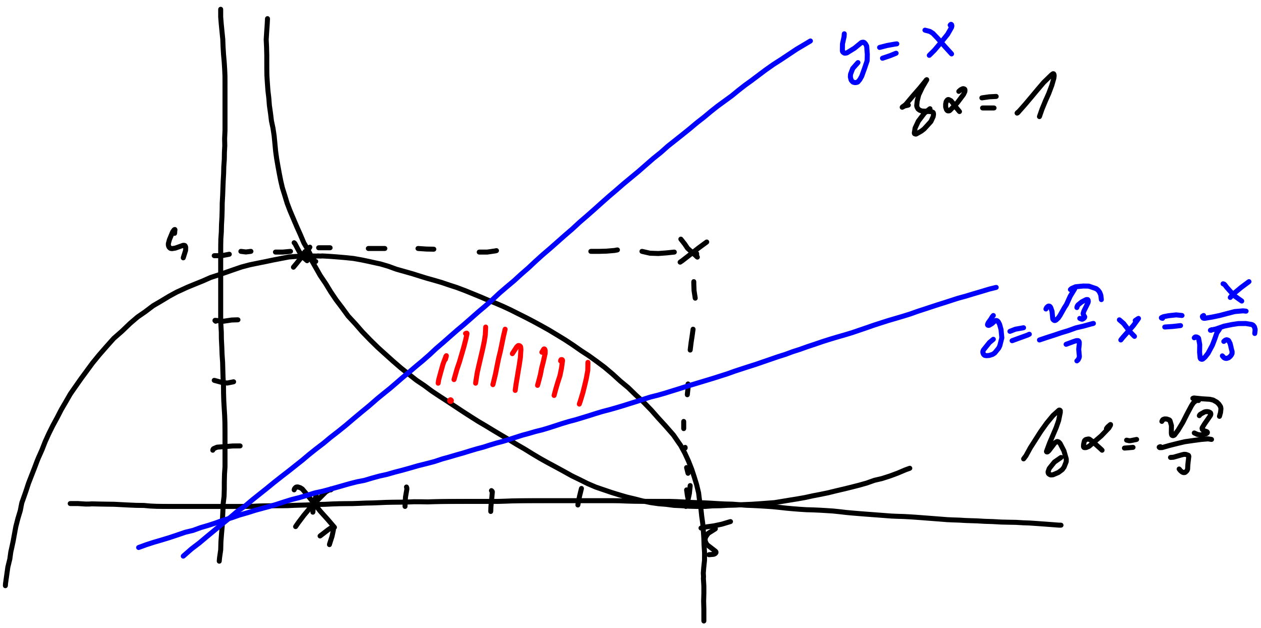
$$y \ge \frac{\sqrt{3}}{3}x, \quad y \le x$$

$$x^2-2x+y^2 \le 15$$

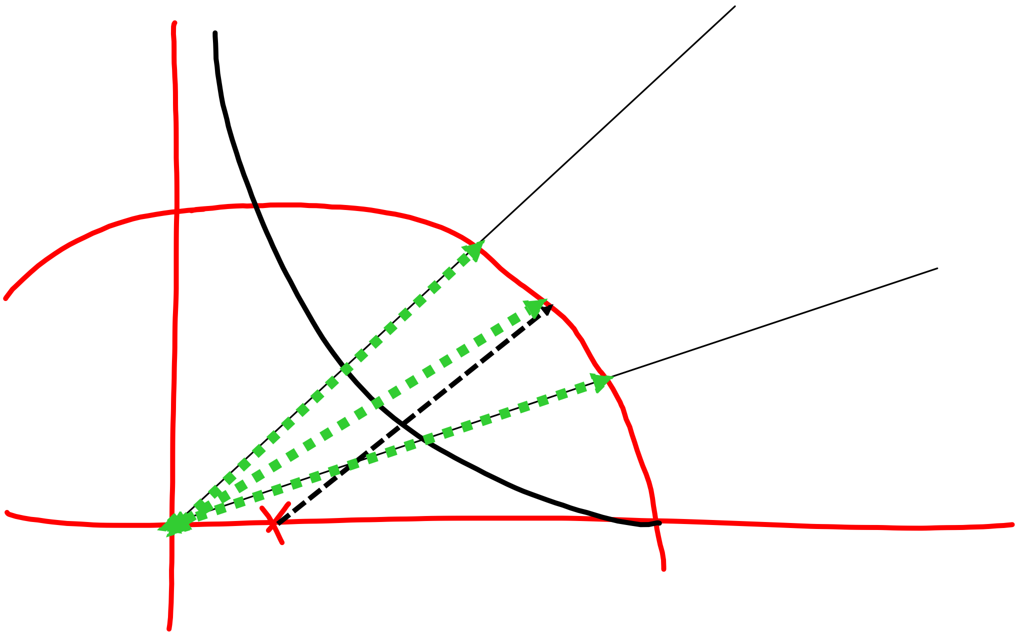
$$(x-1)^2+y^2 \le 16$$

$$x^2-10x+y^2-8y \le -25$$

$$(x-5)^2+(y-4)^2 \le 16$$



y = x
tg α = 1
y = √3/3 x = x/√3
tg α = √3/3



$x = \rho \cdot \cos\varphi$

$y = \rho \cdot \sin\varphi$

$|J| = \rho$

$\varphi \in [\frac{\pi}{3}, \frac{\pi}{4}]$

$\rho \in [?, \boxed{?}]$

$\boxed{?}$ $x^2+y^2 \leq 15+2x$

$\rho^2 \leq 15 + 2\cdot\rho\cdot\cos\varphi$

$\rho^2 - 2\cdot\rho\cdot\cos\varphi - 15 = 0$

$D = 4\cdot\cos^2\varphi + 60 > 0$

$$\rho_{1,2} = \frac{2\cdot\cos\varphi \pm \sqrt{4\cdot\cos^2\varphi+60}}{2}$$

$= \boxed{\cos\varphi + \sqrt{\cos^2\varphi+15}}$

$= \cancel{\cos\varphi - \sqrt{\cos^2\varphi+15}}$

≤ 1 $\quad < 0 \quad$ > 1

2. $x^2+y^2 \leq 10x+8y-25$

$\rho^2 \leq 10\rho\cdot\cos\varphi+8\cdot\rho\cdot\sin\varphi-25$

$\rho^2-(10\cdot\cos\varphi+8\cdot\sin\varphi)\cdot\rho+25=0$

$\underbrace{D=(10\cos\varphi+8\cdot\sin\varphi)^2-100 \geq 0 \ ?}$

$$D'(\varphi) = 2\cdot(10\cdot\cos\varphi + 8\cdot\sin\varphi)\cdot(-10\sin\varphi + 8\cdot\cos\varphi) =$$

$$= -200\cdot\cos\varphi\cdot\sin\varphi + 160\cos^2\varphi - 160\sin^2\varphi$$

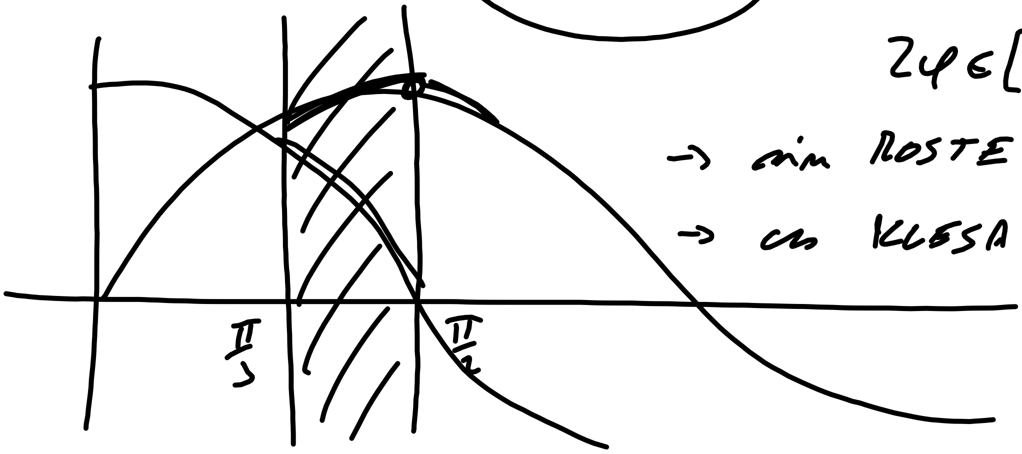
$$+ 128\sin\varphi\cdot\cos\varphi =$$

$$= -72\cdot\sin\varphi\cdot\cos\varphi + 160\cdot(\cos^2\varphi - \sin^2\varphi) =$$

$$= -36\cdot\sin 2\varphi + 160\cdot\cos 2\varphi$$

$D'(\varphi) = 160 \cdot \cos 2\varphi - 36 \cdot \sin 2\varphi \quad , \quad \varphi \in \left[\frac{\pi}{6}, \frac{\pi}{4}\right]$

$2\varphi \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right]$



→ sin ROSTE

→ cos KLESÁ

$$\sin\frac{\pi}{3} \leq \sin 2\varphi \leq \sin\frac{\pi}{2}$$
$$\cos\frac{\pi}{3} \geq \cos 2\varphi \geq \cos\frac{\pi}{2}$$

$$\frac{\sqrt{3}}{2} \leq \sin 2\varphi \leq 1 \quad / \cdot (-1)$$
$$\frac{1}{2} \geq \cos 2\varphi \geq 0$$

$$-\frac{\sqrt{3}}{2} \geq -\sin 2\varphi \geq -1$$
$$\frac{1}{2} \geq \cos 2\varphi \geq 0$$

$$160 \cdot 0 + 36 \cdot (-1) \leq 160 \cdot \cos 2\varphi + 36 \cdot (-\sin 2\varphi) \leq$$
$$\leq 160 \cdot \frac{1}{2} + 36 \cdot \left(-\frac{\sqrt{3}}{2}\right)$$

$$36 \leq D'(\varphi) \leq 80 - 18\sqrt{3} \quad (\doteq -48{,}82)$$

$$D'(\varphi) = \underbrace{160 \cos 2\varphi - 36 \cdot \sin 2\varphi = 0}$$

$$\operatorname{tg} 2\varphi = \frac{\sin 2\varphi}{\cos 2\varphi} = \frac{160}{36} = \frac{40}{9}$$

$$2\varphi \doteq \operatorname{arctg} \frac{40}{9} \doteq 1{,}349$$

$$\varphi \doteq 0{,}215\pi$$

GLOB. EXTR.: STAC. BOD $\varphi_0 = 0{,}215\pi$

HRAN. B. $\frac{\pi}{4}, \frac{\pi}{6}$

$$\left.\begin{array}{l} D(\varphi_0) = \ldots = 64 \\ D\left(\frac{\pi}{6}\right) = \ldots = 60 \\ D\left(\frac{\pi}{4}\right) = \ldots = 62 \end{array}\right\} > 0 \quad \min > 0$$

$\Rightarrow 2\pi$ <u>KOŽ.</u>

$$\varrho_{1,2} = \frac{(10\cdot\cos\varphi + 8\cdot\sin\varphi) \pm \sqrt{(10\cdot\cos\varphi + 8\cdot\sin\varphi)^2 - 100}}{2}$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \int_{5\cdot\cos\varphi + 4\cdot\sin\varphi - \sqrt{(5\cdot\cos\varphi + 4\cdot\sin\varphi)^2 - 25}}^{\cos\varphi + \sqrt{\cos^2\varphi + 15}} f(\varrho\cdot\cos\varphi, \varrho\cdot\sin\varphi)\cdot\varrho \; d\varrho \, d\varphi$$