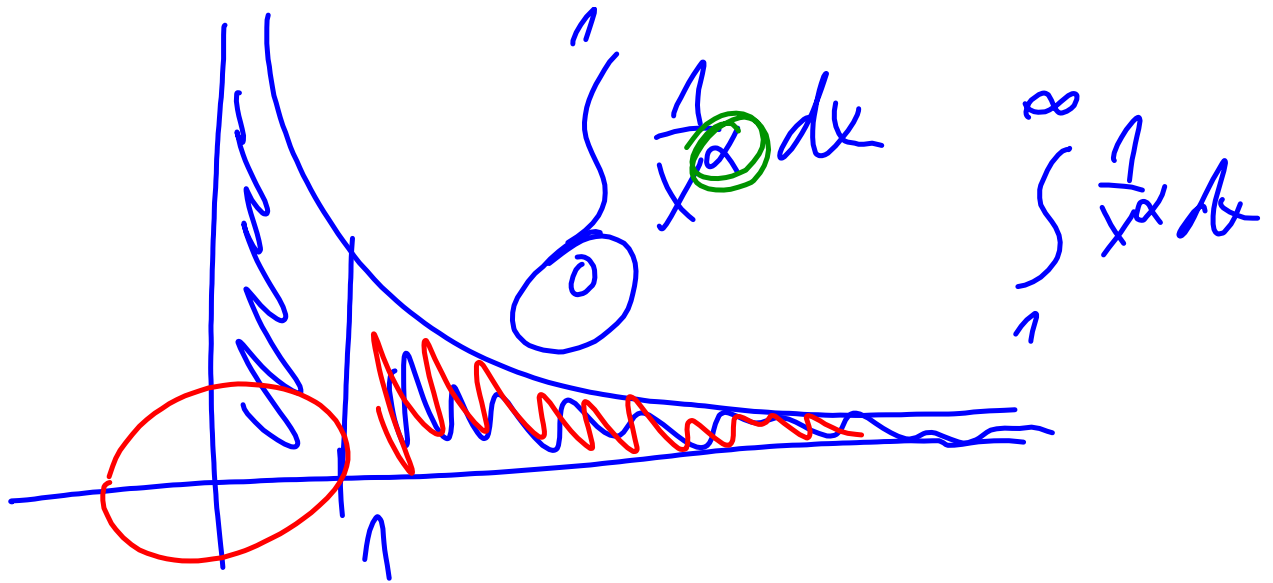
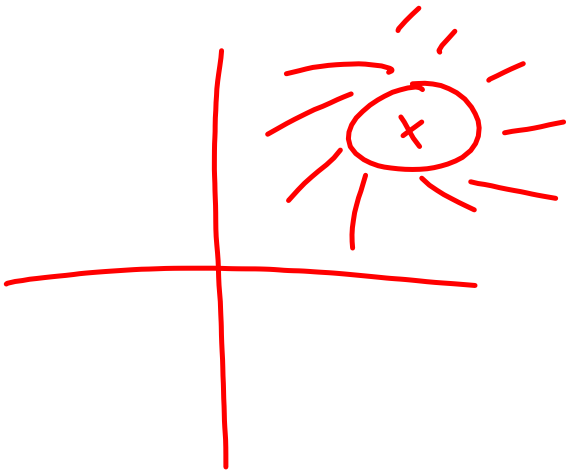


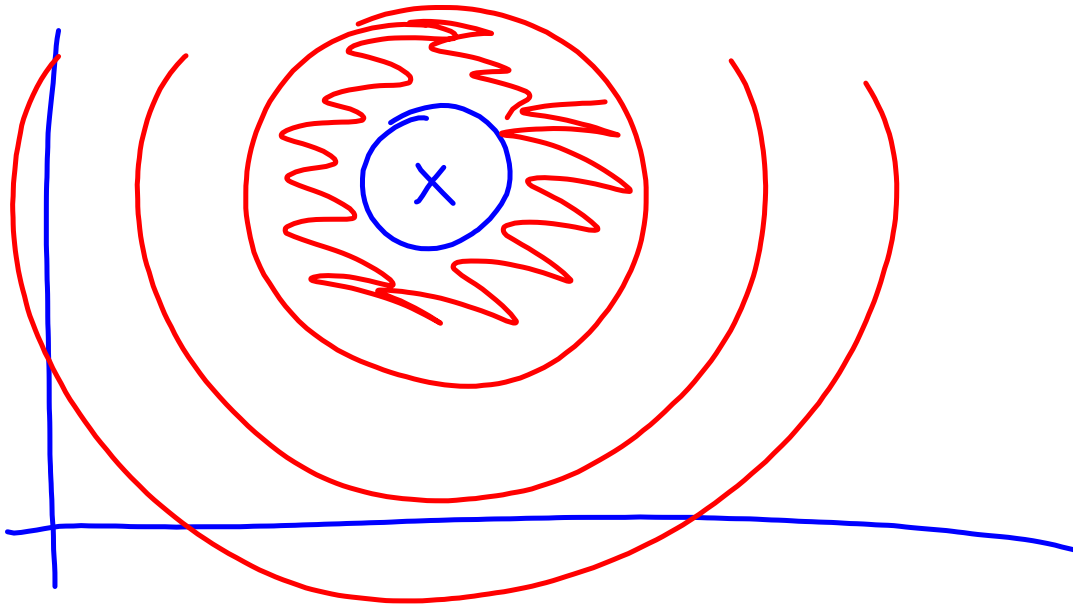
$$\iint \sin x^2 \cdot \cos y^2 dx dy$$

(Note: In the original image, the term $\cos y^2$ is circled in red, and a red arrow points from it to the $\int \cos y^2$ term in the second equation. A green box encloses the $\int \sin x^2 dx$ term, with a green arrow pointing from the $dx dy$ of the first equation to the dx of the boxed term.)

$$\int \cos y^2 \left[\int \sin x^2 dx \right] dy$$





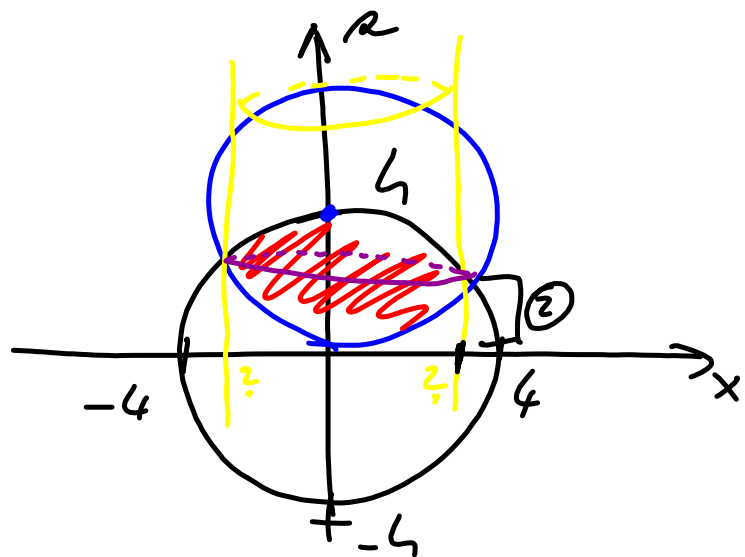


$$V = ?$$

$$\left. \begin{aligned} x^2 + y^2 + z^2 &= 16 \\ x^2 + y^2 + z^2 &= 8z \end{aligned} \right\} \textcircled{*}$$

$$x^2 + y^2 + z^2 - 8z = 0$$

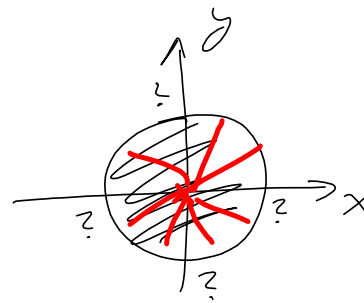
$$x^2 + y^2 + (z - 4)^2 = 16$$



a) VALCOLÉ'

$$\varphi \in [0, 2\pi]$$

$$\rho \in [0, \sqrt{12}]$$



$$\begin{array}{l|l} 8a = 16 & x^2 + y^2 + 4 = 16 \\ a = 2 & x^2 + y^2 = 12 \end{array}$$

$$r \in [4 - \sqrt{16 - \rho^2}, \sqrt{16 - \rho^2}]$$

Dokazí: $(r-4)^2 = 16 - x^2 - y^2$

$$r - 4 = \pm \sqrt{16 - x^2 - y^2}$$

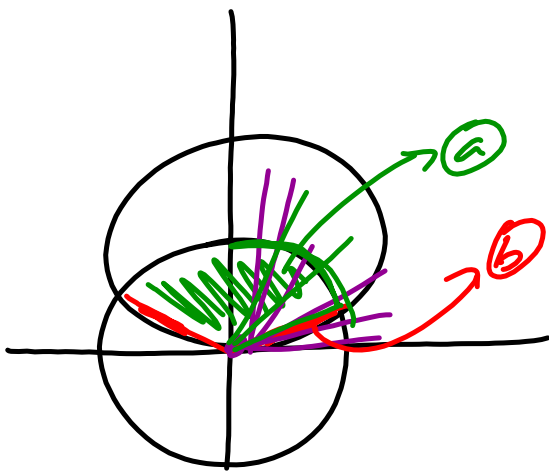
$$r = \pm \sqrt{16 - x^2 - y^2} + 4$$

Hořai: $r^2 = 16 - x^2 - y^2$

$$r = \pm \sqrt{16 - x^2 - y^2}$$

$$\int_0^{\sqrt{12}} \int_0^{2\pi} \int_{4-\sqrt{16-\rho^2}}^{\sqrt{16-\rho^2}} \rho \cdot 1 \, dz \, d\varphi \, d\rho = \dots$$

$$\dots = \underline{\underline{\frac{80}{3} \pi}}$$



a) $\varphi \in [0, 2\pi]$

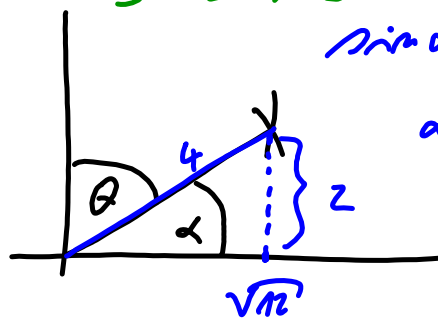
$\theta \in [0, \frac{\pi}{3}]$

$\rho \in [0, 4]$

$\sin \alpha = \frac{2}{4} = \frac{1}{2}$

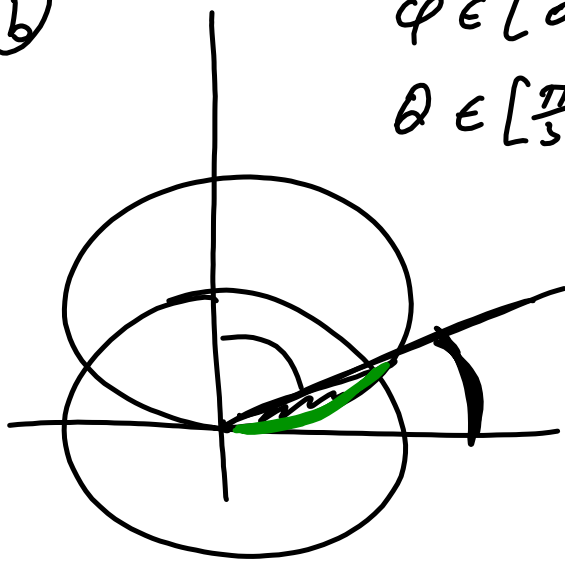
$\alpha = \frac{\pi}{6}$

$\theta = \frac{\pi}{2} - \frac{\pi}{6}$



$$\textcircled{a} = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^4 r^2 \cdot \sin\theta \, dr \, d\theta \, d\varphi = \dots = \underline{\underline{\frac{64}{3}\pi}}$$

⑥



$$\begin{array}{l} \varphi \in [0, 2\pi] \\ \theta \in [\frac{\pi}{3}, \frac{\pi}{2}] \end{array} \Bigg| \begin{array}{l} \rho \in [0, 8 \cdot \cos \theta] \\ \rho^2 = 8r \end{array}$$

$$\rho^* = 8 \cdot \cos \theta$$

$$\textcircled{b} = \int_0^{2\pi} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_0^{2 \cos \theta} \rho^2 \sin \theta \, d\rho \, d\theta \, d\varphi =$$
$$= \dots = \frac{16}{3} \pi$$

$$\textcircled{a} + \textcircled{b} = \frac{80}{3} \pi$$

$$\iint_A f(x,y) dA, \quad A: x^2 + y^2 \leq 15 + 2x$$

A

$$x^2 + y^2 \leq 10x + 8y - 25$$

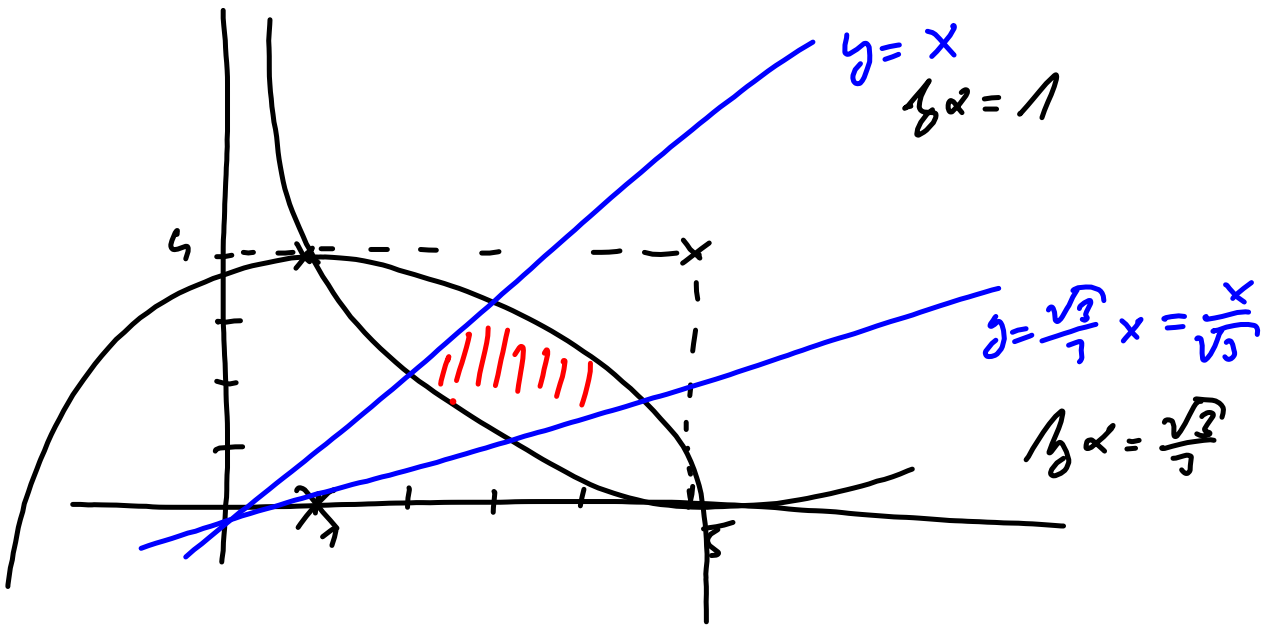
$$y \geq \frac{\sqrt{3}}{3}x, \quad y \leq x$$

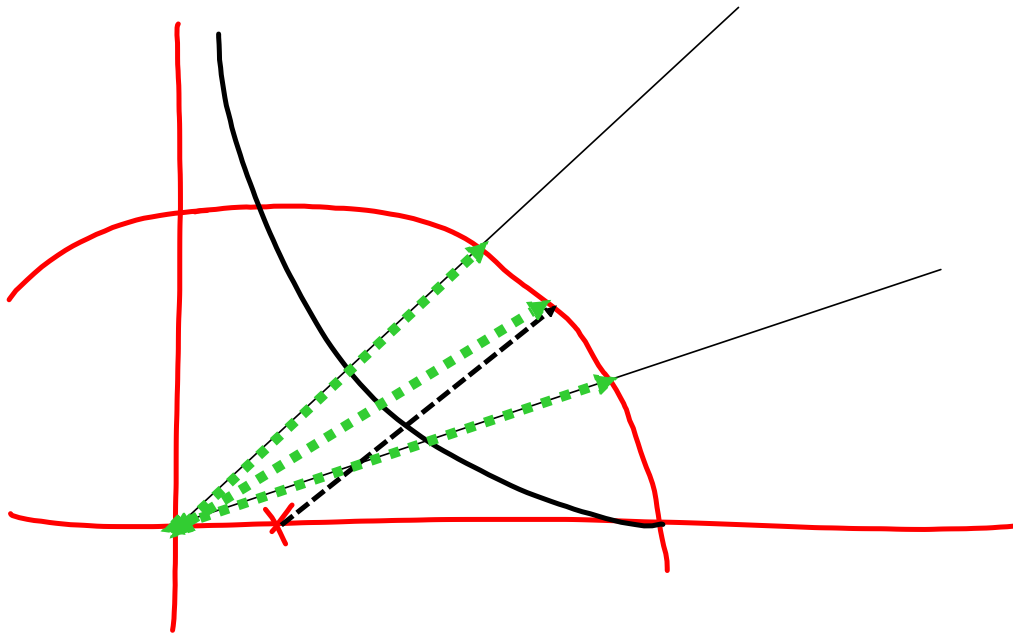
$$x^2 - 2y + y^2 \leq 15$$

$$(x-1)^2 + y^2 \leq 16$$

$$x^2 - 10x + y^2 - 8y \leq -25$$

$$(x-5)^2 + (y-4)^2 \leq 16$$





$$\begin{array}{l|l} x = \rho \cdot \cos \varphi & \varphi \in \left[\frac{\pi}{3}, \frac{\pi}{4} \right] \\ y = \rho \cdot \sin \varphi & \\ |\rho| = \rho & \rho \in [\textcircled{?}, \boxed{?}] \end{array}$$

$$\boxed{?} \quad x^2 + y^2 \leq 15 + 2y$$

$$y^2 \leq 15 + 2y - x^2$$

$$y^2 - 2y - x^2 - 15 = 0$$

$$D = 4 \cdot x^2 + 60 \quad \text{Ⓢ}$$

$$y_{1,2} = \frac{2 \pm \sqrt{4x^2 + 60}}{2}$$

$$= \boxed{x \pm \sqrt{x^2 + 15}}$$

$$= \cancel{x - \sqrt{x^2 + 15}}$$

$\swarrow \quad \quad \quad \searrow$
 $\leq 1 \quad \quad \quad < 0 \quad \quad \quad > 1$

$$\textcircled{2} \quad x^2 + y^2 \leq 10x + 8y - 25$$

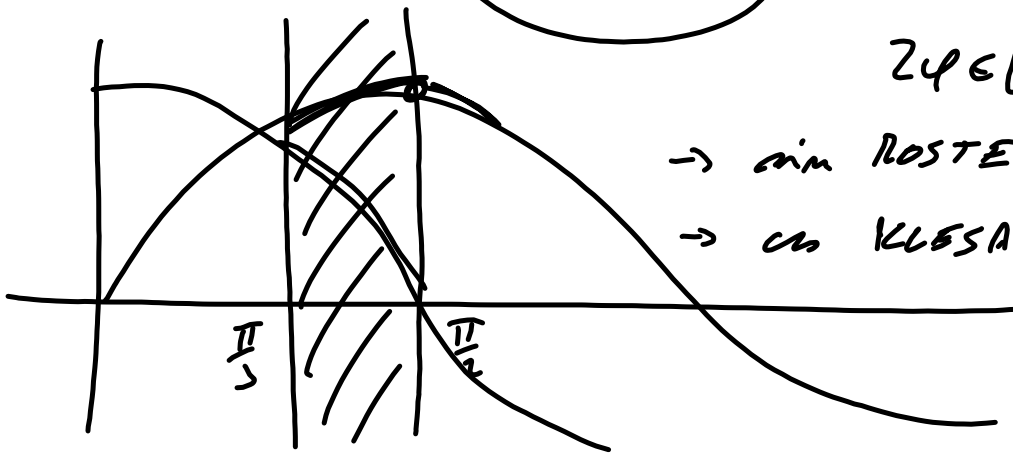
$$\rho^2 \leq 10\rho \cdot \cos \varphi + 8\rho \cdot \sin \varphi - 25$$

$$\rho^2 - (10 \cos \varphi + 8 \sin \varphi) \rho + 25 = 0$$

$$D = (10 \cos \varphi + 8 \sin \varphi)^2 - 100 \geq 0 \quad \text{?}$$

$$\begin{aligned}
 D'(q) &= 2 \cdot (10 \cdot \underline{\cos q} + 8 \cdot \underline{\sin q}) \cdot (-10 \underline{\sin q} + 8 \cdot \underline{\cos q}) = \\
 &= -200 \cdot \cos q \cdot \sin q + 160 \cos^2 q - 160 \sin^2 q \\
 &\quad + 128 \sin q \cdot \cos q = \\
 &= -72 \cdot \sin q \cdot \cos q + 160 \cdot (\cos^2 q - \sin^2 q) = \\
 &= -36 \cdot \sin 2q + 160 \cdot \cos 2q
 \end{aligned}$$

$$D|q| = 160 \cdot \cos 2\varphi - 36 \cdot \sin 2\varphi, \quad \varphi \in \left[\frac{\pi}{6}, \frac{\pi}{4}\right]$$



$$2\varphi \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right]$$

-> sin ROSTE

-> cos KLESA'

$$\sin \frac{\pi}{3} \leq \sin 2\varphi \leq \sin \frac{\pi}{2}$$

$$\cos \frac{\pi}{3} \geq \cos 2\varphi \geq \cos \frac{\pi}{2}$$

$$\frac{\sqrt{3}}{2} \leq \sin 2\varphi \leq 1 \quad (\cdot (-1))$$

$$\frac{1}{2} \geq \cos 2\varphi \geq 0$$

$$\begin{aligned} -\frac{\sqrt{3}}{2} &\geq -\sin 2\varphi \geq -1 \\ \frac{1}{2} &\geq \cos 2\varphi \geq 0 \end{aligned}$$

$$\begin{aligned} 160 \cdot 0 + 36 \cdot (-1) &\leq 160 \cdot \cos 2\varphi + 36 \cdot (-\sin 2\varphi) \leq \\ &\leq 160 \cdot \frac{1}{2} + 36 \cdot \left(-\frac{\sqrt{3}}{2}\right) \end{aligned}$$

$$36 \leq D|q| \leq 80 - 18\sqrt{3} \quad (\approx -48,823)$$

$$D'(q) = \underbrace{160 \cos^2 q - 36 \cdot \sin^2 q = 0}$$

$$\tan^2 q = \frac{\sin^2 q}{\cos^2 q} = \frac{160}{36} = \frac{40}{9}$$

$$2q = \arctan \frac{40}{9} \doteq 1,349$$

$$q \doteq 0,215 \pi$$

$$S_{1,2} = \frac{(10 \cdot \cos \varphi + 8 \cdot \sin \varphi) \pm \sqrt{(10 \cdot \cos \varphi + 8 \cdot \sin \varphi)^2 - 100}}{2}$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \int_{5 \cdot \cos \varphi + 4 \cdot \sin \varphi - \sqrt{(5 \cdot \cos \varphi + 4 \cdot \sin \varphi)^2 - 25}}^{5 \cdot \cos \varphi + \sqrt{5 \cdot \cos^2 \varphi + 15}} f(r, \varphi) \cdot r \, dr \, d\varphi$$

