Elements of monoidal topology

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1. Pre-requisites in the language of instruction

The course requires some basic knowledge on category theory, general topology, and quantales. All the additional required concepts will be introduced during the lecture course. The course language is English.

2. Course objectives

The course aims at introducing its listeners into the theory of monoidal topology, which is an approach to general topology based in category theory and quantales. Course attendants will get to know how to represent a number of well-known mathematical structures, e.g., preordered sets, (generalized) metric spaces, topological spaces, approach spaces, and closure spaces as particular categorical structures and how then to describe and study the properties of these categorical structures using the standard tools of category theory.

3. Learning outcomes

On finishing the course its listeners should be able to apply methods of category theory to different mathematical settings, e.g., general topology. Since the influence of category-theoretic tools in modern mathematics is growing, course attendants will be more competitive in their knowledge of pure mathematics.

4. Course contents

The course consists of 14 lectures on the following topics.

1. Monads and their algebras.
2. Quantale-valued relations and lax extensions of monads.
3. The categories $(\mathbb{T},V)$-**Cat** and V-**Cat**. Examples of the category V-**Cat**.
4. Fundamental examples of the category $(\mathbb{T},V)$-**Cat**.
5. Properties of the category $(\mathbb{T},V)$-**Cat** I: Eilenberg-Moore algebras and topological categories.
6. Properties of the category $(\mathbb{T},V)$-**Cat** II: induced preorders and algebraic functors.
7. Properties of the category $(\mathbb{T},V)$-**Cat** III: change-of-base functors.
8. $(\mathbb{T},V)$-categories as generalized spaces.
9. Generalized Kuratowski-Mrówka theorem I: proper, closed, and perfect maps.
10. Generalized Kuratowski-Mrówka theorem II: proper $(\mathbb{T},V)$-functors and compact $(\mathbb{T},V)$-categories.
11. Symmetric monoidal closed structure on the category V-**Cat**.
12. The category V-**Mod** of V-categories and V-modules.
13. Topological spaces via neighborhood filters.
14. Power-enriched monads and Kleisli monoids.

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5. Recommended or required reading

The main source of information for the course is "D. Hofmann, G. J. Seal, and W. Tholen (eds.), *Monoidal Topology: A Categorical Approach to Order, Metric and Topology*, Cambridge University Press, 2014" (freely available at `http://sweet.ua.pt/dirk/artigos/HST14_Monoidal_topology_A_categorical_approach_to_order_metric_and_topology.pdf`). Additional recommended references include the following:

- category theory: "J. Adámek, H. Herrlich, and G. E. Strecker, *Abstract and Concrete Categories: the Joy of Cats*, Repr. Theory Appl. Categ. **17** (2006), 1–507" (this reference is freely available at `http://www.tac.mta.ca/tac/reprints/articles/17/tr17.pdf`);
- quantales: "K. I. Rosenthal, *Quantales and Their Applications*, Pitman Research Notes in Mathematics, vol. 234, Addison Wesley Longman, 1990";
- general topology: "R. Engelking, *General Topology*, Sigma Series in Pure Mathematics, vol. 6, Heldermann Verlag, 1989"(`https://www.heldermann.de/SSPM/SSPM06/sspm06.htm`).

The transcripts of the course lectures can be found at `https://home.czu.cz/solovjovs/ke-stazeni` under "Elements of monoidal topology".

6. Planned learning activities and teaching methods

The course consists of lectures only. It is planned to have one lecture (two hours) per week. There will be no seminars or any additional activities in the course.

7. Assessment methods and criteria

The course will end with an oral examination. Course attendants will be asked three questions on the topics of the course and will be given 40 minutes to prepare the answers. Every question will require an answer of moderate length. The examination result will depend on the number of correct answers to the questions as follows:

- zero correct answers: "failed" (exam mark "F");
- one correct answer: "good" (exam mark "C");
- two correct answers: "very good" (exam mark "B");
- three correct answers: "excellent" (exam mark "A").

Course attendants will have three attempts to pass the exam.

8. Examination dates

The following are the planned course examination dates for the spring semester of 2022.

Date	Time	Place
June 2, 2022 (Thursday)	15:00 – 17:00	M5, 01013
June 16, 2022 (Thursday)	15:00 – 17:00	M5, 01013
June 30, 2022 (Thursday)	15:00 – 17:00	M5, 01013