$AA^{(-1)}$ je matice kolmé projekce z $\mathbb{K}^k$ do $\mathrm{Im}\,\varphi$

$\varphi(x)=Ax$

$$\min_{x\in\mathbb{K}^n}\|Ax-b\| = \min_{y\in\mathrm{Im}\,\varphi}\|y-b\| = \|P_{\mathrm{Im}\,\varphi}b-b\| \;!$$

kolmá projekce

$$\|P_{\mathrm{Im}\,\varphi}\, b - b\| = \| A \underbrace{A^{(-1)} b}_{\underline{\underline{x}}} - b\|$$

r, $Ar = 0$

$$A\underbrace{(\underbrace{A^{(-1)} b}_{x} + r)} = AA^{(-1)} b + \underset{0}{\underset{\parallel}{Ar}}$$

$$A = PSQ^* = \underbrace{PSP^*}_{R}\underset{I_m}{}\underbrace{PQ^*}_{U}$$

$$(PSP^*)^* = (P^*)^* \cdot S^* \cdot \overset{R}{P^*} =$$

$$= P\underbrace{\begin{pmatrix} \begin{smallmatrix} >0 & \\ & >0 \end{smallmatrix} & 0 \\ 0 & 0 \end{pmatrix}}_{S}P^* \;;\quad (PQ^*)(PQ^*)^* =$$

$$= P\underbrace{Q^*Q}_{I_m}P^* = PP^* = I_m$$

$$A = RU$$

$$\langle Rx, x\rangle = \langle P\underline{S}P^* x, x\rangle =$$

$$\langle SP^* x, P^* x\rangle = \langle Sy, y\rangle =$$

$$= \sigma_1 y_1^2 + \dots + \sigma_n y_n^2 \geq 0$$

$$AA^* = (RU)(RU)^* = R\underbrace{UU^*}_{I_n}R^* =$$

$$= R \cdot R = \underline{\underline{R^2}}$$

A je invertibilní

$\Rightarrow S = \begin{pmatrix} \rho_1 & & 0 \\ & \ddots & \\ 0 & & \rho_n \end{pmatrix}, \underset{?.}{\underline{\underline{\rho_i >}}} 0$

$\Rightarrow \exists S^{-1}$

$A = R_1 U_1 = R_2 U_2$

$AA^* = R_1^2 = R_2^2$

vlastní čísla AA^* jsou $\rho_1^2, \dots, \rho_n^2 > 0$

s vlastními vektory $u_1, \dots, u_n$

$AA^* u_i = \rho_i^2 u_i = R_1^2 u_i = R_2^2 u_i$

Odtud naz

$R_1 u_1 = \rho_1 u_1$

$\vdots$

$R_1 u_n = \rho_n u_n$

$R_2 u_1 = \rho_1 u_1$

$\vdots$

$R_2 u_n = \rho_n u_n$

$\Rightarrow R_1 = R_2$

$A = R_1 U_1$

$A = R_1 U_2$

$$\underset{U_2}{\underset{\parallel}{R_1^{-1} A}} = U_1$$

$$\left(\begin{array}{ccc|ccc|c}
\lambda_1 & 1 & 0 & & & & \\
 & \ddots & \ddots & & 0 & & \\
0 & & \lambda_1 & & & & \\ \hline
 & & & \lambda_2 & 1 & 0 & \\
 & 0 & & & \ddots & 1 & 0 \\
 & & & 0 & & \lambda_2 & \\ \hline
 & & & & & & \ddots
\end{array}\right)$$

$J_n :$

e_n

$\downarrow$

e_{n-1}

$\downarrow$

$\vdots$

e_1

$\downarrow$

0

$$J_n(\lambda) = J_n + \lambda \cdot I_n$$

$$\begin{pmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ 0 & & & 0 \end{pmatrix} \underset{e_i}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}} = \underset{e_{i-1}}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}} \leftarrow (i-1)\ \text{řádce}$$

$(\varphi - \lambda \mathrm{id}_V)(u_i) = u_{i-1}$, $2 \le i \le k$

λ je vl. číslo $(\varphi - \lambda \mathrm{id}_V)(\underset{\text{vlastní}}{u_1}) = 0$

$B = (u_k, u_{k-1}, \dots, u_1)$

$U = [u_k, \dots, u_1]$ je invariantní vůči φ

$\varphi(u_1) = \lambda u_1 \in U$,

$(\varphi - \lambda \mathrm{id}_V)(u_k) = u_{k-1} \Rightarrow \varphi(u_k) = u_{k-1} + \lambda u_k \in U$

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$u_k, \ldots\ldots, u_1$ je řetězec

$\Downarrow$

$u_{k-1}, \ldots, u_1$ je též řetězec.

$\boxed{k=1}$ $\boxed{u_1 \neq 0}$, $x u_1 = \varphi(u_1)$

předp., že pro všechna $l \leq k-1$ tvrzení platí; tj. $u_{k-1}, \ldots, u_1$ jsou $\underline{LN}$.

Chceme ukázat, že i $u_k, \underline{\ldots, u_1}$ je LN.

$$u_k = \sum_{i=1}^{k-1} c_i u_i$$

$$\begin{tikzcd} u_k \arrow[d, mapsto] \\ u_{k-1} \arrow[d, mapsto] \\ {} \end{tikzcd}$$

$$(\varphi - \lambda\, id_V)^{k-1}(u_k)$$

$$\|$$

$$u_1$$

$$= \sum_{i=1}^{k} c_i \underbrace{(\varphi - \lambda\, id_V)^{k-1}(u_i)}_{0} = 0 = u_1$$

$$\boxed{\text{spor}}\ !!$$