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Physical laboratory 3

Task E Zeeman effect

Once a man was walking on a street in Copenhagen (it was W. Pauli) with his face so gloomy that an unknown older lady worriedly asked him after the reason for his anxiety. " But, madam," he answered, " I cannot understand the anomalous Zeeman effect." After its discovery, the Zeeman effect was indeed quite a riddle suggesting something about the atomic structure, but it was yet challenging to solve this enigma. The spectral lines of atoms were splitting into a different number of components. The change in the energy of the radiation was directly proportional to the magnetic induction. However, the proportionality constant was sometimes an integer multiple and sometimes a rational multiple of a different constant, the so-called Bohr magneton. Different components exhibited different polarisation, as well as their numbers were different. This was moreover dependent on the direction from which the observation was made. Today, the change of the energy of atoms in a magnetic field discovered in the Zeeman effect is used not only in optical spectroscopy. It also forms the basis of electron paramagnetic resonance, polarisation spectroscopy, different laser tuning and cooling using the adiabatic demagnetisation of paramagnetics.

Tasks

1. Verify the behaviour of the Fabry-Perot interferometer. Show that the measured radii of different interference rings of a single wavelength correspond to a suitable formula from the manual.

2. Determine the value of the Bohr magneton using the shift of the wavenumbers in the normal Zeeman effect. Turn the magnetic field on only after getting clearance from the teacher!

3. Determine which components of the split spectral lines are emitted in the direction perpendicular to the magnetic induction direction and which are emitted in the direction parallel to the magnetic induction direction. Measure the polarisation of all the different line components in both directions mentioned above.

Think before the laboratory:

- How do you construct a telescope from two converging lenses with focal lengths of $5\,\mathrm{cm}$ and $30\,\mathrm{cm}$?

- Why do energy levels of electrons change in a magnetic field?

- How do you determine the energy difference of two photons of two close spectral lines from the interference pattern provided by the Fabry-Perot interferometer?

Figure 1: Energy level diagram of cadmium.

- How do you measure the radiation if it has circular polarisation and only a linear polarisation filter and a quarter-wave plate is available? You can find some fundamental facts about quarter-wave plates in the addendum to this manual.

Theory of Zeeman effect

The Zeeman effect is the name for the splitting of spectral lines emitted by atoms in magnetic fields. This splitting occurs due to the energy changes of the individual atomic energy levels caused by the external magnetic field. Pieter Zeeman first described this effect in 1896, and it brought in new insights into electron energy levels in the atom and about the quantum nature of momentum projection in a selected direction. When observing radiation emitted by an atom transition between two singlet states, we will see the so-called normal Zeeman effect. Singlet states are states where the spins of all the electrons compensate, and the total spin of the atom is zero. The normal Zeeman effect is caused only by the interaction of the orbital magnetic momentum of the atom with the external magnetic field. If the total spin of the atom is non-zero, the effect of the spin in the external magnetic field also has to be counted towards the energy of the atom. When observing spectral lines of transitions between states where at least one is not a singlet, we will observe a more complicated line splitting - the so-called anomalous Zeeman effect. We will not take into account any possible atomic nucleus' spin in this task.

We will observe the Zeeman effect on selected spectral lines of cadmium. The electron configuration of cadmium in the ground state is (Kr) $4d^{10} 5s^2$, and it is a singlet. The ground state and the excited states of cadmium with the corresponding energies are shown in figure 1. The

cadmium states are denoted by something like $2\,^3\mathrm{S}_1$. In such a case, the letter "S" denotes the total orbital momentum of the atom represented by the quantum number L. "S" specifically denotes a state with $L = 0$, "P" would denote a state with $L = 1$, "D" with $L = 2$, "F" with $L = 3$ etc. The superscript "3" in the upper left is called the multiplicity, and it can be calculated from the quantum number S showing the magnitude of the total electron spin of the atom as $2S + 1$. Singlet states carry the multiplicity of 1. The subscript "1" on the bottom right shows the quantum number of the total momentum of the atom J. It can have the values of $L+S$, $L+S-1$ etc. up to $|L - S|$. The leading number "2" is connected to the principal quantum number of the excited electron – electron in its ground state is denoted by the number "1". Therefore, an electron in the $2\,^3\mathrm{S}_1$ state is excited to the 6s level (the ground state has no electrons higher than 5s).

In a magnetic field, the cadmium energy levels split and their energy shifts according to the projection of the atomic magnetic moment in the external magnetic field direction. When we describe the motion of a single electron in an atom using a wave function and the laws of quantum mechanics, we find out that it can exist only in the states described by an integer value of the so-called orbital angular momentum (or azimuthal) quantum number l. In these states the electron orbital momentum $\vec{l}_1$ magnitude is $|\vec{l}_1| = \hbar\sqrt{l(l + 1)}$. This single electron would have the magnetic moment due to its orbital motion (at the moment not considering spin) of

$$\vec{\mu}_1 = -\frac{e}{2m_e}\,\vec{l}_1,$$

where e denotes the elementary charge and m_e the electron mass. The magnetic moment magnitude will be

$$|\vec{\mu}_1| \;=\; \frac{e\hbar}{2m_e}\sqrt{l(l + 1)} \;=\; \mu_B\sqrt{l(l + 1)},$$

where

$$\mu_B \;=\; \frac{e\hbar}{2m_e} \;\approx\; 9{,}274{.}10^{-24}\,\mathrm{Am}^2 \tag{1}$$

is called the Bohr magneton. The projection of the magnetic moment $\vec{\mu}_1$ in a selected direction can only have the values of $\mu_B.m_l$, where m_l is an arbitrary integer in the interval of $-l$ to $+l$. Similarly for a multielectron system with the total orbital momentum of

$$\vec{L}_{tot} = \sum_i \vec{l}_i$$

we can write

$$|\vec{L}_{tot}| \;=\; \hbar\sqrt{L(L + 1)} \tag{2}$$
$$|\vec{\mu}_L| \;=\; \mu_B\sqrt{L(L + 1)}, \tag{3}$$

where μ_L is the orbital magnetic momentum.

The total atomic momentum $\vec{J}_{tot}$ is in the case of the LS bond a sum of the total orbital and total spin $(\vec{S}_{tot})$ momentum

$$\vec{J}_{tot} = \vec{L}_{tot} + \vec{S}_{tot}. \tag{4}$$

We again describe its magnitude by the quantum number J as:

$$|\vec{J}_{tot}| = |\vec{L}_{tot} + \vec{S}_{tot}| = \hbar\sqrt{J(J + 1)}. \tag{5}$$

The momentum $\vec{J}_{tot}$ is connected to the magnetic momentum $\vec{\mu}_J$. The magnitude of the mean magnetic momentum can be described as

$$\mu_\parallel = g_J\mu_B\sqrt{J(J + 1)}, \tag{6}$$
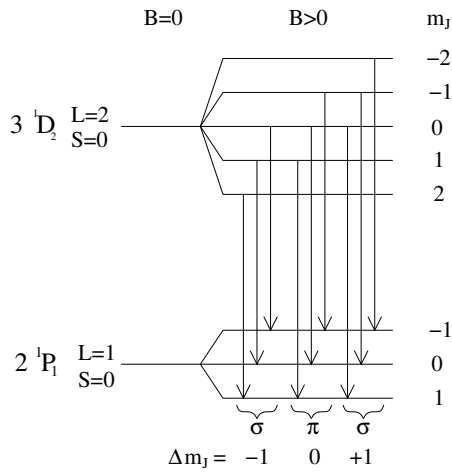
where g_J is the Landé g-factor.

Figure 2: Splitting of the $3\,^1D_2$ and $2\,^1P_1$ cadmium states in a magnetic field. The arrows denote electric dipole transitions.
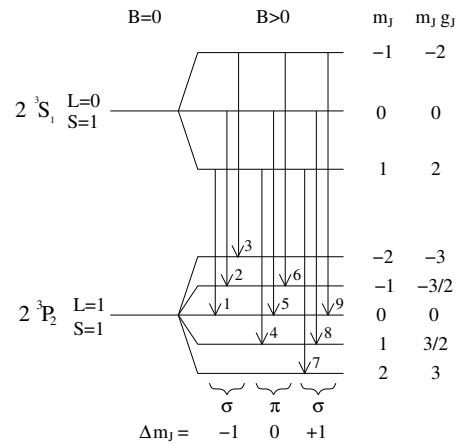

Figure 3: Splitting of the $2\,^3S_1$ and $2\,^3P_2$ cadmium states in a magnetic field. The arrows denote electric dipole transitions.

Normal Zeeman effect

The normal Zeeman effect occurs for transitions between singlet states, where the total electron spin $\vec{S}_{tot}$ is zero. The total magnetic momentum of the atom can then be written simply as $\vec{\mu}_J = \vec{\mu}_L$, $J = L$ a $g_J = 1$. The projection of the total magnetic momentum in the magnetic field direction can take on values of $m_J \cdot g_J \cdot \mu_B$, where the magnetic quantum number m_J can be an arbitrary integer between $-J$ and $+J$. The energy of the interaction between the magnetic momentum $\vec{\mu}_J$ and the external magnetic field with the induction $\vec{B}$ has the magnitude of

$$E_{m_J} = -m_J\, g_J\, \mu_B\, B. \tag{7}$$

In this laboratory, we will observe the normal Zeeman effect on the cadmium $3\,^1D_2 \rightarrow 2\,^1P_1$ transition corresponding to the wavelength of $643.847\,\mathrm{nm}$. In the initial state $J = L = 2$ and the magnetic quantum number m_{J1} can take on values of -2, -1, 0, 1, 2. In the final state $J = L = 1$ and the magnetic quantum number m_{J1} can take on values of -1, 0, 1. As the Landé g-factor $g_J = 1$ in both states, the energy difference between two neighbouring energy sub-levels in both states is $\mu_B \cdot B$. The corresponding energy level splitting is shown in figure 2. The energy of the photon emitted in such a transition changes by

$$E_{m_{J1}} - E_{m_{J2}} = (m_{J2} - m_{J1})\, \mu_B\, B \tag{8}$$

As the dipole moment transition selection rules allow only the Δm_J difference of -1, 0, 1, the observed line will split into three lines in a magnetic field. The centre line with $\Delta m_J = 0$ is denoted as π, and its wavelength does not change in the magnetic field. The side-lines with $\Delta m_J = \pm 1$ are denoted σ, and their photons have their energy shifted by $\pm \mu_B B$ from the central π line. All of these lines are emitted in the direction perpendicular to the magnetic induction. However, only some of them are emitted in the direction parallel to the magnetic field. Not only the number of the lines but also their polarisation is dependent on the emission direction. The polarisation can change all the way from linear to circular. Measuring the line polarisation and determining which lines are emitted into which directions is one of the tasks of this laboratory. Although the changes in the polarisation and the different number of lines in different directions might seem confusing at first glance, both can be understood even in the classical conception of an electron oscillating in a magnetic field.

Anomalous Zeeman effect

The situation is more complicated in the case of the anomalous Zeeman effect. This is due to the existence of a non-zero total electron spin $\vec{S}_{tot} = \sum_i \vec{s}_i$. Its magnitude and magnetic momentum are described by the quantum number S as

$$|\vec{S}_{tot}| = \hbar \sqrt{S(S+1)} \tag{9}$$
$$|\vec{\mu}_S| = g_s \, \mu_B \, \sqrt{S(S+1)}, \tag{10}$$

where $g_s = 2.0023$ is called the gyromagnetic factor. We obtain the total momentum of the atom as the sum of its orbital and spin momenta according to (4) and (5). The total magnetic momentum of the atom $\vec{\mu}_J = \vec{\mu}_L + \vec{\mu}_S$ is not parallel to the total momentum of the atom $\vec{J}_{tot}$ due to the existence of the gyromagnetic factor of 2.0023. We are, however, interested only in the mean value of the magnetic momentum. This value is at the same time the projection of the $\vec{\mu}_J$ vector in the direction of the $\vec{J}_{tot}$ vector. We can calculate its value from geometric considerations

$$\mu_\parallel = \frac{3J(J+1) + S(S+1) - L(L+1)}{2\sqrt{J(J+1)}} \, \mu_B$$

valid in the $g_s \approx 2$ approximation. From the definition of the Landé g-factor

$$\mu_\parallel = g_J \, \mu_B \, \sqrt{J(J+1)}$$

we obtain

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}. \tag{11}$$

As the projection of $\mu_\parallel$ into the direction of the magnetic field can take on values of $m_J \cdot g_J \cdot \mu_B$ with the magnetic quantum number $m_J = J, J-1, \ldots, -J$, we once again obtain formula (7) for the description of the change of the energy of the atom in a magnetic field.

In this laboratory, we will study the anomalous Zeeman effect on the $2\,^3\mathrm{S}_1 \rightarrow 2\,^3\mathrm{P}_2$ cadmium transition with the emission line with the wavelength of $508.588\,\mathrm{nm}$. The initial state is characterised by quantum numbers $L = 0$, $S = 1$ a $J = 1$ and the Landé g-factor $g_{J1} = 2$ and the final state by $L = 1$, $S = 1$, $J = 2$ and $g_{J2} = 3/2$. The magnetic quantum number m_J can have the values of -1, 0, 1 in the initial state and -2, -1, 0, 1, 2 in the final state. The diagram of the split atomic levels of the $2\,^3\mathrm{S}_1$ and $2\,^3\mathrm{P}_2$ states together with dipole transitions allowed by the transition rule $\Delta m_J = $ -1, 0, 1 is shown in figure 3. It can be seen that the observed line will split into nine components (three π and six σ). The energy shift of the different components is described by

$$E_{m_{J1}} - E_{m_{J2}} = (m_{J2}\, g_{J2} - m_{J1}\, g_{J1}) \, \mu_B \, B \tag{12}$$

The properties of the individual lines can be deduced from the following table.

Number	1	2	3	4	5	6	7	8	9
Δm_J	1	1	1	0	0	0	-1	-1	-1
$m_{J2}\, g_{J2} - m_{J1}\, g_{J1}$	-2	-3/2	-1	-1/2	0	1/2	1	3/2	2

Fabry-Perot interferometer

As the individual line components are very close in energy and wavelength, a high-resolution device is needed for their analysis. Therefore a Fabry-Perot interferometer is used. In this case, it is a $3\,\mathrm{mm}$ silica plate covered on both sides by a thin film reflecting $90\,\%$ of the incident light. The Fabry-Perot interferometer is often used in spectroscopy, telecommunications, and laser technology for wavelength measuring or filtering a specific wavelength. For example, it is used for the choice of different wavelengths on the recipient's side of a wave multiplex, it forms the basis of interference
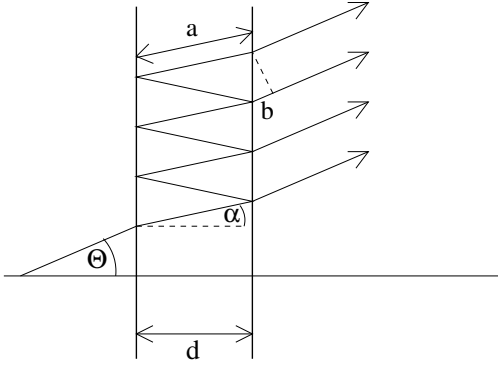
Figure 4: Beam paths in the Fabry-Perot interferometer. The optical path difference of two neighbouring beams is $2na - b$.
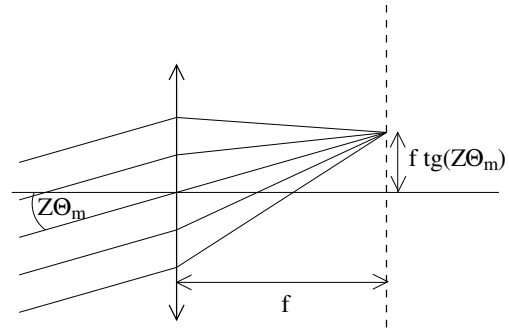
Figure 5: Interference pattern displaying by a converging lens.

standards and some dichroic filters, it is also used to choose a single-mode from all the modes of the resonance chamber of a single-mode laser. Owing to its high resolution, the Fabry-Perot interferometer allows for the measurement of narrow spectral lines. The way how to measure the difference of the close spectral lines is described in the following paragraphs.

The path of the individual rays in the Fabry-Perot interferometer is shown in figure 4. For the greatest intensity of the radiation passing through the FP interferometer, the phase difference of the individual rays needs to be integer times 2π. Because the optical path difference of two neighbouring rays is $2nd \cos \alpha$, the constructive interference condition can be written as

$$m\lambda = 2nd \cos \alpha_m, \tag{13}$$

where m is an arbitrary integer, λ is the wavelength of the light, $2 = 1 + 1$, n the refractive index of silicate glass (1.4519 for the wavelength of $509\,\text{nm}$ and 1.4560 for $644\,\text{nm}$) and d the thickness of the interferometer. As the condition (13) is fulfilled for only certain angles α_m, the monochromatic radiation after passing the interferometer will form a pattern consisting of light and dark circles. Let us denote

$$M = \frac{2nd}{\lambda}.$$

It can be seen that the maxima of interference are formed for $m \leq M$. We can rewrite (13) for small angles α_m as

$$m = M \cos \alpha_m \approx M \left(1 - \frac{\alpha_m^2}{2}\right)$$

and for the angles fulfilling the constructive interference condition we obtain

$$\alpha_m \approx \sqrt{\frac{2(M - m)}{M}} \tag{14}$$

The light is refracted after exiting the interferometer, and the constructively interfering rays exit at the angle Θ_m. The interference pattern is then Z-times magnified by a telescope and then displayed by a converging lens with the focal distance f onto a chip of a CCD camera (see fig. 5), where it forms light rings with radii

$$r_m = f \, \text{tg}(Z\Theta_m) \approx fZ\Theta_m \approx fZ \, n\alpha_m \tag{15}$$

We can get the final formula or the radii of the interference rings from the last to equations. Let us observe rays propagating at a small angle Θ_m. Their m is, however, a large number that is difficult to determine exactly. It is therefore practical to use the index of $p = m_{max} - m + 1$

numbering the rings from the smallest one to larger ones (m_{max} is the largest index m fulfilling the $m \leq M$ condition). Combining (14) and (15) using new indexing we obtain

$$r_p^2 = 2(fZn)^2 \left(1 - \frac{m_{max} + 1}{M} + \frac{p}{M} \right) \tag{16}$$

We can measure the distance between two close spectral lines by comparing the difference of the interference ring radii of these lines with the difference of the neighbouring rings of a single spectral line. To do this we rewrite (16) using wavenumber $\tilde{\lambda} = 1/\lambda$ as

$$r_p^2 = 2(fZn)^2 \left(1 - \frac{m_{max} + 1}{2nd\tilde{\lambda}} + \frac{p}{2nd\tilde{\lambda}} \right)$$

and for the difference of the squares of the radii of two spectral lines (r_a a r_b) with close wavenumbers $\tilde{\lambda}_a$ a $\tilde{\lambda}_b$ we obtain

$$r_{b,p}^2 - r_{a,p}^2 = 2(fZn)^2 \, \frac{m_{max} + 1 - p}{2nd} \left(\frac{1}{\tilde{\lambda}_a} - \frac{1}{\tilde{\lambda}_b} \right) \tag{17}$$

Next we calculate the difference of the squares of radii of two neighbouring rings of a single spectral line

$$r_{a,p+1}^2 - r_{a,p}^2 = 2(fZn)^2 \, \frac{1}{2nd\tilde{\lambda}_a} \tag{18}$$

and by dividing (17) by (18) we obtain

$$\frac{r_{b,p}^2 - r_{a,p}^2}{r_{a,p+1}^2 - r_{a,p}^2} = (m_{max} + 1 - p) \frac{\tilde{\lambda}_b - \tilde{\lambda}_a}{\tilde{\lambda}_b}$$

We can easily measure the left side of the equation. As $m_{max} \approx M \gg 1$ a $\tilde{\lambda}_a \approx \tilde{\lambda}_b$, we can rewrite the previous equation as

$$\frac{r_{b,p}^2 - r_{a,p}^2}{r_{a,p+1}^2 - r_{a,p}^2} = 2nd \, (\tilde{\lambda}_b - \tilde{\lambda}_a). \tag{19}$$

This formula enables us to easily measure the difference of the wavenumbers of two close spectral lines.

Experimental setup

The cadmium lamp and the magnets are placed on a rotating table (see fig. 6). The lamp overheats if the magnetic field is turned on for too long. Therefore, the magnetic field needs to be applied for the shortest time possible! The radiation is exiting from the lamp through the gap between the electromagnets and through a hole in the magnet holder, enabling to observe the emission lines from a direction perpendicular to and parallel with the vector of the magnetic induction. The electromagnets can be powered by a DC current of up to $10\,\text{A}$. The relation between the current going through the electromagnets and the magnetic induction magnitude in the gap between the electromagnets can be found in a table uploaded to the study materials in the information system. You can use the following excerpt as a rough guide:

I [A]	2.97	5.05	6.06	7.1	8.06	8.96	9.95
B [mT]	188	319	381	444	500	549	594

An iris used for observing the light emitted in the direction perpendicular to the magnetic field is placed behind the rotating table. The hole in the magnetic field holder serves as the iris when observing the light emitted in the direction parallel to the magnetic field. Next, a converging lens

with a focal length of 5 cm is placed. It creates light with nearly parallel beams that are entering the Fabry-Perot interferometer.

The student builds the optical path between the interferometer and a camera. The interference pattern needs to be magnified by a telescope. You can use two converging lenses with focal lengths of 5 cm and 30 cm to build the telescope. A CCD camera captures the final image at the end of the optical path. This image is being sent to a computer. Moreover, a polarising filter and a quarter-wave plate, used for the determination of the line polarisation, can be placed in the optical path. As cadmium emits many different wavelengths, the resulting interference pattern is unclear. Therefore, we need to filter out only the measured optical line. Use the red filter placed into a slot at the end of the Fabry-Perot interferometer to observe the 644 nm for the normal Zeeman effect. Use the green interference filter that can be mounted on any lens to observe the anomalous Zeeman effect.

Experimental procedure

- Turn the table with the magnets and the cadmium lamp to allow the light emitted perpendicularly to the magnetic field to go through the optical path. The iris should be placed 5 cm in front of the converging lens in the optical path. While the magnetic field is turned off, place the two converging lenses behind the Fabry-Perot interferometer. The lenses with the focal lengths of 5 cm and 30 cm should form a magnifying telescope making the interference clearly visible on the CCD. Cover the optical path from the exterior lighting for consecutive measurements.

- Image as many interference circles of the red (643.847 nm) or green (508.588 nm) cadmium line as possible. Block the other lines by using the red or the green interference filter. Verify that the interference ring radii are in accordance with the theoretical expectations shown in section 2. It is advisable to use the line showing the higher number of interference rings.

- You will be using the magnetic field in the following tasks. As the cadmium lamp overheats in such strong magnetic fields, it is necessary to measure the following tasks swiftly. Do not set the current going through the electromagnets above 10 A.

- While observing the light emitted perpendicularly to the magnetic field, image the interference patterns of the studied line at different currents through the electromagnets. You will measure the spectral line splitting and calculate the Bohr magneton from these images. Determine the Bohr magneton from the red line only. Determine the polarisation of all the components of the split spectral line. You have the linear polarisation filter and a quarter-wave plate available for this task. Do this for the normal Zeeman effect of the red line as well as the anomalous Zeeman effect of the green line. Turn the magnetic field off while not acquiring images!
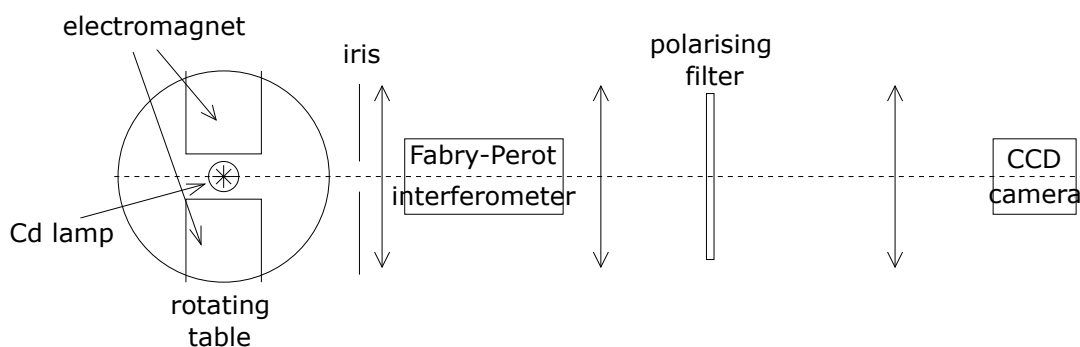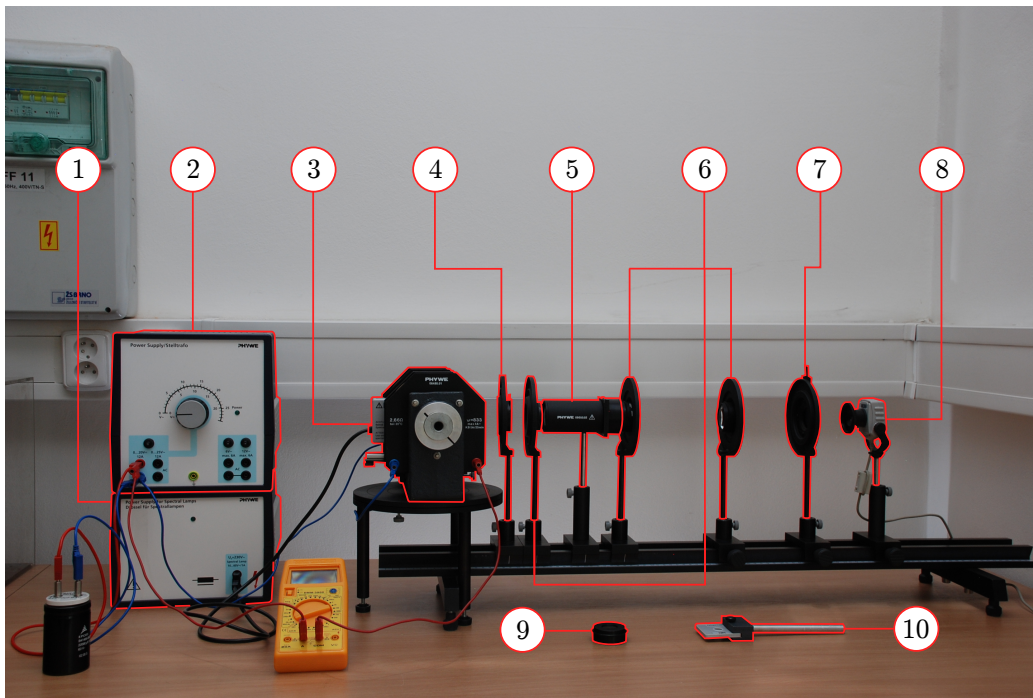

Figure 6: Diagram of the experimental system.

Figure 7: Experimental system: 1 - Cadmium lamp power source, 2 - Electromagnet power source, 3 - Cadmium lamp with electromagnets, 4 - Iris, 5 - Fabry-Perot interferometer, 6 - Lenses, 7 - Polarising filter, 8 - CCD camera, 9 - Interference filter, 10 - Quarter-wave plate.

- When the magnetic field is turned off, rotate the table so that the emission going through the hole in the magnet holder is aimed at the optical path. You do not need to use the iris now. Check if the interference pattern is still visible on the screen.

- Determine which components of both lines (643.847 nm and 508.588 nm) are emitted in the direction parallel to the magnetic field and what is their polarisation.

- You can use "Motic images" to evaluate the acquired interference patterns. The images are saved by "file – capture window – capture" and the radii of the interference rings can be directly measured by "measure – circle" or "circle (3 points)". Measure three line triplets for each of 10 different magnetic fields. Compare the calculated value of the Bohr magneton with the tabulated value (1).

Addendum: Waveplates

Waveplates are simple devices made of optically anisotropic materials, i.e. materials with the refraction index depending on the polarisation of the light. The waveplate will slow the radiation polarised in the y axis compared to the radiation polarised in the x axis by a phase difference δ given by the refractive indexes of the phase plate material for the two polarisations $(n_y - n_x)$, the plate thickness (d_f) and the wavelength of the radiation as

$$\delta = \frac{2\pi(n_y - n_x)d_f}{\lambda}.$$

For example, a waveplate made of mica having refractive indexes of 1.594 a 1.599 for 633 nm radiation would cause a phase shift by π having a thickness of only 63 μm. The phase shift δ of many materials can be affected, e.g. by an electric field. This is used in optoelectronics and imaging technology.

The x and y axes are called the fast axis and the slow axis of the waveplate. We can describe the radiation by the Jones vector with its components being complex amplitudes of projection of the wave's electric field into these axes. The waveplate is characterised by the Jones matrix

$$F = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\delta} \end{bmatrix}$$

Nothing interesting happens if the incident radiation is polarised in the direction of the principal axes of the plate. The situation is more interesting if the radiation is polarised in a general direction or has a circular polarisation. Practically we use two main waveplate types:

- A quarter-wave plate shifts the phase by $\delta = \pi/2$. A linearly polarised light with its polarisation axis being turned by $45°$ compared to the principal axes of the plate changes to a circularly polarised, while previously circularly polarised light becomes linearly polarised.

- A half-wave plate with $\delta = \pi$ rotates the plane of linearly polarised light, changes right-circularly polarised light into left-circularly polarised light and vice versa.