Elements of monoidal topology: exam questions

Sergejs Solovjovs*

Department of Mathematics, Faculty of Engineering, Czech University of Life Sciences Prague (CZU)
Kamýcká 129, 16500 Prague - Suchdol, Czech Republic

Exam questions: set 1

(1) Give the definition and an example of *monad*.
(2) Give the definition of the category V-**Rel**. Describe the embedding of the category **Set** into V-**Rel**.
(3) Give the definition and an example of the category V-**Cat**.

Exam questions: set 2

(1) Give the definition and an example of *unital commutative quantale*.
(2) Give the definition and an example of *lax extension* of a functor on the category **Set** to the category V-**Rel**.
(3) Give the definition and an example of the category $(\mathbb{T}, V)$-**Cat**.

Exam questions: set 3

(1) Give the definition of the *powerset monad* $\mathbb{P}$ on the category **Set**. Give an example of the category $(\mathbb{P}, V)$-**Cat**.
(2) Give the definition and an example of *lax extension* of a monad on the category **Set** to the category V-**Rel**.
(3) Give the definition of *topological category*. Describe initial structures in the category $(\mathbb{T}, V)$-**Cat**.

Exam questions: set 4

(1) Give the definition of the *ultrafilter monad* β on the category **Set**. Give an example of the category (β, V)-**Cat**.
(2) Describe and give an example of the *induced preorder functor* $(\mathbb{T}, V)$-**Cat** $\xrightarrow{Ind}$ **Prost**.
(3) Give the definition and an example of *morphism of lax extensions of monads*.

Exam questions: set 5

(1) Give the definition and an example of the *Eilenberg-Moore category* $\mathbf{X}^{\mathbb{T}}$ of a monad $\mathbb{T}$ on a category $\mathbf{X}$.
(2) Describe the embedding of the Eilenberg-Moore category $\mathbf{Set}^{\mathbb{T}}$ into the category $(\mathbb{T}, V)$-**Cat**.
(3) Describe the *algebraic functor* $(\mathbb{T}, V)$-**Cat** $\xrightarrow{A_e}$ V-**Cat** and its left adjoint functor V-**Cat** $\xrightarrow{A^\circ}$ $(\mathbb{T}, V)$-**Cat**.

*Tel.: (+420) 224 383 239
Email address: solovjovs@tf.czu.cz (Sergejs Solovjovs)
URL: http://home.czu.cz/solovjovs (Sergejs Solovjovs)

Exam questions: set 6

(1) Give the definition and an example of *lax homomorphism of unital quantales compatible with lax extensions of monads*.
(2) Describe and give an example of the *change-of-base functor* induced by a lax homomorphism of unital quantales compatible with lax extensions of monads.
(3) Given a monad $\mathbb{T}$ on the category **Set**, describe a 2-functor from the 2-quasicategory $\mathbf{Quant}(\mathbb{T})$ to the 2-quasicategory **CAT**.

Exam questions: set 7

(1) Give the definition of *closed*, *proper*, and *perfect maps*. Give the definition of *proper* $(\mathbb{T}, V)$*-functor*.
(2) Describe and give an example of the functor $(\mathbb{T}, V)$-**Cat** $\xrightarrow{G}$ V-**Cat** as an extension of the induced preorder functor.
(3) Give the definition of *compact topological space*. Give the definition of *compact* $(\mathbb{T}, V)$*-category*.

Exam questions: set 8

(1) Give the definition and an example of *ultrafilter* on a set.
(2) Give the definition of the category **App** and represent it as a category $(\mathbb{T}, V)$-**Cat**.
(3) Give the definition of the category **Cls** and represent it as a category $(\mathbb{T}, V)$-**Cat**.

Exam questions: set 9

(1) Give the definition and an example of *flat* lax extension of a functor on the category **Set** to the category V-**Rel**.
(2) Give the definition and an example of *fibration*.
(3) Describe and give an example of *discrete* and *indiscrete* $(\mathbb{T}, V)$-category structures on a set.

Exam questions: set 10

(1) Describe the Alexandroff topology functor **Prost** $\to$ **Top** and represent it as a left adjoint functor to an algebraic functor $(\mathbb{T}, V)$-**Cat** $\xrightarrow{A_e}$ V-**Cat**.
(2) Given a lax homomorphism of unital quantales $V \xrightarrow{\varphi} W$, describe the lax functor V-**Rel** $\xrightarrow{\varphi}$ V-**Rel**W.
(3) Give the definition and an example of *taut* functor on the category **Set**.

Name, surname: **Učo:**

Exam questions: set 1

(1) Give the definition and an example of *monad*.
(2) Give the definition of the category V-**Rel**. Describe the embedding of the category **Set** into V-**Rel**.
(3) Give the definition and an example of the category V-**Cat**.

Name, surname: **Učo:**

Exam questions: set 2

(1) Give the definition and an example of *unital commutative quantale*.
(2) Give the definition and an example of *lax extension* of a functor on the category **Set** to the category V-**Rel**.
(3) Give the definition and an example of the category $(\mathbb{T}, V)$-**Cat**.

Name, surname: **Učo:**

Exam questions: set 3

(1) Give the definition of the *powerset monad* $\mathbb{P}$ on the category **Set**. Give an example of the category $(\mathbb{P}, V)$-**Cat**.
(2) Give the definition and an example of *lax extension* of a monad on the category **Set** to the category V-**Rel**.
(3) Give the definition of *topological category*. Show that the category $(\mathbb{T}, V)$-**Cat** is a topological construct.

Name, surname: **Učo:**

Exam questions: set 4

(1) Give the definition of the *ultrafilter monad* β on the category **Set**. Give an example of the category (β, V)-**Cat**.

(2) Describe the *induced preorder functor* $(\mathbb{T}, V)$-**Cat** $\xrightarrow{Ind}$ **Prost**.

(3) Give the definition and an example of *morphism of lax extensions of monads*.

Name, surname: **Učo:**

Exam questions: set 5

(1) Give the definition and an example of the *Eilenberg-Moore category* $\mathbf{X}^{\mathbb{T}}$ of a monad $\mathbb{T}$ on a category $\mathbf{X}$.
(2) Describe the embedding of the Eilenberg-Moore category $\mathbf{Set}^{\mathbb{T}}$ into the category $(\mathbb{T}, V)$-**Cat**.
(3) Describe the *algebraic functor* $(\mathbb{T}, V)$-**Cat** $\xrightarrow{A_e}$ V-**Cat** and its left adjoint functor V-**Cat** $\xrightarrow{A^\circ}$ $(\mathbb{T}, V)$-**Cat**.

Name, surname: **Učo:**

Exam questions: set 6

(1) Give the definition and an example of *lax homomorphism of unital quantales compatible with lax extensions of monads*.

(2) Describe the *change-of-base functor* induced by a lax homomorphism of unital quantales compatible with lax extensions of monads.

(3) Given a monad $\mathbb{T}$ on the category **Set**, describe a 2-functor from the 2-quasicategory $\mathbf{Quant}(\mathbb{T})$ to the 2-quasicategory **CAT**.

Name, surname: **Učo:**

Exam questions: set 7

(1) Give the definition of *closed*, *proper*, and *perfect maps*. Give the definition of *proper* $(\mathbb{T}, V)$*-functor*.

(2) Describe the functor $(\mathbb{T}, V)$-**Cat** $\xrightarrow{G}$ V-**Cat** as an extension of the induced preorder functor. Describe the respective functor **Top** $\xrightarrow{G}$ **Prost**.

(3) Give the definition of *compact topological space*. Give the definition of *compact* $(\mathbb{T}, V)$*-category*.

Name, surname: **Učo:**

Exam questions: set 8

(1) Give the definition and an example of *ultrafilter* on a set.
(2) Give the definition of the category **App** and represent it as a category $(\mathbb{T}, V)$-**Cat**.
(3) Give the definition of the category **Cls** and represent it as a category $(\mathbb{T}, V)$-**Cat**.

Name, surname: **Učo:**

Exam questions: set 9

(1) Give the definition and an example of *flat* lax extension of a functor on the category **Set** to the category V-**Rel**.
(2) Give the definition and an example of *fibration*.
(3) Describe and give an example of *discrete* and *indiscrete* $(\mathbb{T}, V)$-category structures on a set.

Name, surname: **Učo:**

Exam questions: set 10

(1) Describe the Alexandroff topology functor **Prost** $\to$ **Top** and represent it as a left adjoint functor to an algebraic functor $(\mathbb{T}, V)$-**Cat** $\xrightarrow{A_e}$ V-**Cat**.
(2) Given a lax homomorphism of unital quantales $V \xrightarrow{\varphi} W$, describe the lax functor V-**Rel** $\xrightarrow{\varphi}$ V-**Rel**W.
(3) Give the definition and an example of *taut* functor on the category **Set**.
