Exam Problems: Physics in Spacetime, Spring 2023

1. Protons (p^+) are bombarded with pions ($\pi^{\pm}$). What is the minimum energy that the pions need to have, according to an observer at rest with respect to the proton, for the reaction

$$\pi^- + p^+ \to \pi^+ + \pi^- + n^0$$

to be possible? (Set $M_{\pi^\pm} = m$ and $M_{p^+} = M_{n^0} = M$.)

2. Pulsars are neutron stars which emit radio pulses at regular intervals. Alice and Bob count pulses from a very distant pulsar in the y-direction. Alice travels in the x-direction with velocity $u = 24/25$ relative to Bob for seven years, then reverses and returns with the same velocity, while Bob stays at home. At the end of the trip they have counted the same number of pulses. When Alice returns she has aged by 14 years. How much has Bob aged?

Find the angle to the x-axis at which Alice observes the pulsar (for Bob the angle is $90°$).

3. A light source is moving away from an observer U with velocity v. Another observer V is also moving away from U with velocity v, but in a direction perpendicular to the direction of motion of the light source. When the source emits a light signal it is as far from the origin as V is when she receives it. Draw a spacetime diagram to illustrate the situation.

Calculate the Doppler shift ω_V/ω_0 where ω_0 is the angular frequency measured by an observer at rest with respect to the light source.

4. Consider a photon-drive rocket, driven by shooting a powerful laser (of fixed power and wavelength) out of its tail. According to an observer the rocket's relative velocity is zero at $\tau = \tau_i$ and v at $\tau = \tau_f$, where τ refers to the rocket's proper time. What is the ratio of the masses M_i/M_f of the rocket at the two times?