$\vec{v}_1$

$[\vec{v}_1] = V_1$

$\|A\vec{v}_1\|$

W_k

$W_k \cap V_{k-1}^{\perp} = \{0\}$

$\cancel{k} + n - \cancel{k} + 1$

$= \underline{n+1}$

$$\begin{pmatrix} \sigma_1 & & & \\ & \ddots & & 0 \\ & & \sigma_r & \\ 0 & & & 0 \end{pmatrix} \qquad \underline{A = P\Sigma Q^T}$$

$$A \doteq \sum_{j=1}^{?} \sigma_j u_j v_j^T \quad / \, v_i$$

$$A v_i = \sigma_i u_i \qquad u_i \not\perp u_k$$

$$u_i \perp u_k \quad i \neq k \qquad i < k$$

$$u_i^T \cdot u_k \neq 0$$

$$A \cdot \frac{(v_i + \epsilon v_k)}{\sqrt{1+\epsilon^2}} = \frac{\sigma_i u_i + \epsilon \sigma_k u_k}{\sqrt{1+\epsilon^2}}$$

$$\frac{v_i + \varepsilon v_j}{\sqrt{1+\varepsilon^2}} \perp v_1, \dots, v_{i-1}$$

$$\left\| \frac{A v_i + \varepsilon A v_j}{\sqrt{1+\varepsilon^2}} \right\|^2 = \frac{\sigma_i^2 + 2\varepsilon \cancel{u_i^T u_j}^{0} + \varepsilon^2 \sigma_j^2}{1+\varepsilon^2}$$

$$(1+\varepsilon^2)\overset{\sigma_i^2}{\overset{\vee}{\sigma_i^2}} < \sigma_i^2 + \varepsilon^2 \sigma_j^2 + 2\varepsilon u_i^T u_j$$

$$\varepsilon^2 (\underbrace{\sigma_i^2 - \sigma_j^2}_{\geq 0}) < 2\varepsilon u_i^T u_j$$

$$\left(\begin{array}{c|c} \begin{matrix}1 & 0\\ 0 & 1\end{matrix} & 0 \\ \hline 0 & 0 \end{array}\right)$$

1) $A^{(-1)} = A^{-1}$

2) $\left(A^{(-1)}\right)^{(-1)} = A$

, $A = P \cdot \left(\begin{array}{c|c} D & 0 \\ \hline 0 & 0 \end{array}\right) Q^*$

$A^{(-1)} = Q \left(\begin{array}{c|c} D^{-1} & 0 \\ \hline 0 & 0 \end{array}\right) P^*$

", $\left(A^{(-1)}\right)^{(-1)} = P \left(\begin{array}{c|c} D & 0 \\ \hline 0 & 0 \end{array}\right) Q^* = A$

3) $Q\left(\begin{array}{c|c} D^{-1} & 0 \\ \hline 0 & 0 \end{array}\right) P^* P \left(\begin{array}{c|c} D & 0 \\ \hline 0 & 0 \end{array}\right) Q^*$

$= Q\left(\begin{array}{c|c} \begin{matrix} 1 & 0 \\ 0 & 1 \\ & & 1 \end{matrix} & 0 \\ \hline 0 & 0 \end{array}\right) Q^*$

4) $\varphi(x) = Ax$, $\varphi^{(-1)}(y) = A^{(-1)}y$

$\varphi^{(-1)} \circ \varphi = A^{(-1)} A x \quad (\mathrm{Ker}(\varphi))^{\perp}$

$x - A^{(-1)}Ax \perp (\mathrm{Ker}\,\varphi)^{\perp}$

$\Leftrightarrow \quad x - A^{(-1)}Ax \in \mathrm{Ker}\,\varphi$

$\Leftrightarrow Ax - AA^{(-1)}Ax = 0 \Leftrightarrow$

$A = AA^{(-1)}$

$$P\left(\begin{array}{c|c} D & 0 \\ \hline 0 & 0 \end{array}\right)\underline{Q^*\,Q}\left(\begin{array}{c|c} D^{-1} & 0 \\ \hline 0 & 0 \end{array}\right)\underline{P^*P}\left(\begin{array}{c|c} D & 0 \\ \hline 0 & 0 \end{array}\right)Q^*$$

$$P\left(\begin{array}{ccc|c} 1 & & & 0 \\ & \ddots & & \\ & & 1 & \\ \hline 0 & & & 0 \end{array}\right)\cdot\left(\begin{array}{c|c} D & 0 \\ \hline 0 & 0 \end{array}\right)Q^* = A.$$

$$A^*A = Q\left(\begin{array}{c|c} D^* & 0 \\ \hline 0 & 0 \end{array}\right) P^* P \left(\begin{array}{c|c} D & 0 \\ \hline 0 & 0 \end{array}\right) Q^*$$

$$= Q\left(\begin{array}{ccc|c} \lambda_1^2 & & & \\ & \ddots & & 0 \\ & & \lambda_r^2 & \\ \hline & & & \end{array}\right) Q^*$$

$$(A^*A)^{-1} = Q\left(\begin{array}{ccc|c} \frac{1}{\lambda_1^2} & & 0 & 0 \\ & \ddots & & \\ & & \frac{1}{\lambda_r^2} & \\ \hline 0 & & & e \end{array}\right) \cdot Q$$

$$(A^*A)^{-1} \cdot A^* = Q\left(\begin{array}{ccc} \frac{1}{\lambda_1^2} & & 0 \\ & \ddots & \\ 0 & & 0 \end{array}\right)\left(\begin{array}{ccc|c} \ddots & & & 0 \\ & \lambda_r & & \\ \hline 0 & & & 0 \end{array}\right) P^* = A^{-1}$$

$$A = \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad A^* \cdot A = \begin{pmatrix} 2 & 2 \\ 2 & 5 \end{pmatrix}$$

$$\det A^*A = 10 - 4 = 6$$

$$(A^*A)^{-1} = \begin{pmatrix} \frac{5}{6} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{pmatrix} \qquad A^{(-1)} = \begin{pmatrix} \frac{1}{6} & \frac{5}{6} & -\frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

$$\|AA^{(-1)}b - b\|$$

$$A\cdot A^* = R\underbrace{UU^*}_{I_n}\overset{R}{\overset{\parallel}{R^*}} = R^2$$