

Physical laboratory 3

Task I Rutherford's experiment

Tasks

1. Observe the number of detected α particles for a sufficient number of gold foil positions. Verify formula 18 for Rutherford's scattering.
2. Verify that the numbers of detected α particles obey Poisson's distribution (11).



Figure 1: Ernest Rutherford, Hans Geiger a Ernest Marsden.

Introduction

Not much was known about the building blocks of matter in 1909. Joseph John Thomson had already discovered corpuscular radiation in 1897. The particles of this radiation have been named electrons. Further measurements of electrons revealed that they are relatively light and negatively electrically charged.

It still had to be resolved how the electrons and the positive matter are placed inside the atom. Thomson suggested a model where the positive matter is evenly distributed inside the atom, and the electrons are placed just like “raisins in pudding”. This model is known as Thomson's pudding model.

In 1909 Hans Geiger and Ernest Marsden, under the guidance of Ernest Rutherford, carried out an experiment where a beam of α particles bombarded a very thin gold foil. A diagram of this experiment is shown in Figure 2. According to Thomson's model, the positively charged α particles should only very slightly scatter from their initial direction by repelling positive charges.

The results of the experiment were surprising. Most of the α particles passed through the foil without being scattered. On the other hand, some particles scattered to large angles were also

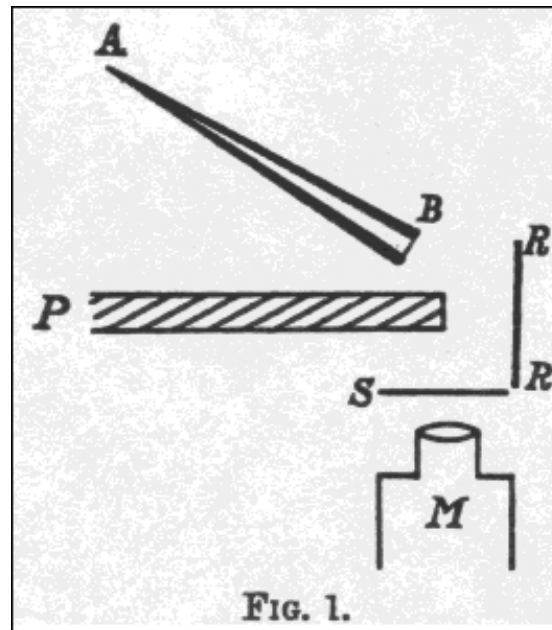


Figure 2: Historical diagram of the original experiment of Geiger and Marsden. AB source of α particles, P lead plate, RR thin foil, S scintillating screen, M detector (source: <http://www.chemteam.info/Chem-History/GM-1909.html>).

detected. For this to happen, the positively charged matter in the atom had to be concentrated into a very small volume. This was the foundation of Rutherford's model of the atom. The model assumed the existence of a very small and heavy positively charged atomic nucleus with electrons orbiting the nucleus as planets with lots of free space around them.

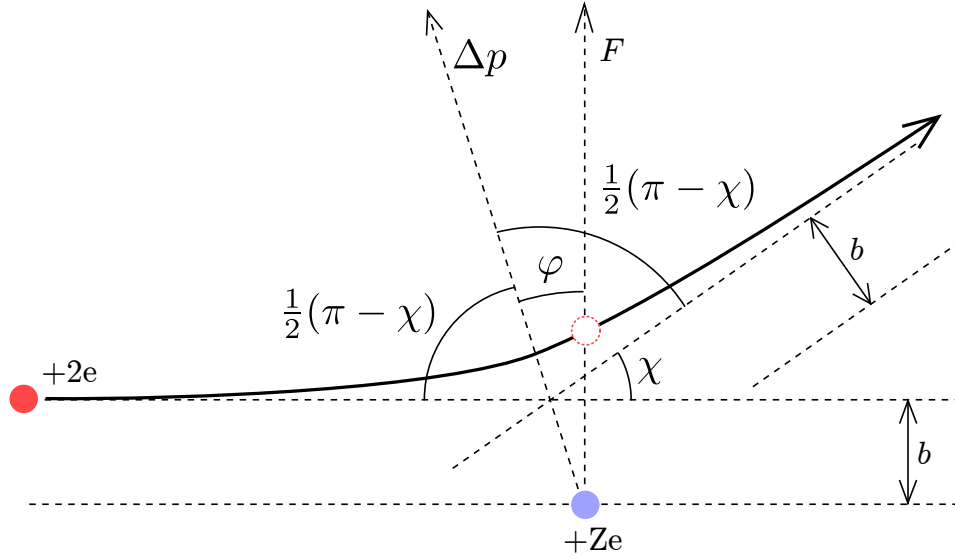
This experiment was crucial for understanding the structure of matter. Most parts of modern physics and chemistry are built upon this model with added quantum theory. And also, the principle of this scattering has found its use. Nowadays, the method of RBS – Rutherford backscattering spectrometry is used to analyse the chemical composition of materials. A beam of high-energy ions is bombarding an unknown sample, and the scattering of these particles on the sample is measured. And, as the scattering is dependent on the number of protons in the nucleus, the chemical composition can be determined.

Scattering theory

The principle of Rutherford's experiment lies in measuring light α particles scattered on heavy atoms such as gold. Owing to the large difference in the atomic masses, the gold atoms can be considered stationary. If the incoming α particle gets close enough to the atomic nucleus (inside its electron shell), it starts to be repulsed by its positive charge. Due to this force, the incoming α particle is scattered by the angle χ from its initial direction. The magnitude of the angle χ depends on how close to the nucleus the α particle gets and on its initial velocity.

The fundamental assumption of the experiment is that the incoming particle transfers no kinetic energy to the nucleus it is scattering on. The magnitude of its initial kinetic energy and the magnitude of the initial momentum are the same as their final magnitudes after the scattering process. The incoming particle, however, scatters by the angle χ . The direction of the momentum, therefore, changes. The magnitude of this change during the whole process is

$$\Delta p = 2mv \sin \frac{\chi}{2}, \quad (1)$$

Figure 3: Schematic of α particle scattering.

where m is the α particle mass and v is its initial (and final) velocity. The momentum change is caused by mutual force F between the nuclei

$$\Delta\vec{p} = \int \vec{F} dt. \quad (2)$$

Figure 3 schematically shows a collision of an α particle with a massive nucleus. The scattering process can be described by the angle φ instead of the time. The magnitude of the momentum change of the α particle can be described as

$$\Delta p = \int_{-\frac{1}{2}(\pi-\chi)}^{\frac{1}{2}(\pi-\chi)} F(\varphi) \cos \varphi \frac{dt}{d\varphi} d\varphi, \quad (3)$$

where $\frac{dt}{d\varphi} = \frac{1}{\omega}$, is the reciprocal of the angular velocity of the particle around the nucleus.

The force acting between the nuclei is an electric force that can be described as $F_E = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{r^2}$, where r is the instantaneous distance between the nuclei. This relation is valid for one of the nuclei being an α particle ($q=2e$) and the other one a nucleus with Z protons. Substituting this into the relation for the momentum change equation we obtain

$$2mv \sin \frac{\chi}{2} = \int_{-\frac{1}{2}(\pi-\chi)}^{\frac{1}{2}(\pi-\chi)} \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{r^2} \frac{1}{\omega} \cos \varphi d\varphi. \quad (4)$$

We can simplify the integral on the right side by realising that the electric force between the nuclei always acts along the connecting line between the nuclei. Therefore, such force acts with no torque on the α particle and the momentum of the α particle is conserved throughout the whole process. The momentum in every point of the α particle's trajectory must be identical to the initial momentum. Therefore, we can write $m\omega r^2 = mv b$, where b is the impact parameter (see Fig. 3) and v is the initial velocity of the α particle. Substituting this into 4 we obtain

$$\frac{4\pi\epsilon_0 m v^2 b}{Ze^2} \sin \frac{\chi}{2} = \int_{-\frac{1}{2}(\pi-\chi)}^{\frac{1}{2}(\pi-\chi)} \cos \varphi d\varphi = 2 \cos \frac{\chi}{2}. \quad (5)$$

This can be rewritten as

$$b = \frac{Z e^2}{4\pi\epsilon_0 E_k} \cotg \frac{\chi}{2}, \quad (6)$$

where $E_k = \frac{mv^2}{2}$ is the kinetic energy of the incoming α particle. This dependence can be understood as the relation between the scattering angle χ and the impact parameter b . An alternative explanation is that all α particles incoming to the area $\sigma = \pi b^2$ around the nucleus are scattered to the angle χ or larger.

In reality, we cannot experiment with a single α particle, and we rather use a beam of α particles of known kinetic energy and their flight direction. We also do not have a single target nucleus, but we use a thin foil. Let us assume that the α particle beam impacts an area S of the foil with the thickness d having (volume) concentration of atomic nuclei n . The incoming α particles interact with nSd target atomic nuclei. The foil area that the α particles need to aim for to scatter by the angle χ or more is $\sigma nS d$. The ratio of the α particles scattered by χ or more to all the α particles is

$$f = \frac{\sigma nS d}{S} = \pi b^2 n d = \pi n d \left(\frac{Z e^2}{4\pi\epsilon_0 E_k} \right)^2 \cotg^2 \frac{\chi}{2}. \quad (7)$$

We do not have detectors for counting particles scattered to angles larger than a certain value in a real experiment. Usually, only detectors capable of counting particles in a small interval of angles $(\chi, \chi + d\chi)$ are available. The number of α particles in this interval can be obtained by differentiating the previous formula (7)

$$df = -\pi n d \left(\frac{Z e^2}{4\pi\epsilon_0 E_k} \right)^2 \cotg \frac{\chi}{2} \sin^{-2} \frac{\chi}{2} d\chi. \quad (8)$$

If we place the foil into the centre of an imaginary spherical surface of the radius r , the particles scattered into the angle interval $(\chi, \chi + d\chi)$ will fly out of this spherical surface through a circular strip with the area of $dS_r = 2\pi r \sin \chi r d\chi = 4\pi r^2 \sin \frac{\chi}{2} \cos \frac{\chi}{2} d\chi$. The number of α particles captured by a detector of unit area per one unit of time placed in the distance r from the foil will be

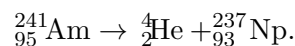
$$N = \frac{N_0 |df|}{dS_r} = N_0 n d \left(\frac{Z e^2}{8\pi\epsilon_0 r E_k} \right)^2 \frac{1}{\sin^4 \frac{\chi}{2}}, \quad (9)$$

where N_0 is the initial number of the α particles hitting the foil. Due to simplicity, we usually do not use the number of scattered particles onto a unit area in the distance r , but rather onto an elementary area dS described by a solid angle $dS = r^2 d\Omega$. The number of α particles scattered to this solid angle is

$$dN = \frac{N_0 |df|}{dS_r} dS = N_0 n d \left(\frac{Z e^2}{8\pi\epsilon_0 E_k} \right)^2 \frac{1}{\sin^4 \frac{\chi}{2}} d\Omega. \quad (10)$$

Poisson distribution testing

Atomic nuclei are composed of protons and neutrons. Only some combinations of the numbers of protons and neutrons are stable. Others are unstable, and they undergo one of thy different types of radioactive decay. An example is Americium ^{241}Am undergoing the α decay



The ${}^4_2\text{He}$ particle is a helium nucleus alternatively called an α particle.

The decay of a nucleus is a random process. It cannot be exactly predicted when it happens. Also, decays of neighbouring nuclei do not directly affect the time of decay of the nucleus. If

the decays do not happen too often, the probability of detecting α particles originating from the decay in a specific time interval obeys the Poisson distribution

$$P(n) = \frac{\lambda^n}{n!} e^{-\lambda}. \quad (11)$$

. Here $P(n)$ is the probability that n decays occur (n α particles are detected), and λ is the mean (expected) number of decays counted during the measurement interval. The use of the Poisson distribution primarily predicts the probability of phenomena occurring in some specific time interval. When we know that a certain phenomenon repeats itself twice per minute on average and that the Poisson distribution can describe its occurrence, we can determine the probability of n during two minutes by selecting $\lambda = 2 \cdot 2 = 4$. For example, the probability of measuring five phenomena in two minutes will be

$$P(5) = \frac{4^5}{5!} e^{-4} \doteq 0.156. \quad (12)$$

This corresponds to the value plotted in Figure 4. It can also be seen that the occurrence of 3 or 4 phenomena is the most probable with the probability of around 20%.

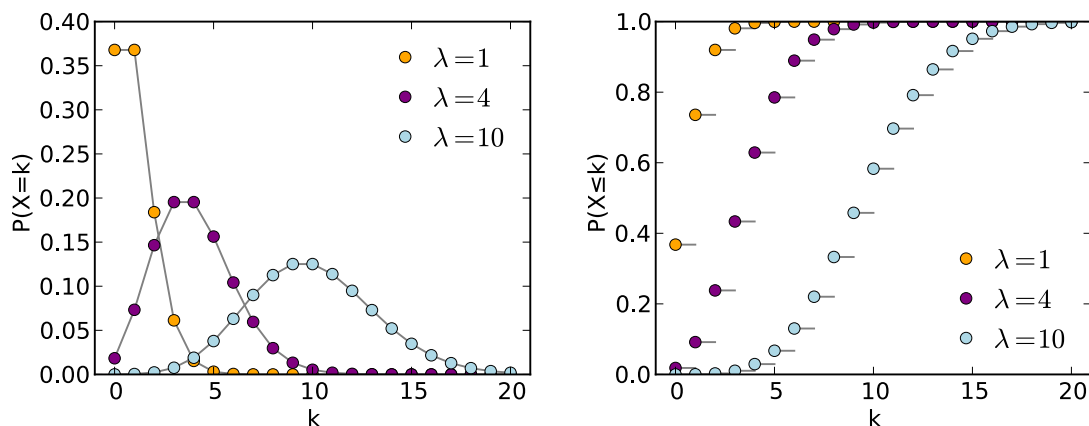


Figure 4: Poisson distribution: probability function and corresponding distribution function.

It is necessary to verify that the Poisson distribution can describe α decay of americium. This can be done in several ways. The easiest one is to measure for a very long time interval and determine the mean value of the detected α particles λ_0 . Then, we can divide this interval into N equally long time intervals and determine the number of detected α particles in each time interval. Then the dependence of the number of intervals with n detected particles on n is plotted. Such dependence can be fitted by the expected shape of the Poisson distribution with the fitting parameter λ . This value can be compared to its theoretical value λ_t calculated as $\lambda_t = \lambda_0 T$, where T is the length of one time interval N .

However, such a method is incorrect from the statistics point of view. It does not provide for a quantitative assessment of the degree of reliability of the obtained result. That's why statistics work with probability. First, we need to formulate a hypothesis we want to test. In our case, the hypothesis is that the detected α particles obey the Poisson distribution. Next, we choose a parameter a that corresponds to the probability of wrongfully rejecting the hypothesis. The usual values of $a = 0.05$ or $a = 0.01$ are chosen. The number a is called the level of reliability.

Next, we need to choose the correct test that we use to check our hypothesis. In the case of testing of distribution functions, the so-called "goodness-of-fit tests" are used. The most used is the χ^2 (chi squared) test, as it can be used to test any discrete distribution. In the χ^2 we compare the measured values to the expected distribution. We work similarly as in the previously described statically incorrect way. However, the test evaluation is different. We take a very

Degrees of freedom	0.05	0.01
1	3.841	6.635
2	5.991	9.210
3	7.815	11.341
4	9.483	13.277
5	11.070	15.086
6	12.592	16.812
7	14.067	18.475
8	15.507	20.090
9	16.919	21.666
10	18.307	23.209
11	19.675	24.725
12	21.026	26.217
13	22.362	27.688
14	23.685	29.141
15	24.996	30.578
16	26.296	32.000
17	27.587	33.409
18	28.868	34.805
19	30.144	36.191
20	31.410	37.566

Table 1: χ^2 test critical values.

First, we will check the conditions for the use of the χ^2 test. The first condition states that we should expect at least 5 scores for every possible value on the dice. This condition is fulfilled. We threw the dice 60 times and for the probability $P(n) = 1/6$ we expect $NP(n) = 10$ scores for every dice value. However, if we threw the dice only 20 times, we would get $NP(n) = 3,33 < 5$. In such a case, we would need to merge several points, and the table could look something like this:

n	1 + 2	3 + 4	5 + 6
$K_j(n)$	5	9	6
$P_j(n)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

The second condition states that we need to compare the whole distribution function. This condition is fulfilled as no other result apart from the 6 different numbers is possible.

Now we can calculate the value of χ^2 according to (14) as

$$\chi^2 = \sum_{j=1}^5 \frac{(K_j(n) - NP_j(n))^2}{NP_j(n)} = \frac{(12 - 10)^2}{10} + \frac{(3 - 10)^2}{10} + \frac{(9 - 10)^2}{10} + \frac{(15 - 10)^2}{10} + \frac{(7 - 10)^2}{10} + \frac{(14 - 10)^2}{10} = 10,4$$

Now we compare this value to the values shown in table 1. For the level of reliability of 0.05 and 5 degrees of freedom we have the critical value of $\chi^2 = 11,07$. As we obtained a lower number we cannot reject the hypothesis that the dice is even.

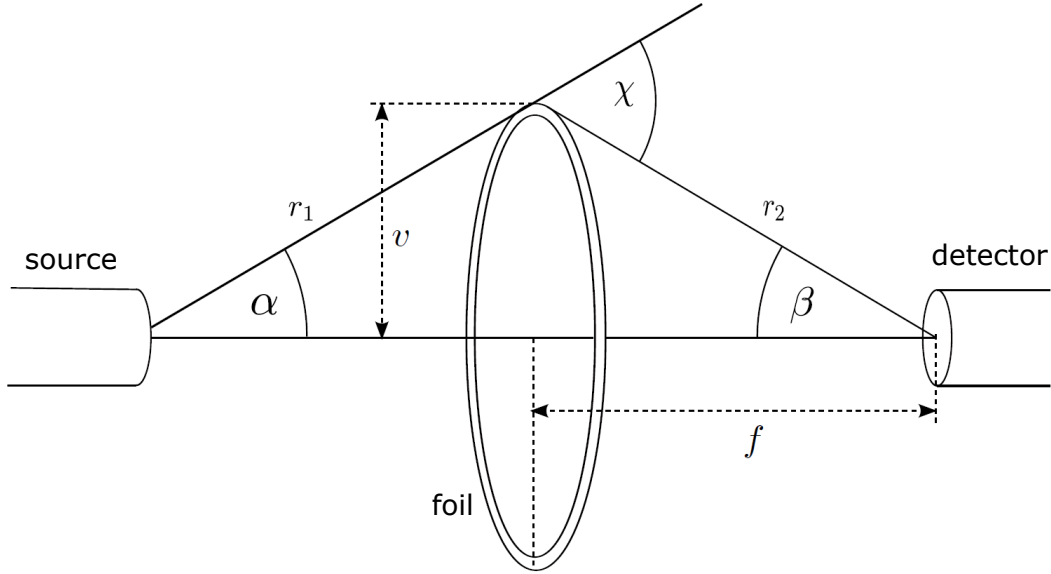


Figure 5: Experimental setup.

Experiment description

The first part of the manual has shown us that the number of α particles scattered under the angle χ to the element of the solid angle $d\Omega$ per a unit of time can be written as

$$dn = N \frac{K_1}{\sin^4 \frac{\chi}{2}} d\Omega, \quad (15)$$

where N is the number of particles hitting the foil, and K_1 is a constant determined by the experimental parameters. A diagram of the experimental setup is shown in Figure 5. Let us assume that the source of the α particles has a relatively small area S_z and that the α particles fly out into all directions equally at the rate of N_0 per unit of time. Let us assume that the gold foil thickness is negligibly small. Then, the α particles hitting the foil are those incoming from the solid angle under which the source is in the line of sight from any point on the foil. This can be written as

$$N = N_0 \frac{S_z \cos \alpha}{r_1^2}. \quad (16)$$

Particles scattered on the foil fly in different directions. Only those scattered to the solid angle

$$\Omega_d = \frac{S_d \cos \beta}{r_2^2} \quad (17)$$

can get to the detector with the area S_d .

To determine the number of particles counted by the detector per unit of time, we should integrate equation 15 within limits for Ω_d . However, for simplification, we can assume that the source and detector areas are small compared to the dimensions of the whole apparatus. We then can consider the scattering angle χ independent of the point on the point of the detector that was hit by the α particle. We can obtain the total number of detected α particles per unit of time simply by multiplying equation 15 by the solid angle Ω_d . If Rutherford's description of the scattering process is valid, the number of detected α particles per unit of time n in our experiment can be calculated according to

$$n = K \frac{\cos \alpha \cos \beta}{r_1^2 r_2^2 \sin^4 \frac{\chi}{2}}, \quad (18)$$

where $K = N_0 S_z S_d K_1$ is a constant given by the experimental setup.

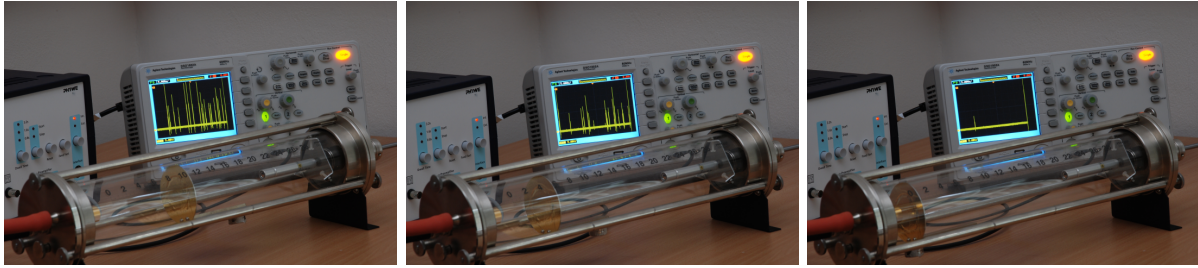


Figure 6: Influence of the gold foil position on the number of detected α particles.

Measurement planning

Before the laboratory, think over what gold foil positions you need to use to verify the formula describing Rutherford's scattering. Before making this choice, it is convenient to plot the dependence of the solid angle on the gold foil position.

Also, think how long you will measure each point – how many α particles you should detect without too high data noise and error. For this, you should understand the properties of the Poisson distribution: the mean value (μ) of a quantity with the Poisson distribution equals the parameter λ . Next, data scattering (s^2) is also equal to λ and the standard deviation (s) is calculated as $s = \sqrt{\lambda}$. Most of the measured points will fall into the interval $< \mu - s; \mu + s >$. (This majority of points roughly equals 68%, although, in the case of the Poisson distribution, this value is strongly dependent also on μ .) The measure of the relative uncertainty of the number of particles is the ratio of s/μ , which, in the case of Poisson distribution, can be calculated as $1/\sqrt{\lambda}$. These facts can help us assess the relative uncertainty of the measured data we can expect for a certain number of detected α particles.

It is moreover necessary to judge the time available for your measurements. When the gold foil is in the centre between the α particle source and the detector, you can expect approximately 30 detected particles per minute. This information, together with the formula (18), is enough to help you judge how time-consuming your measurement plan will be and if it is realistic.

Measurement setup description

The whole measurement setup is shown in Figure 7. It consists of a glass vacuum tube having an α particle source (Americium ^{241}Am , half-life 432.2 years) inside. The α particle detector is on the other side of the tube. A movable foil holder is placed between the source and the detector. A strong magnet on the outside of the tube is used to move the foil holder. **CAUTION! Do not try to remove the magnet from the glass – the glass may break.** The gold foil is in the shape of an annulus with the mean radius of $v = 2$ cm, the distance between the source and the detector is 22.7 cm. A scale on the outer wall of the glass chamber is used to read the position of the foil.

The mean free path of the α particles in the air under ambient pressure is very small. This negatively influences or even completely prohibits the measurement as the α particles scatter not only on the foil but on the gas molecules during their whole path. To avoid such a problem, we need to decrease the pressure in the tube as much as possible. This is done using a membrane vacuum pump (2). The pressure in the tube is measured by a manometer (4). As neither the pump nor the manometer is leak-free, a throttling valve (3) is used to seal the tube. A venting valve is used to vent the tube after the measurement.

α particles arriving on the detector cause rise of electric signals. These are first amplified in a preamplifier (1) and amplifier (5). These impulses are then shown and counted by an oscilloscope (7).

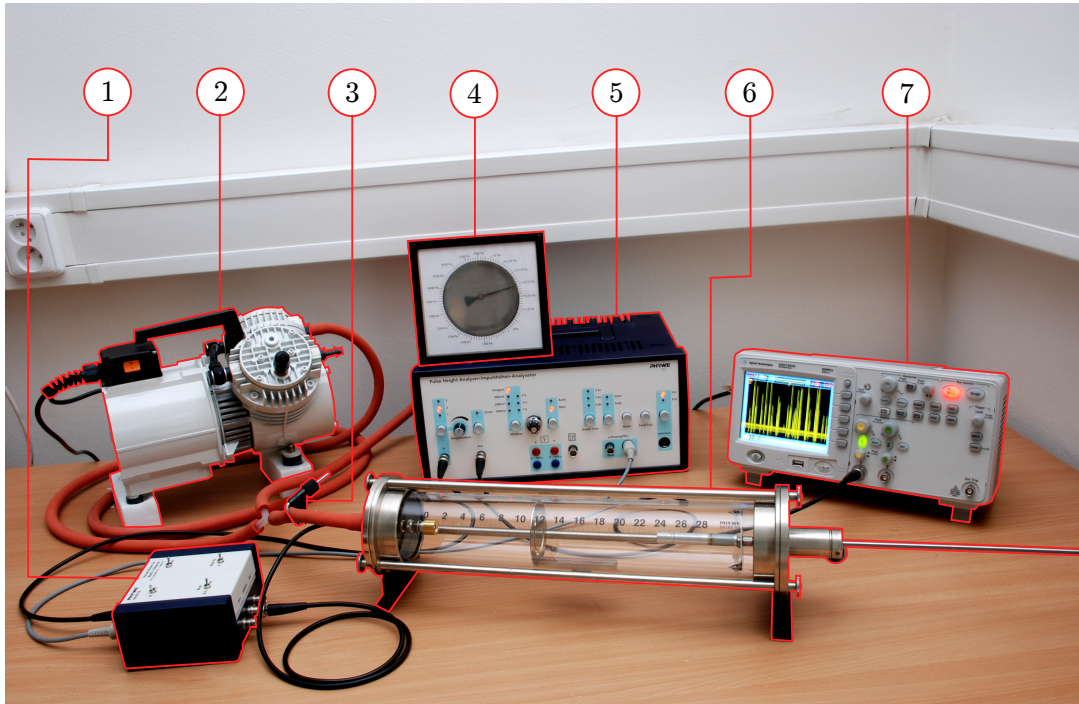


Figure 7: Measurement setup: (1) preamplifier, (2) vacuum pump, (3) throttle valve, (4) manometer, (5) amplifier and discriminator, (6) vacuum tube (containing (a) detector, (b) gold foil and (c) α particle source), (7) oscilloscope.

Experimental procedure

Decrease the pressure in the tube to about 1 kPa using the membrane pump. Seal the tube by the throttling valve and turn off the pump. Turn on the oscilloscope and connect a USB drive (bring your own!). Set a correct measurement time on the oscilloscope and move the x-axis to have the trigger on the left side of the screen. Make sure that after pressing the **Acquire** button, the **acquisition** option in the menu is set to **Peak Detect**. The menu can be closed by pressing the **Menu On/Off** button. Set an appropriate position of the gold foil. Start the measurement on the oscilloscope using the **SINGLE** button. Save the measurement to your USB disc using the **Save/Recall** button.

Measure the dependence of the detected α particles on the scattering angle. Plot this dependence in a graph and verify the formula (18) for the Rutherford scattering.

Before verifying the Poisson distribution of the detected α particles, move the gold foil into the position that corresponds to the maximum number of the detected α particles. Set the longest possible measurement time on the oscilloscope. Carry out the measurement, save the data and repeat the measurement at least three times to have a long combined measurement time interval. When processing the data, you will divide this interval into shorter ones. You will find out if the number of the α particles in the short intervals corresponds to the Poisson distribution.

Move the foil holder close to the source or the detector after the measurement.

References

- [1] Jiří Anděl, *Statistické metody*, Matfyzpress, Praha 2003
- [2] H. Geiger, E. Marsden, *On a Diffuse Reflection of the α -Particles*, Proc. Roy. Soc. vol. 82, pp. 495-500, 1909

- [3] E. Rutherford, The Scattering of α and β Particles by Matter and the Structure of the Atom, Philos. Mag., vol 6, pp.21, 1911