1.) $D(f) = \mathbb{R}^2 \setminus \{\underline{[0,0]}, \dots\}$

$\downarrow$

JE HR. B. ✓

$$\lim_{[x,y]\to[0,0]} \frac{3x}{x^3+y} = \left| \begin{array}{l} x = \varrho\cdot\cos\varphi \\ y = \varrho\cdot\sin\varphi \end{array} \right| =$$

$$= \lim_{\varrho\to 0^+} \frac{3\cdot\cancel{\varrho}\cdot\cos\varphi}{\cancel{\varrho^3}^{\,2}\cdot\cos^3\varphi + \cancel{\varrho}\cdot\sin\varphi} =$$

LIM. NEEX

$\Uparrow$

$$= \lim_{\varrho\to 0^+} \frac{3\cdot\cos\varphi}{\varrho^2\cdot\cos^3\varphi + \sin\varphi} = \frac{3\cdot\cos\varphi}{\sin\varphi} \to \text{ZÁVISÍ NA } \varphi$$

2.) $$\underbrace{1 - x\cdot\cos(2y)}_{P(x,y)}\,dx + \underbrace{x^2 \sin 2y}_{Q(x,y)}\,dy$$

$$P_y \overset{?}{=} Q_x \quad \left|\quad \begin{array}{l} P_y = x\cdot\sin(2y)\cdot 2 \\ Q_x = \sin(2y)\cdot 2x \end{array}\right\} \checkmark$$

$$K(x,y) = \int \left(1 - x\cdot\cos(2y)\right)\,dx =$$

$$= x - \cos(2y)\cdot\frac{x^2}{2} + C(y)$$

$$K_y = \frac{x^2}{\cancel{2}}\cdot\sin(2y)\cdot\cancel{2} + C_y'(y) = x^2\cdot\sin(2y)$$

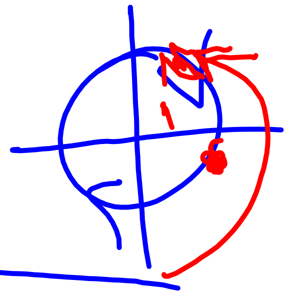
$$C_y' = 0 \qquad \int 0\,dy = c$$

$$K(x,y) = x - \frac{x^2}{2}\cos(2y) + c\,,\quad c\in\mathbb{R}$$

$$\text{Pot.} = -K$$

8.) PARAM. $\underline{x = 3\cdot\cos\varphi}$

$y = 3\cdot\sin\varphi$



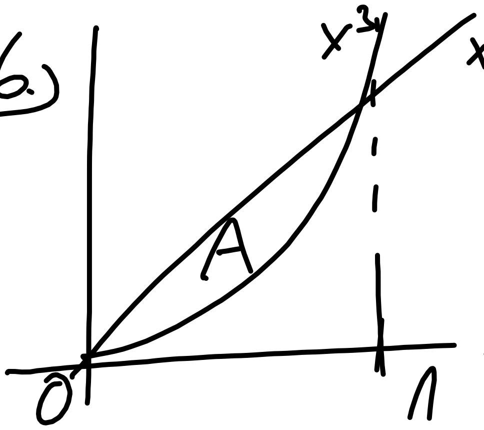
$$I = -\int_0^{2\pi} \frac{3\cdot\cos\varphi + 3\cdot\sin\varphi}{3^2\cdot\cos^2\varphi + 3^2\cdot\sin^2\varphi}\cdot 3\cdot(-\sin\varphi) +$$

$$+ \frac{3\cdot\cos\varphi - 3\cdot\sin\varphi}{3^2\cdot\cos^2\varphi + 3^2\cdot\sin^2\varphi}\, 3\cdot\cos\varphi \; d\varphi =$$

$$= \int_0^{2\pi} \cancel{\cos\varphi\sin\varphi} + \underbrace{\sin^2\varphi + \cos^2\varphi}_{=1} - \cancel{\sin\varphi\cos\varphi}\; d\varphi$$

$$= 2\pi$$

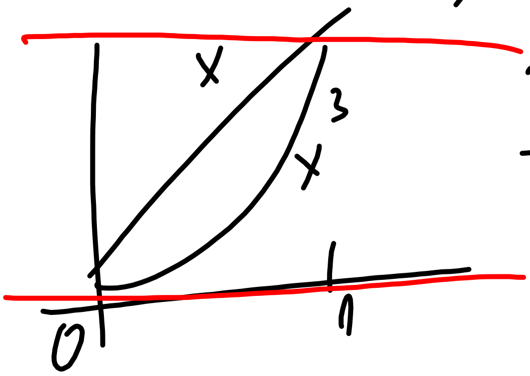
6.)



$$I=\int_0^1\int_{x^3}^{x} x^3 y \, dy \, dx = \int_0^1 x^3 \cdot \left[\frac{y^2}{2}\right]_{x^3}^{x} dx =$$

$$=\frac{1}{2}\int_0^1 x^5 - x^9 \, dx = \frac{1}{2}\cdot\left[\frac{x^6}{6}-\frac{x^{10}}{10}\right]_0^1 =$$

$$=\frac{1}{2}\cdot\left(\frac{1}{6}-\frac{1}{10}\right)-\frac{1}{2}\cdot 0 = \frac{1}{2}\cdot\frac{5-3}{30} = \underline{\underline{\frac{1}{30}}}$$



$$I=\int_0^1\int_y^{\sqrt[3]{y}} x^3 y \, dx \, dy$$

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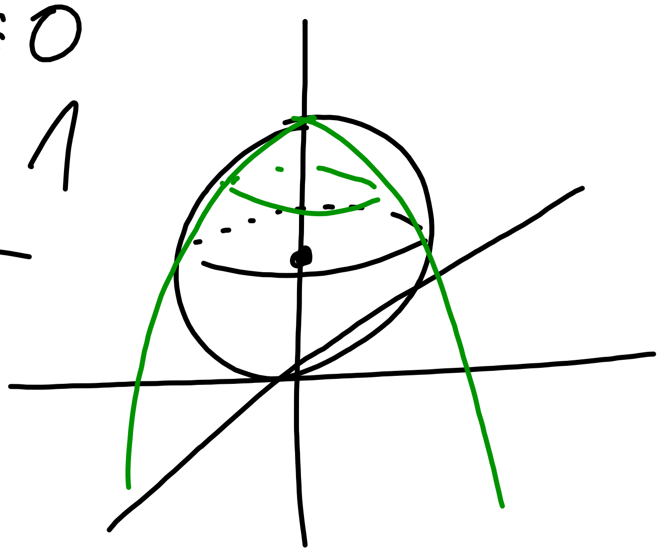
7.) $x^2+y^2+z^2-2z \leq 0$

$x^2+y^2+(z-1)^2 \leq 1$

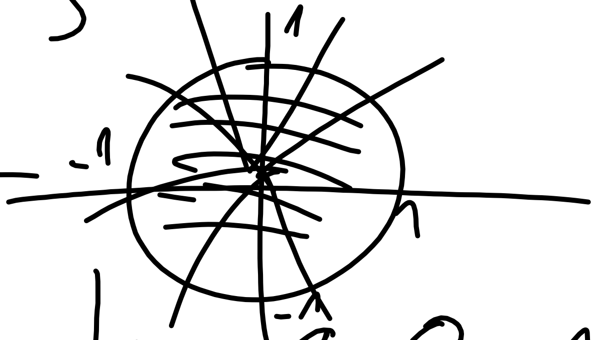
$z-2 \leq -x^2-y^2$



$$x = \rho \cdot \cos\varphi \qquad \varphi \in [0, 2\pi]$$

$$y = \rho \cdot \sin\varphi \qquad \rho \in [0, 1]$$

$$z = z$$



$$x^2 + y^2 = 2 - z$$

$$2 - z + z^2 = 2z \qquad D = 9 - 8 = 1$$

$$z^2 - 3z + 2 = 0 \qquad z_{1,2} = \frac{3 \pm 1}{2} = \begin{cases} = 2 \\ = 1 \end{cases}$$

$$z \in \left[1-\sqrt{1-\rho^2}, 2-\rho^2\right]$$

$$x^2+y^2+(z-1)^2=1$$

$$\rho^2+(z-1)^2=1$$

$$z-1=\pm\sqrt{1-\rho^2}$$

$$z=1\pm\sqrt{1-\rho^2}$$

$$z=2-(x^2+y^2)$$

$$z=2-\rho^2$$

$$V=\int_0^1\int_0^{2\pi}\int_{1-\sqrt{1-\rho^2}}^{2-\rho^2} 1\cdot\rho \, dz\, d\varphi\, \cancel{d\rho}$$

$$=\int_0^1\int_0^{2\pi} \rho\cdot\left(\underbrace{2-\rho^2-1}_{1-\rho^2}+\sqrt{1-\rho^2}\right) d\varphi\, d\rho$$

$$\left|\begin{array}{l} t=1-\rho^2 \\ dt=-2\rho\, d\rho \end{array}\right| = -\frac{1}{2}\int_1^0\int_0^{2\pi} t+\sqrt{t}\, d\varphi\, dt$$

$$=\frac{1}{2}\cdot\int_0^1 (t+\sqrt{t})\cdot[\varphi]_0^{2\pi}\, dt = \pi\cdot\left[\frac{t^2}{2}+\frac{2}{3}t^{\frac{3}{2}}\right]_0^1$$

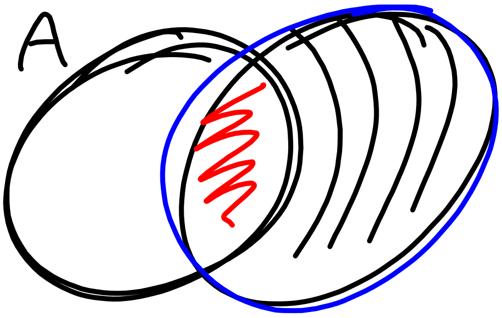
$$=\pi\cdot\left(\frac{1}{2}+\frac{2}{3}\right)=\pi\,\frac{7}{6}$$

5.) (ii) $A \cap B = \emptyset$

$$m_*(A) + m_*(B) \leq m_*(A \cup B) \leq$$

$$\leq m^*(A \cup B) \leq m^*(A) + m^*(B)$$

(ii) $m(A\cap B) > 0$



$$A\cup B = \underbrace{A\cup(B\setminus A)}$$

$$m(A\cup B) = m(A) + m(B\setminus A)$$

$$B\setminus A = B\setminus(A\cap B)$$

$$B = (B\setminus A)\cup(A\cap B)$$

$$m(B) = m(B\setminus A) + m(A\cap B)$$

$$m(A\cup B) = m(A) + m(B) - \cancel{m(A\cap B)}$$

3) $f_x = 4x \cdot e^{-x^2-y^2} + (2x^2+3y^2)\cdot e^{-x^2-y^2}\cdot(-2x)$

$= e^{-x^2-y^2}\cdot 2x\cdot(2-2x^2-3y^2)$

$f_y = 6y\cdot e^{-x^2-y^2} + (2x^2+3y^2)\, e^{-x^2-y^2}\cdot(-2y)$

$= 2y\cdot e^{-x^2-y^2}\cdot(3-2x^2-3y^2)$

STAC. B.

$f_x = 0$	$x \cdot (2-2x^2-3y^2) = 0$
$f_y = 0$	$y \cdot (3-2x^2-3y^2) = 0$
$[0,0]$	$2x^2+3y^2 = 2 \wedge = 3$ $\times$

$x=0 \Rightarrow 3-3y^2=0, \quad y^2=1 \quad [0,\pm 1]$

$y=0 \Rightarrow 2-2x^2=0, \quad x^2=1 \quad [\pm 1, 0]$

$[0,0], [\pm 1, 0], [0, \pm 1]$

$$D^2 f(x,y) = D(x,y) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

$$D(0,0) = \begin{pmatrix} 4 & 0 \\ 0 & 6 \end{pmatrix}$$

POZ. DEF. $\Rightarrow$ L. MIN.

$$D(0,\pm 1) = \begin{pmatrix} -\frac{2}{e} & 0 \\ 0 & -\frac{12}{e} \end{pmatrix}$$

NEG. DEF. $\Rightarrow$
$\Rightarrow$ L. MAX

$$D(\pm 1, 0) = \begin{pmatrix} -\frac{8}{e} & 0 \\ 0 & \frac{2}{e} \end{pmatrix}$$

INDEF.
$\Downarrow$
NENÍ

kvě 10-19:24

4.) $L(x, y, \lambda) = \cos^2 x + \cos^2 y + \lambda \cdot (4x - 4y - \pi)$

$L_x = 2 \cdot \cos x \cdot (-\sin x) + 4\lambda = 0$

$L_y = 2 \cdot \cos y \cdot (-\sin y) - 4\lambda = 0$

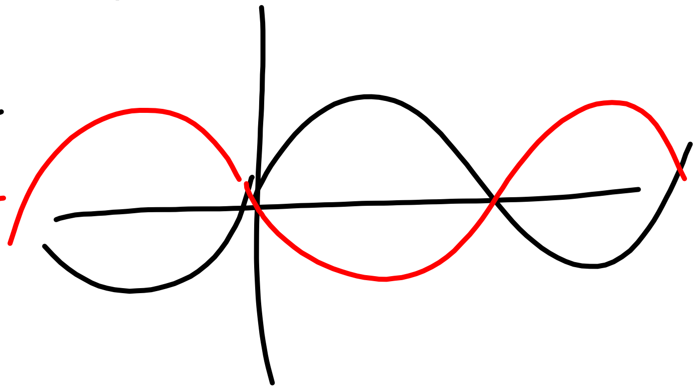
$L_\lambda = 4x - 4y - \pi = 0$

$$\sin 2x = 4\lambda \quad \ldots (1)$$
$$\sin 2y = -4\lambda \quad \ldots (2)$$
$$4x - 4y = \pi \quad \ldots (3)$$

$$(1)+(2) \Rightarrow \sin 2x + \sin 2y = 0$$

$$\sin 2x = -\sin 2y$$

$$2x = -2y + 2k\pi$$



$x = -y + k\pi$ ... DOS. DO (3)

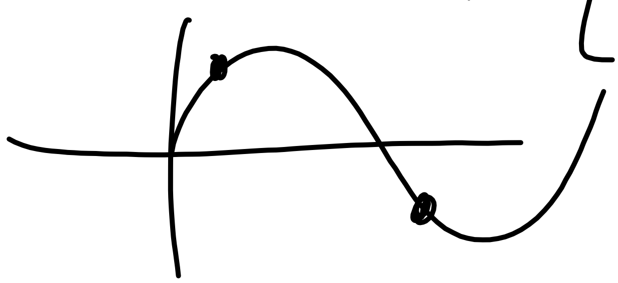
$$4 \cdot (-y + k\pi) - 4y = \pi$$

$$-8y = \pi - 4k\pi$$

$$y = -\frac{\pi}{8} + k\frac{\pi}{2}$$

$$\left[\frac{\pi}{8} + k\frac{\pi}{2}, -\frac{\pi}{8} + k\frac{\pi}{2}\right] \quad (1) \Rightarrow \lambda =$$

$$= \frac{1}{4} \cdot \sin\left(\frac{\pi}{4} + k\pi\right) = \begin{cases} = \frac{1}{4} \cdot \frac{\sqrt{2}}{2}, & k \text{ SUDÉ} \\ = \frac{1}{4} \cdot \left(-\frac{\sqrt{2}}{2}\right), & k \text{ LICHÉ} \end{cases}$$



$$D^2 \mathcal{L}(\text{SUDÉ } 2) = \begin{pmatrix} -\sqrt{2} & 0 \\ 0 & -\sqrt{2} \end{pmatrix}$$

NEG. D.

MAX.

LICHÉ k

$k = 2m+1$

$$\nabla^2 L = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$$

POZ. DEF.

$\Rightarrow$ MIN.