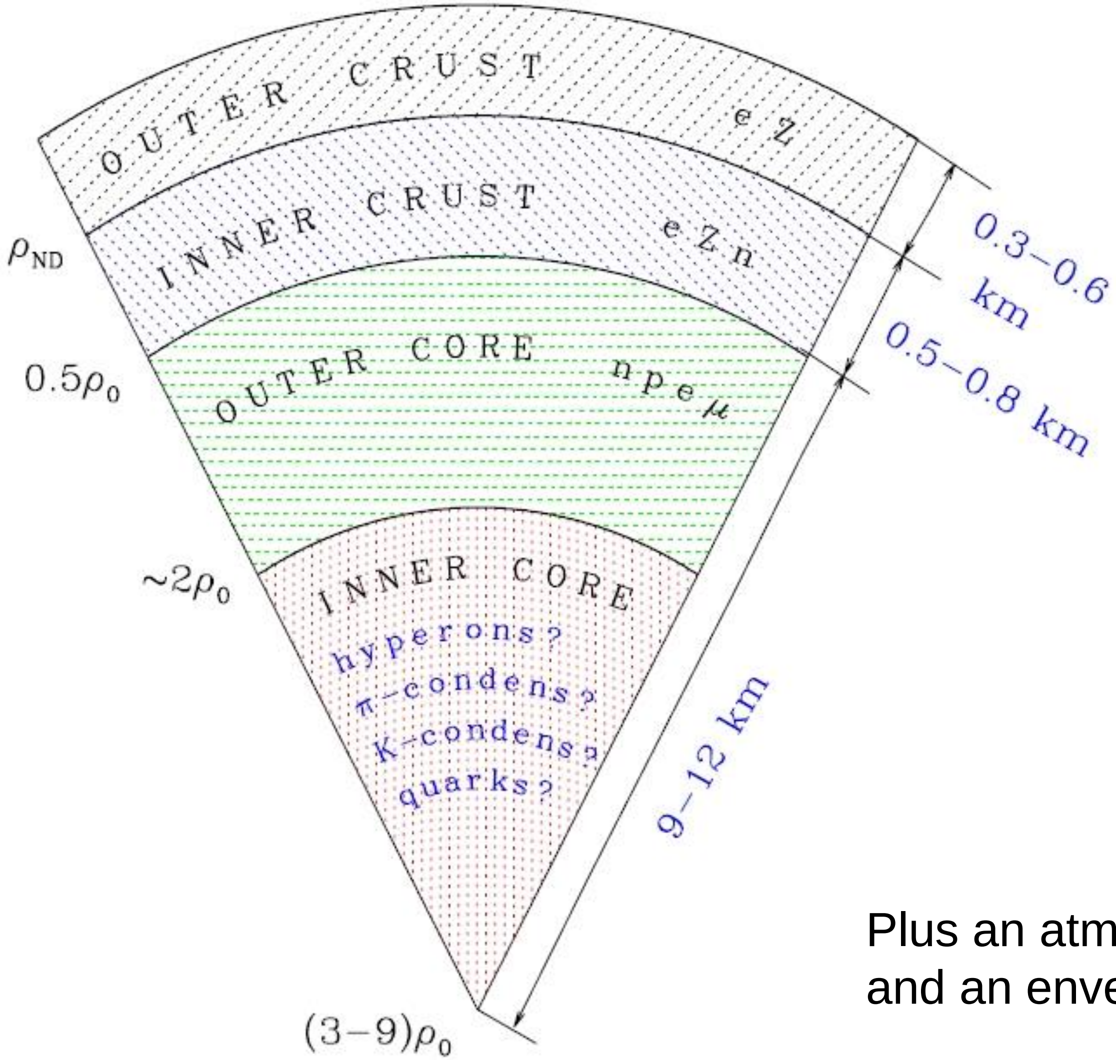
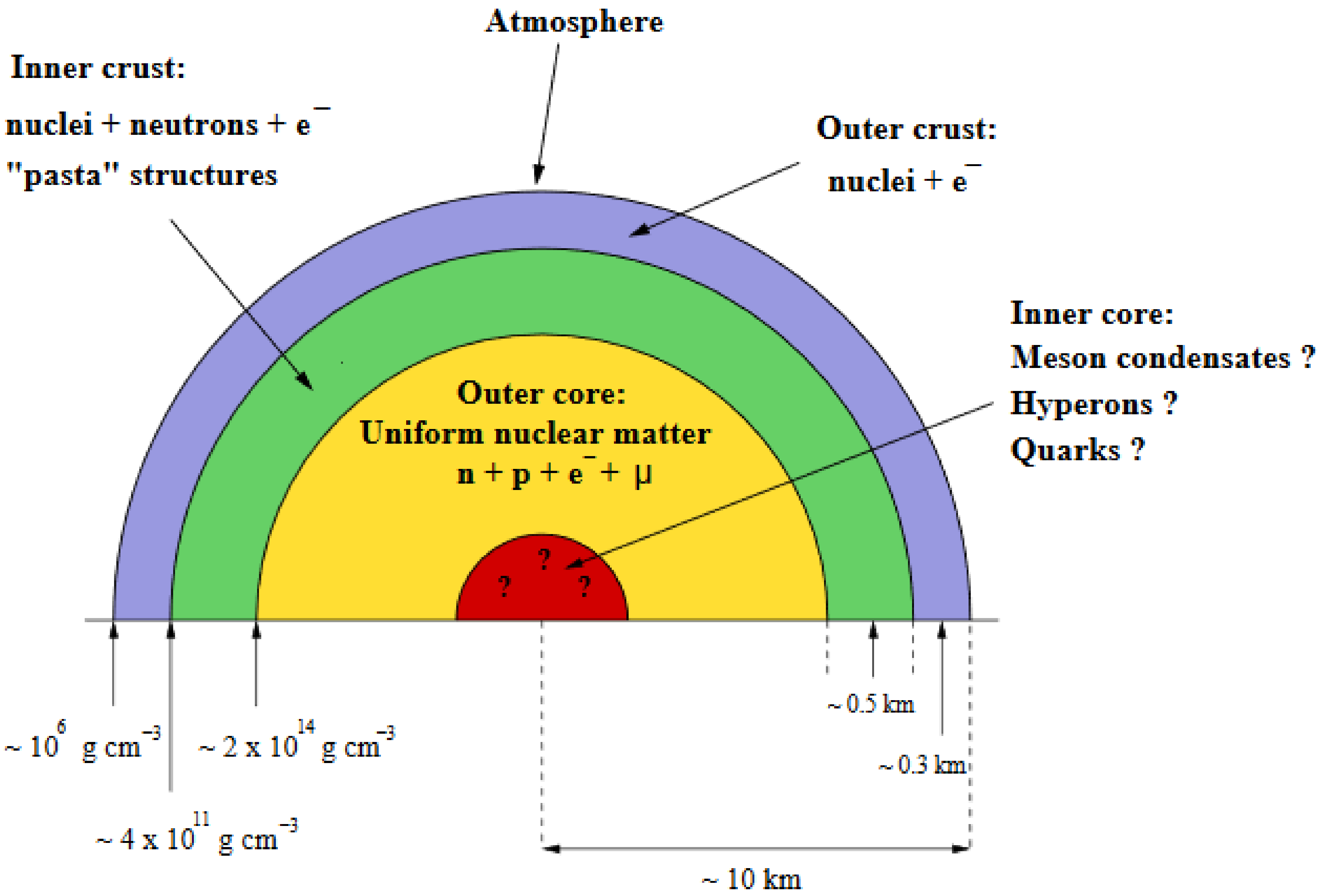
# Neutron Star EoS, masses and radii

# Structure and layers

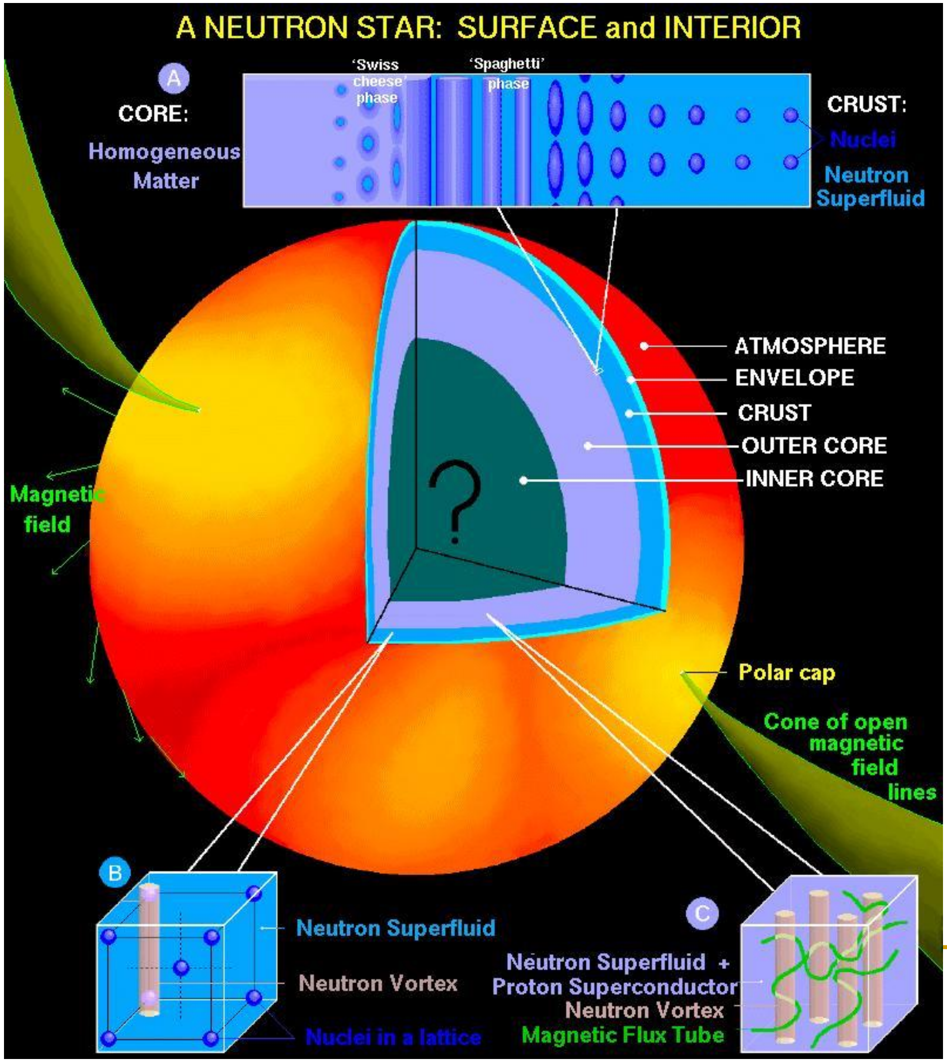



Plus an atmosphere
and an envelope ...

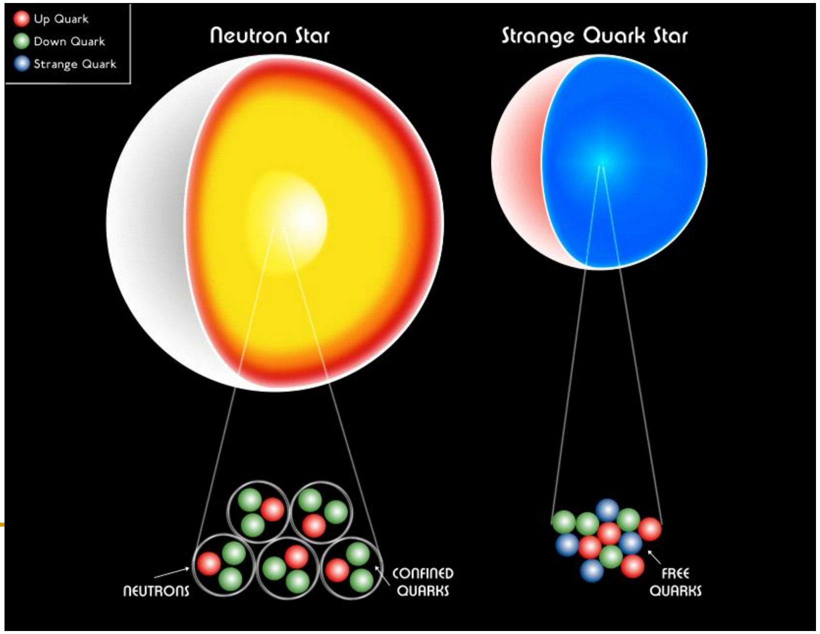

Atmosphere
Inner crust:
nuclei + neutrons + e⁻
"pasta" structures
Outer crust:
nuclei + e⁻
Inner core:
Meson condensates ?
Hyperons ?
Quarks ?
Outer core:
Uniform nuclear matter
n + p + e⁻ + μ
? ? ?
~ 10^6 g cm^-3
~ 2 x 10^14 g cm^-3
~ 4 x 10^11 g cm^-3
~ 0.5 km
~ 0.3 km
~ 10 km

2007.04427

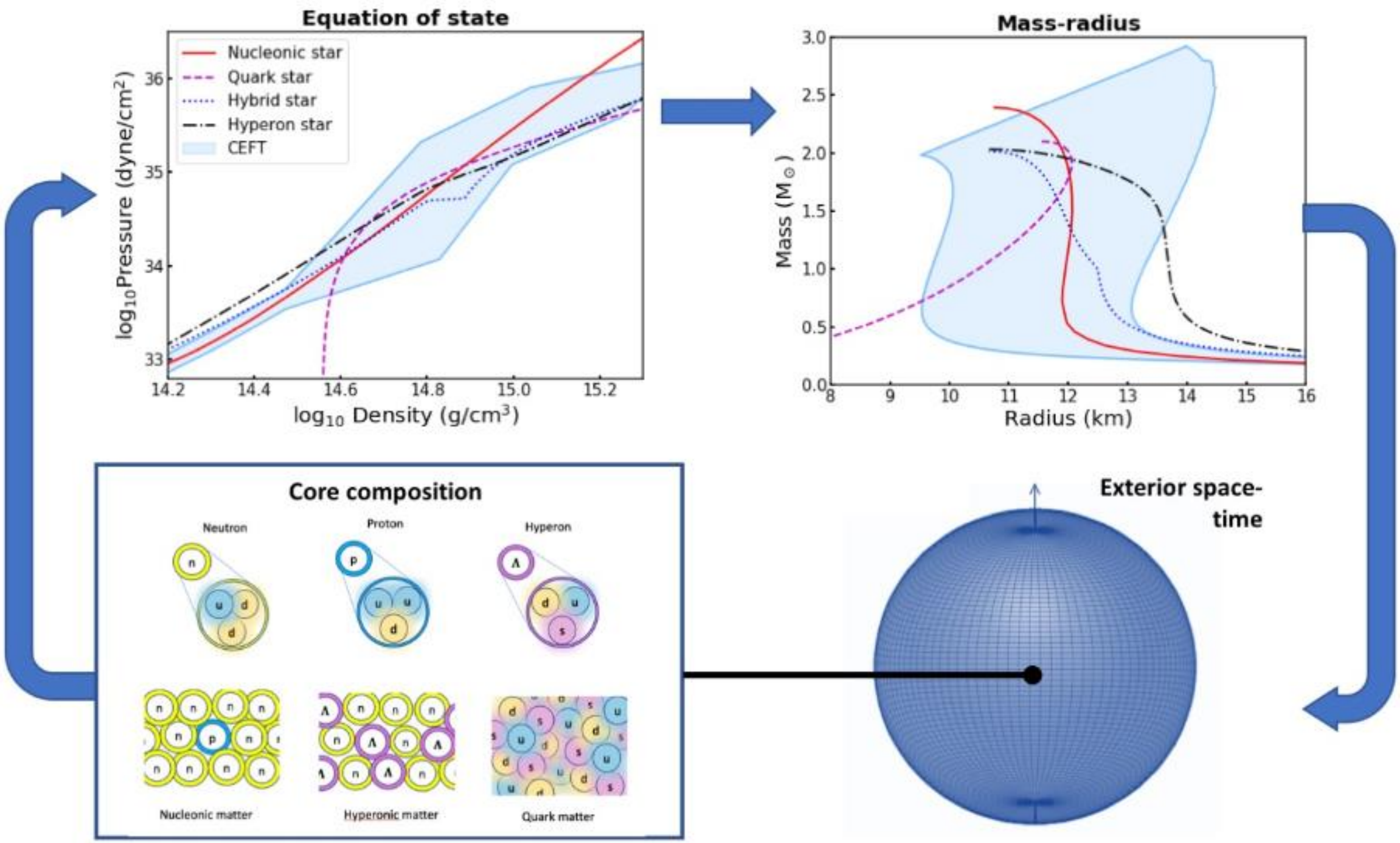
# Neutron star interiors




Radius: 10 km
Mass: 1-2 solar
Density: above the nuclear
Strong magnetic fields



# Why important?




1903.04648, a short review on M-R measurements related to EoS

# Astrophysical point of view

**Astrophysical appearence of NSs**
**is mainly determined by:**

- **Spin**
- **Magnetic field**
- **Temperature**
- **Velocity**
- **Environment**

The first four are related to the NS structure!

# Equator and radius

$$ds^2=c^2dt^2e^{2\Phi}-e^{2\lambda}dr^2-r^2[d\theta^2+\sin^2\theta d\varphi^2]$$

In flat space $\Phi(r)$ and $\lambda(r)$ are equal to zero.

- $t=const,\ r= const,\ \theta=\pi/2,\ 0<\Phi<2\pi \Rightarrow l=2\pi r$
- $t=const,\ \theta=const,\ \varphi=const,\ 0<r<r_0 \Rightarrow dl=e^{\lambda}dr \Rightarrow l=\int_0^{r_0} e^{\lambda}dr\neq r_0$

# Gravitational redshift

<1

$$d\tau = dt\, e^{\Phi}, \qquad \nu_r = \frac{dN}{d\tau} = e^{-\Phi}\frac{dN}{dt}$$

Frequency emitted at r

$$r \to \infty \qquad \Phi \to 0 \qquad \nu_\infty = \frac{dN}{dt}$$

Frequency detected by an observer at infinity

$$\nu_\infty = \nu_r\, e^{\Phi} \quad \Rightarrow \quad \Phi(r)$$

This function determines gravitational redshift

$$e^{2\lambda} \equiv \frac{1}{1 - \dfrac{2Gm}{c^2 r}}$$

It is useful to use m(r) – gravitational mass inside r – instead of λ(r)

# Outside of the star

$$r > R \Rightarrow m(r) = M = \text{const}$$

$$e^{2\Phi} = 1 - \frac{2GM}{c^2 r} = 1 - \frac{r_g}{r}, \qquad r_g = \frac{2GM}{c^2}$$

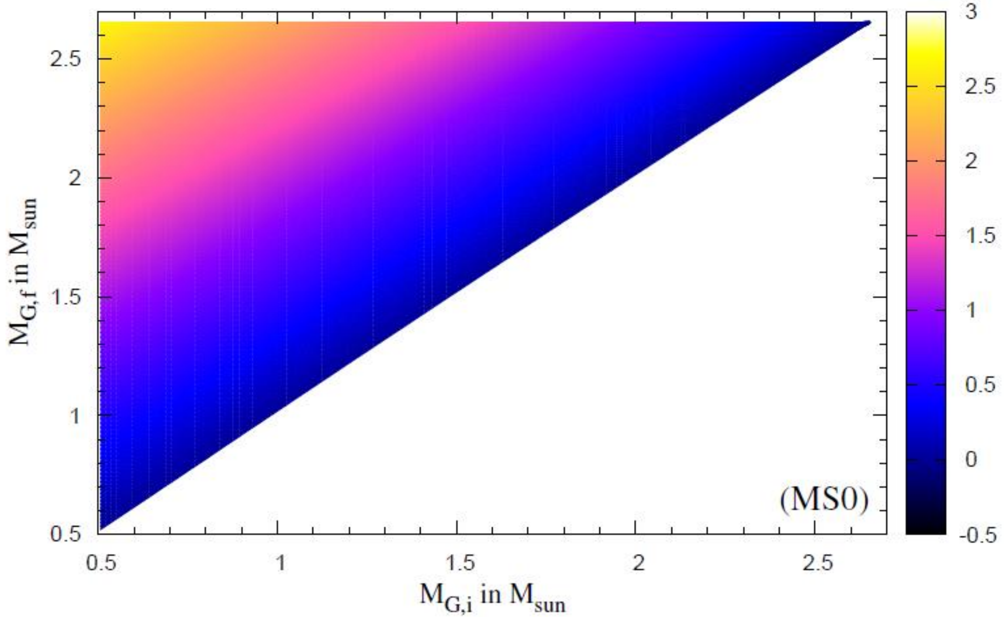
$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2\, dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2\, d\Omega^2$$

$$\nu_\infty = \nu_r \sqrt{1 - \frac{r_g}{r}}$$ → redshift

Bounding energy → $\Delta M = M_b - M \sim 0.2\ M_{\text{sun}}$

Apparent radius → $R_\infty = R / \sqrt{1 - r_g / R}$

# Bounding energy




| $M_{G,i}$ $(M_\odot)$ | $\Delta M_G$ $(M_\odot)$ | $M_{B,i}$ $(M_\odot)$ | | $M_{acc}$ $(\Delta M_B)$ $(M_\odot)$ | |
|---|---|---|---|---|---|
| | | APR | MS0 | APR | MS0 |
| 1.4 | 0.57 | 1.554 | 1.525 | 0.768 | 0.712 |
| 1.5 | 0.47 | 1.681 | 1.647 | 0.641 | 0.591 |
| 1.6 | 0.37 | 1.811 | 1.767 | 0.511 | 0.470 |
| 1.7 | 0.27 | 1.943 | 1.892 | 0.379 | 0.345 |
| 1.8 | 0.17 | 2.080 | 2.018 | 0.242 | 0.219 |
| 1.9 | 0.07 | 2.221 | 2.146 | 0.101 | 0.091 |

If you drop a kilo on a NS, then you increase its mass for < kilo

$M_{acc}$ is shown with color

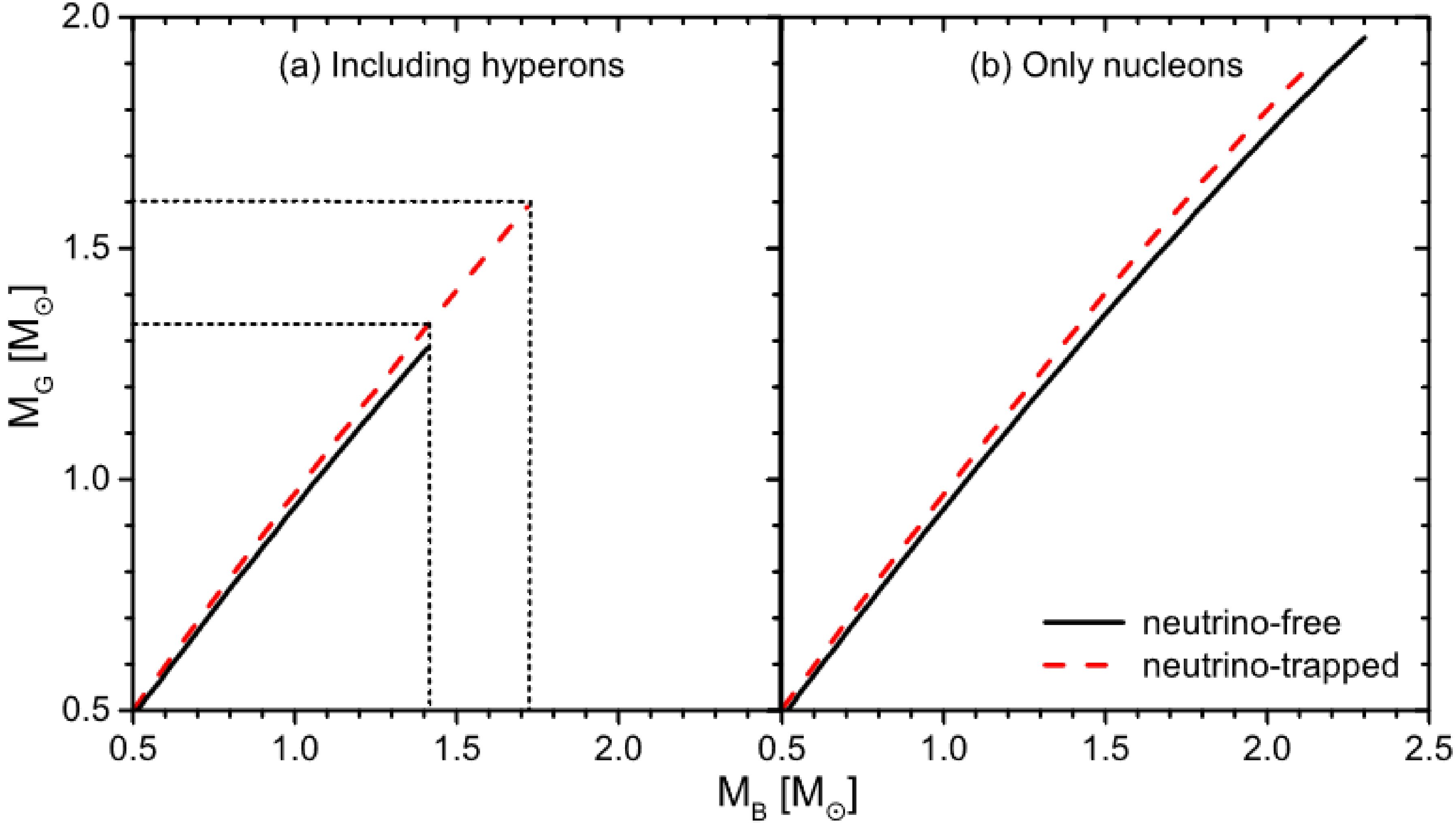
$M_{acc}=\Delta M_G+\Delta BE/c^2=\Delta M_B$

BE- binding energy

$BE=(M_B-M_G)c^2$

1102.2912

# Gravitational mass vs. baryonic




2105.03747

# NS Masses

- Stellar masses are directly measured in binary systems
- Accurate NS mass determination for PSRs in relativistic systems by measuring PK corrections
- Gravitational redshift may provide M/R in NSs by detecting a *known* spectral line,

$$E_\infty = E(1-2GM/Rc^2)^{1/2}$$

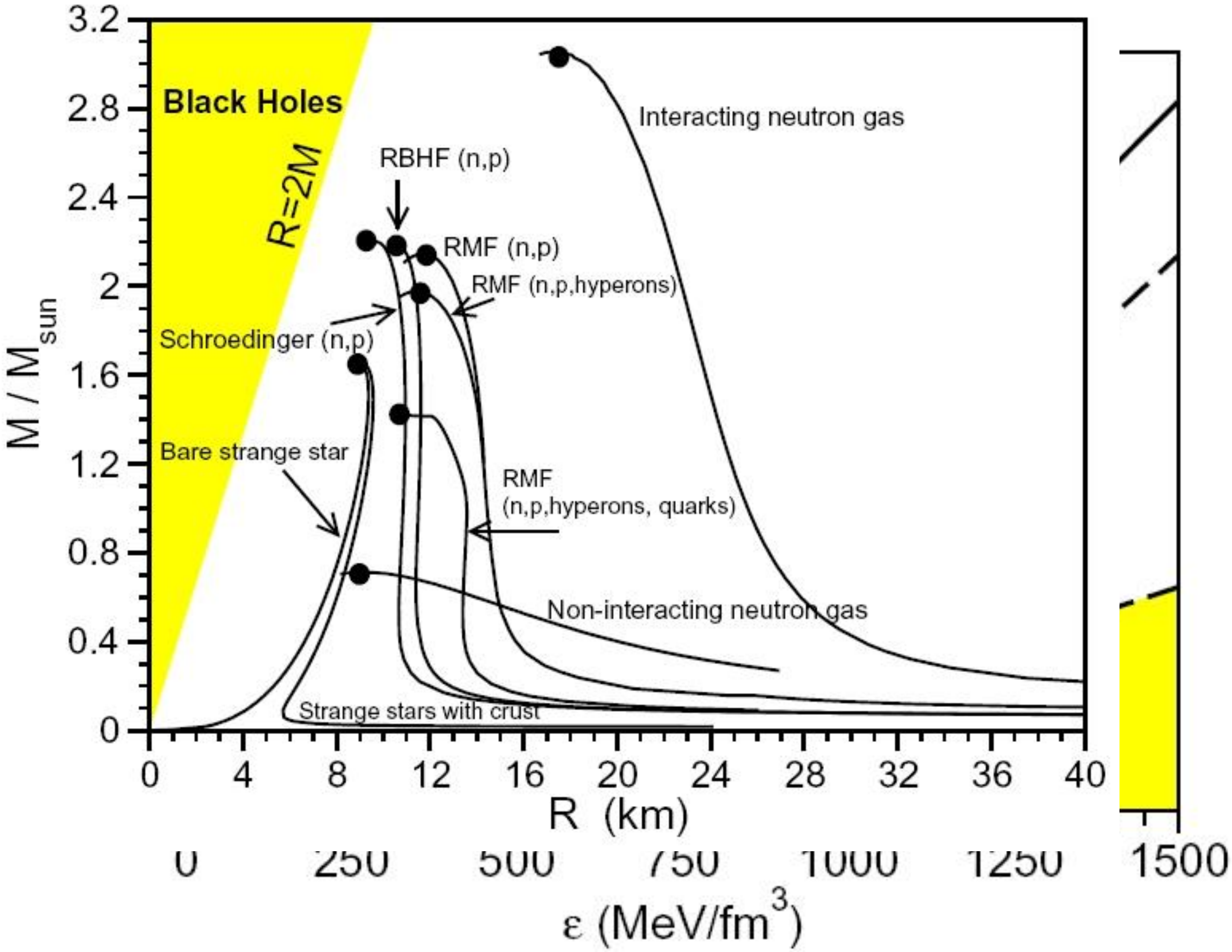
# TOV equation

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik}$$

$$\begin{cases} (1) \quad \dfrac{dP}{dr} = -\dfrac{G\rho m}{r^2}\left(1+\dfrac{P}{\rho c^2}\right)\left(1+\dfrac{4\pi r^3 P}{mc^2}\right)\left(1-\dfrac{2Gm}{rc^2}\right)^{-1} \\ (2) \quad \dfrac{dm}{dr} = 4\pi r^2 \rho \\ (3) \quad \dfrac{d\Phi}{dr} = -\dfrac{1}{\rho c^2}\dfrac{dP}{dr}\left(1+\dfrac{P}{\rho c^2}\right)^{-1} \\ (4) \quad P = P(\rho) \end{cases}$$

***Tolman (1939)***
***Oppenheimer-Volkoff (1939)***

# EoS

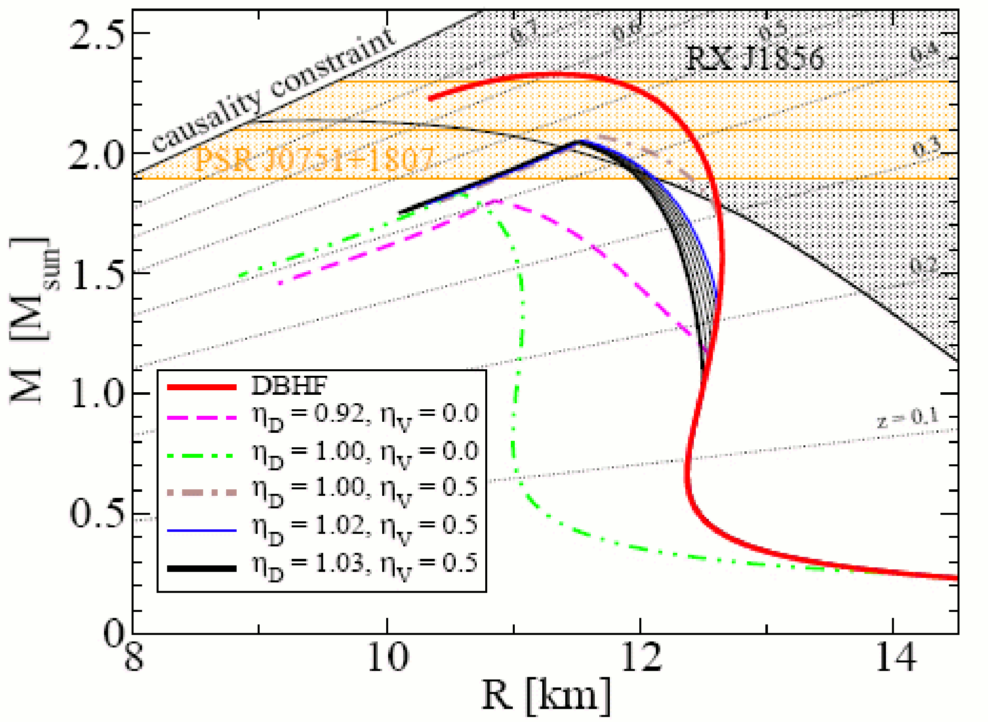



(Weber et al. ArXiv: 0705.2708 )

# Mass-radius

Mass and radius
are marcoscopical
potentially measured
parameters.
Thus, it is important
to formulate EoS
in terms of these
two parameters.

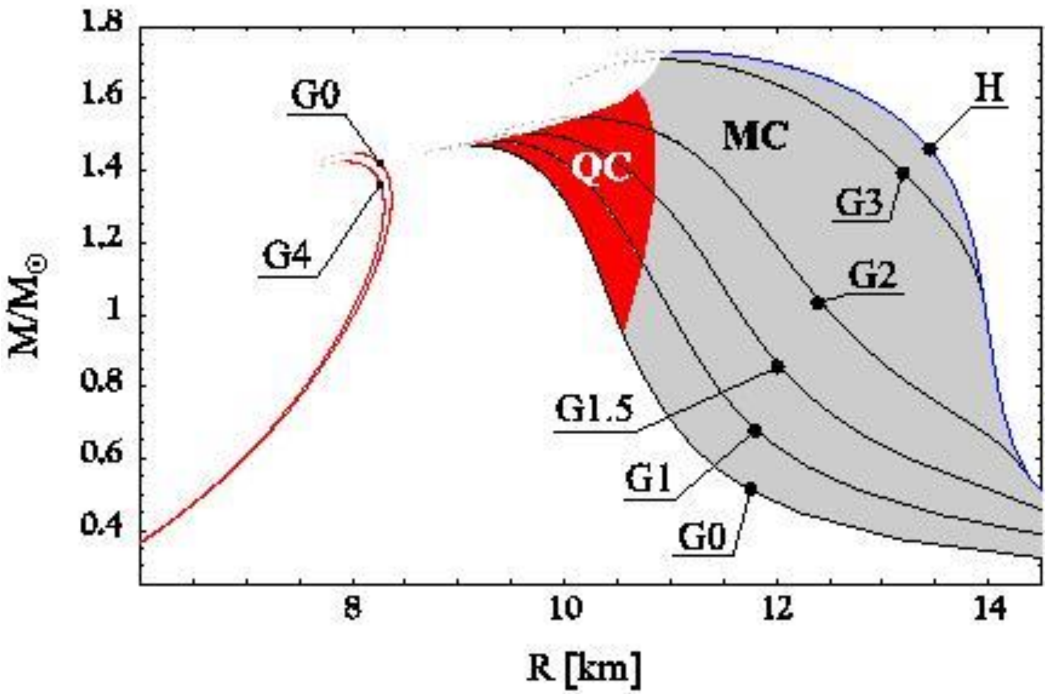
Mass-radius relations for CSs
with possible phase transition
to deconfined quark matter.




About hyperon stars see a
review in 1002.1658.
About strange stars and some other
exotic options – 1002.1793

astro-ph/0611595

# Mass-radius relation

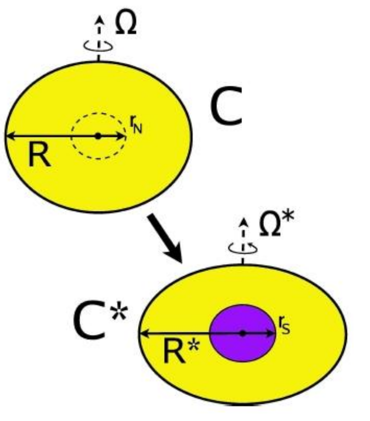



## Main features

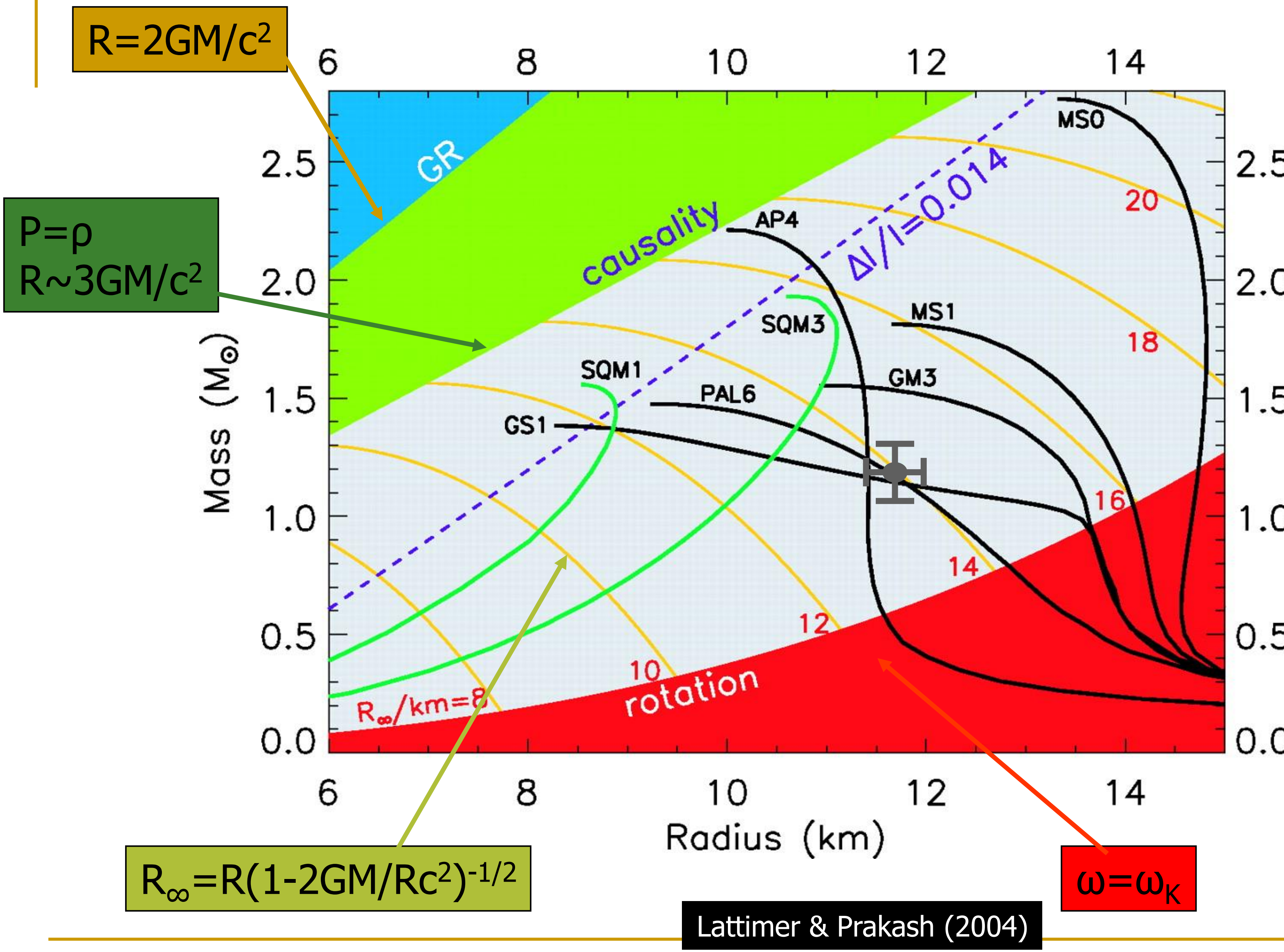
- Max. mass
- Diff. branches (quark and normal)
- Stiff and soft EoS
- Small differences for realistic parameters
- Softening of an EoS with growing mass

Rotation is neglected here.
Obviously, rotation results in:
- larger max. mass
- larger equatorial radius

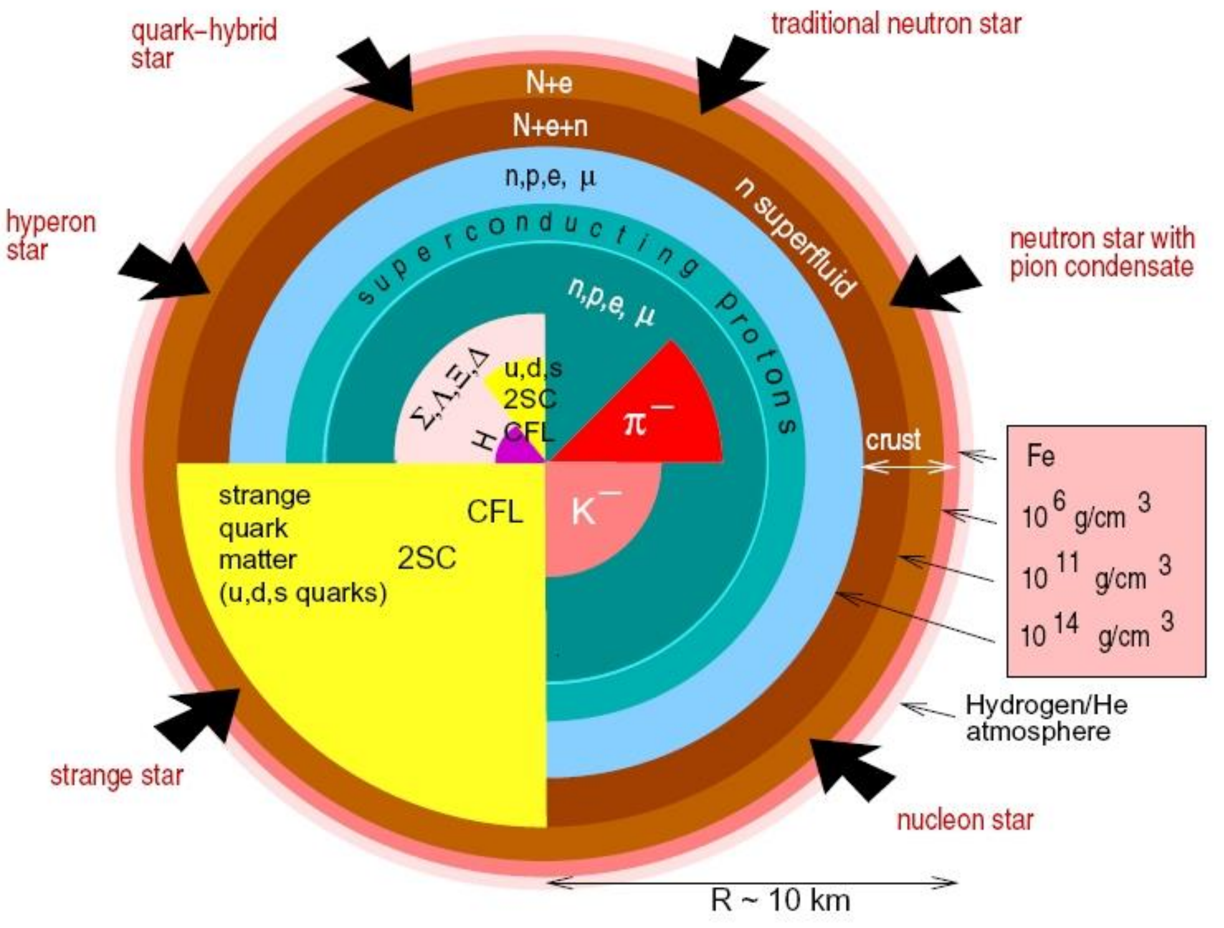
Spin-down can result in phase transition, as well as spin-up (due to accreted mass), see 1109.1179




Haensel, Zdunik
astro-ph/0610549

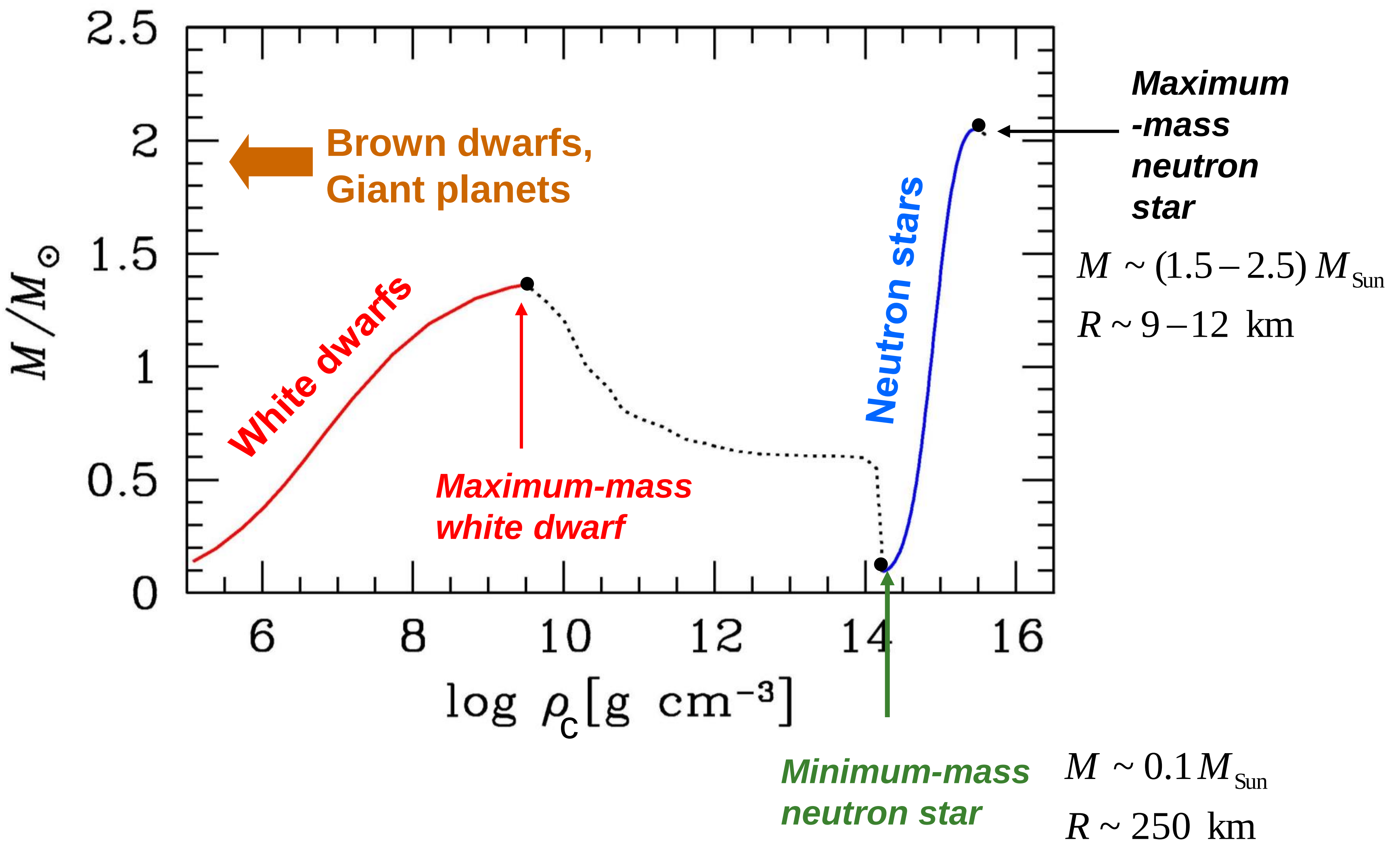

R=2GM/c²
P=ρ
R~3GM/c²
GR
causality
ΔI/I=0.014
AP4
MS0
MS1
SQM3
SQM1
GM3
PAL6
GS1
20
18
16
14
12
10
R∞/km=8
rotation
Mass (M⊙)
Radius (km)
R∞=R(1-2GM/Rc²)^-1/2
ω=ω_K
Lattimer & Prakash (2004)

# NS interiors: resume




(Weber et al. ArXiv: 0705.2708)

# Neutron stars and white dwarfs



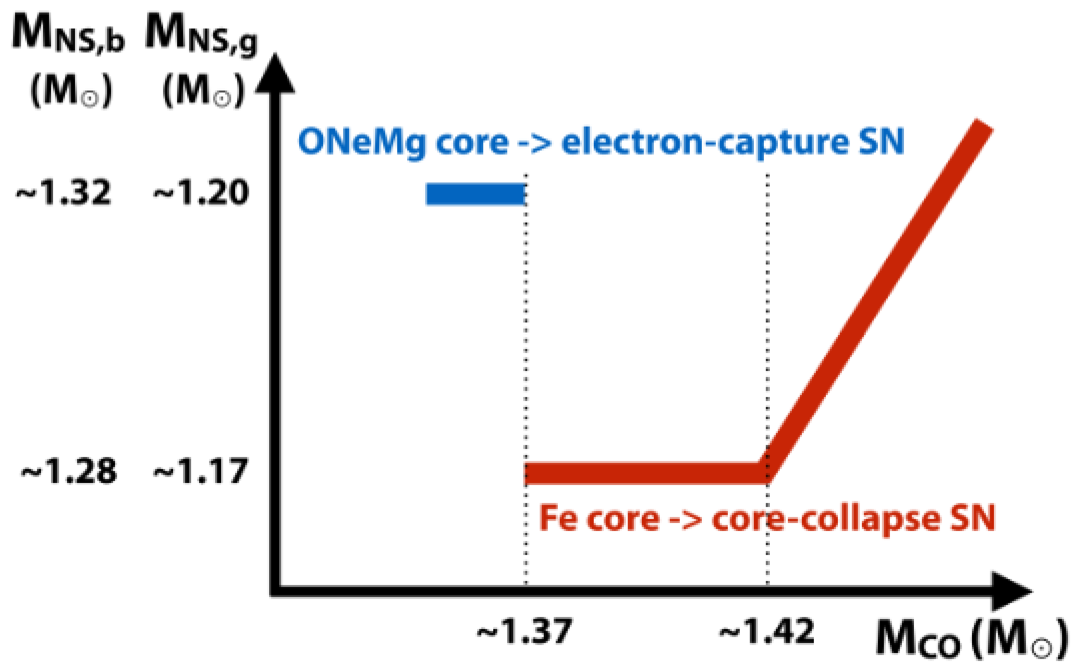

Remember about the difference between baryonic and gravitational masses in the case of neutron stars!

# Minimal mass

In reality, the minimal mass is determined by properties of protoNSs.
Being hot, lepton rich they have much higher limit: about 0.7 solar mass.

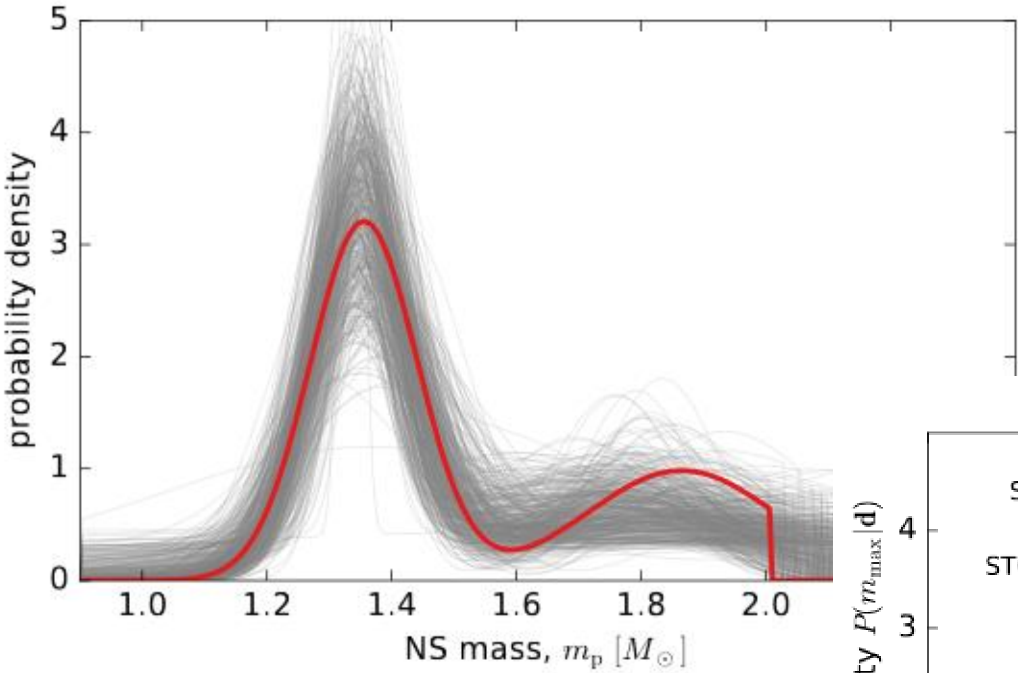
Stellar evolution does not produce NSs with baryonic mass less than about 1.1-1.2 solar.

Fragmentation of a core due to rapid rotation potentially can lead to smaller masses, but not as small as the limit for cold NSs.

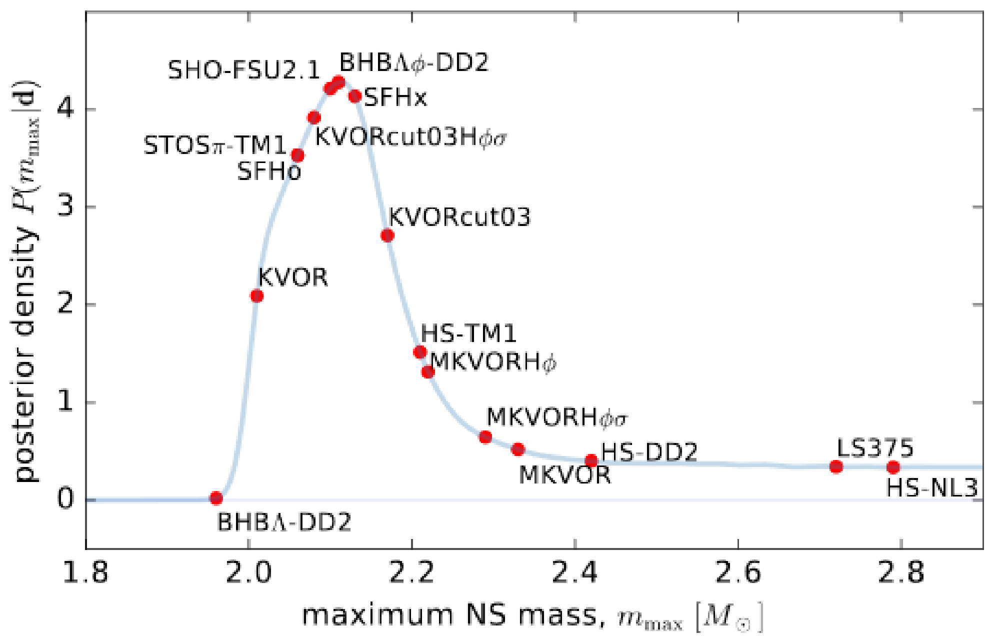



1808.02328

# Maximum mass and cut-off

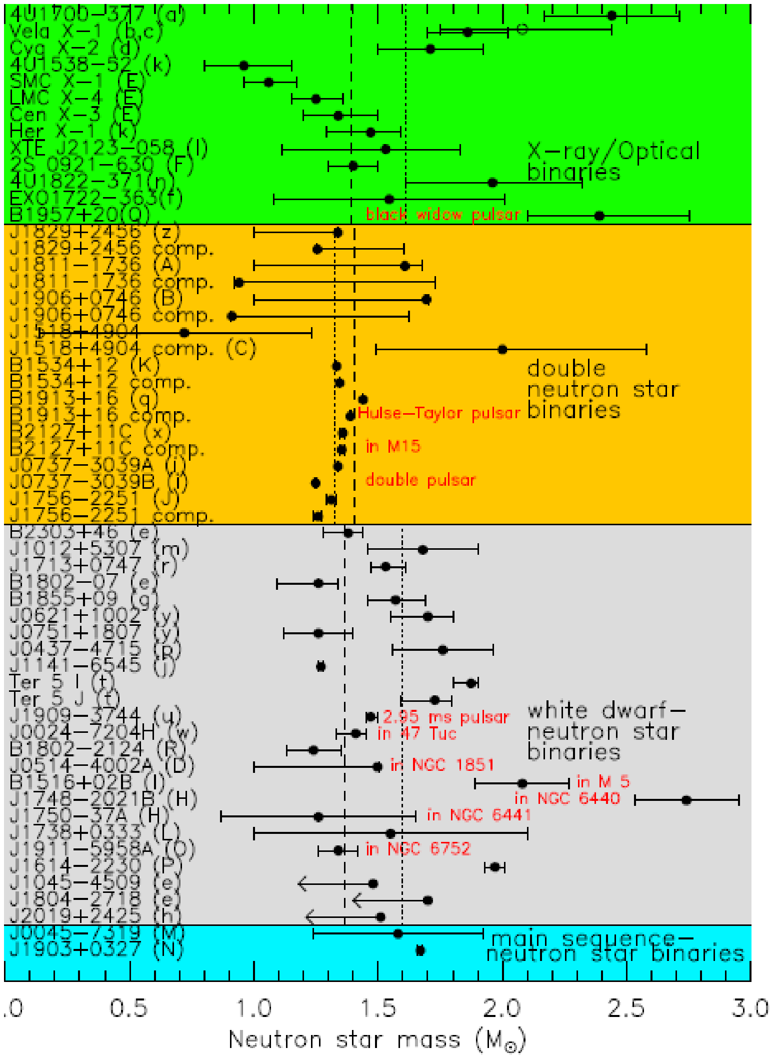



Two gaussians and a hard cut at Mmax




1709.07889

# Neutron star masses

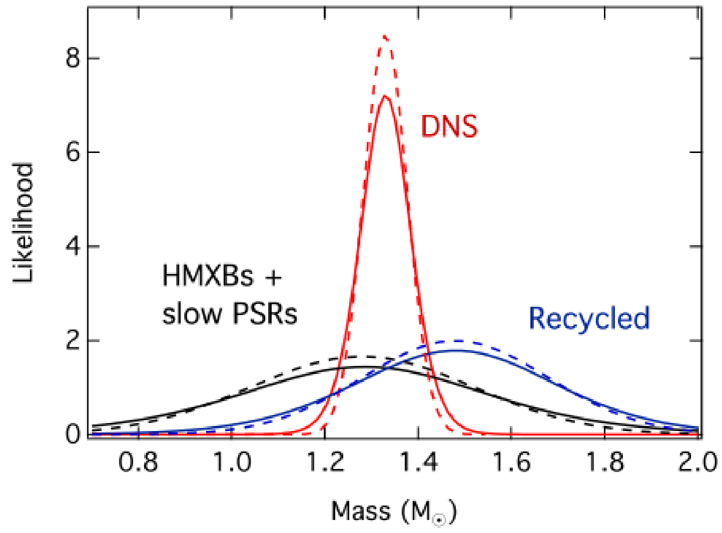


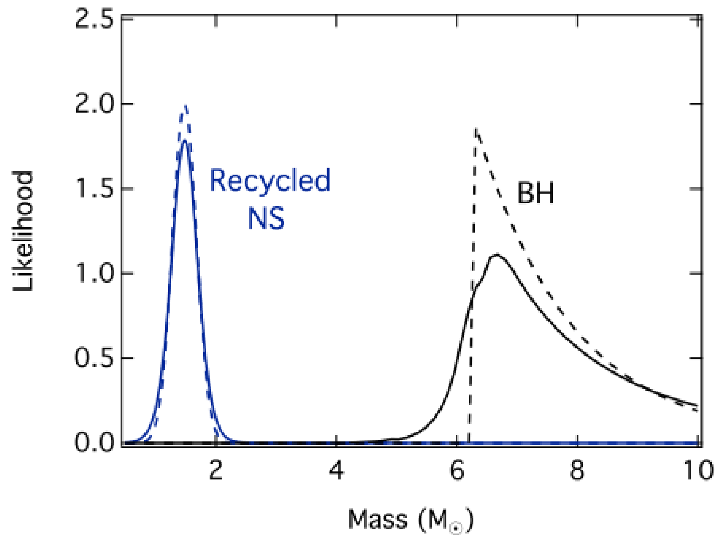

| Object | Mass ($M_\odot$) | Reference | Object | Mass ($M_\odot$) | Reference |
|---|---|---|---|---|---|
| *X-Ray/Optical Binaries (mean = 1.609 $M_\odot$, weighted mean = 1.393 $M_\odot$)* | | | | | |
| 4U1700-377 | $2.44^{+0.27}_{-0.27}$ | a (10) | Vela X-1 | $1.86^{+0.16}_{-0.16}$ | b, c (11, 12) |
| Cyg X-2 | $1.71^{+0.21}_{-0.21}$ | d (13) | 4U1538-52 | $0.96^{+0.19}_{-0.16}$ | k (14) |
| SMC X-1 | $1.06^{+0.11}_{-0.10}$ | E (15) | LMC X-4 | $1.25^{+0.11}_{-0.10}$ | E (15) |
| Cen X-3 | $1.34^{+0.16}_{-0.14}$ | E (15) | Her X-1 | $1.47^{+0.12}_{-0.18}$ | k (14) |
| XTE J2123-058 | $1.53^{+0.30}_{-0.42}$ | l (16, 17) | 2S 0921-630 | $1.4^{+0.1}_{-0.1}$ | F (18) |
| 4U 1822-371 | $1.96^{+0.36}_{-0.35}$ | n (19) | EXO 1722-363 | $1.545^{+0.465}_{-0.465}$ | f (47) |
| B1957+20 | $2.39^{+0.36}_{-0.29}$ | Q (49) | | | |
| *Neutron Star – Neutron Star Binaries (mean = 1.325 $M_\odot$, weighted mean = 1.403 $M_\odot$)* | | | | | |
| J1829+2456 | $1.338^{+0.002}_{-0.338}$ | z (20) | J1829+2456 (c) | $1.256^{+0.346}_{-0.003}$ | z (20) |
| J1811-1736 | $1.608^{+0.066}_{-0.608}$ | A (21) | J1811-1736 (c) | $0.941^{+0.787}_{-0.021}$ | A (21) |
| J1906+07 | $1.694^{+0.012}_{-0.694}$ | B (22) | J1906+07 (c) | $0.912^{+0.710}_{-0.004}$ | B (22) |
| J1518+4904 | $0.72^{+0.51}_{-0.58}$ | C (23) | J1518+4904 (c) | $2.00^{+0.58}_{-0.51}$ | C (23) |
| 1534+12 | $1.3332^{+0.0010}_{-0.0010}$ | K (24) | 1534+12 (c) | $1.3452^{+0.0010}_{-0.0010}$ | K (24) |
| 1913+16 | $1.4398^{+0.0002}_{-0.0002}$ | q (25) | 1913+16 (c) | $1.3886^{+0.0002}_{-0.0002}$ | q (25) |
| 2127+11C | $1.358^{+0.010}_{-0.010}$ | x (26) | 2127+11C (c) | $1.354^{+0.010}_{-0.010}$ | x (26) |
| J0737-3039A | $1.3381^{+0.0007}_{-0.0007}$ | i (27) | J0737-3039B | $1.2489^{+0.0007}_{-0.0007}$ | i (27) |
| J1756-2251 | $1.312^{+0.017}_{-0.017}$ | J (28) | J1756-2251 (c) | $1.258^{+0.017}_{-0.017}$ | J (28) |
| *Neutron Star – White Dwarf Binaries (mean = 1.599 $M_\odot$, weighted mean = 1.362 $M_\odot$)* | | | | | |
| B2303+46 | $1.38^{+0.06}_{-0.10}$ | e (29) | J1012+5307 | $1.68^{+0.22}_{-0.22}$ | m (30) |
| J1713+0747 | $1.53^{+0.08}_{-0.04}$ | r (31, 51) | B1802-07 | $1.26^{+0.08}_{-0.17}$ | e (29) |
| B1855+09 | $1.57^{+0.12}_{-0.11}$ | g (32, 51) | J0621+1002 | $1.70^{+0.10}_{-0.17}$ | y (33) |
| J0751+1807 | $1.26^{+0.14}_{-0.14}$ | y (33) | J0437-4715 | $1.76^{+0.20}_{-0.20}$ | p (34) |
| J1141-6545 | $1.27^{+0.01}_{-0.01}$ | j (35) | Ter 5 I | $1.874^{+0.32}_{-0.068}$ | t (36) |
| Ter 5 J | $1.728^{+0.066}_{-0.136}$ | t (36) | J1909-3744 | $1.47^{+0.03}_{-0.02}$ | u (37) |
| J0024-7204H | $1.41^{+0.04}_{-0.08}$ | w (38) | B1802-2124 | $1.24^{+0.11}_{-0.11}$ | R (50) |
| J0514-4002A | $1.497^{+0.008}_{-0.497}$ | D (39) | B1516+02B | $2.08^{+0.19}_{-0.19}$ | I (40) |
| J1748-2021B | $2.74^{+0.21}_{-0.21}$ | H (41) | J1750-37A | $1.26^{+0.39}_{-0.39}$ | H (41) |
| J1738+0333 | $1.55^{+0.55}_{-0.55}$ | L (42) | J1911-5958A | $1.34^{+0.08}_{-0.08}$ | O (43) |
| J1614-2230 | $1.97^{+0.04}_{-0.04}$ | P (48) | J1045-4509 | $< 1.48$ | e (29) |
| J1804-2718 | $< 1.70$ | e (29) | J2019+2425 | $< 1.51$ | h (44) |
| *Neutron Star – Main Sequence Binaries* | | | | | |
| J0045-7319 | $1.58^{+0.34}_{-0.34}$ | M (45) | J1903+0327 | $1.67^{+0.01}_{-0.01}$ | N (46) |

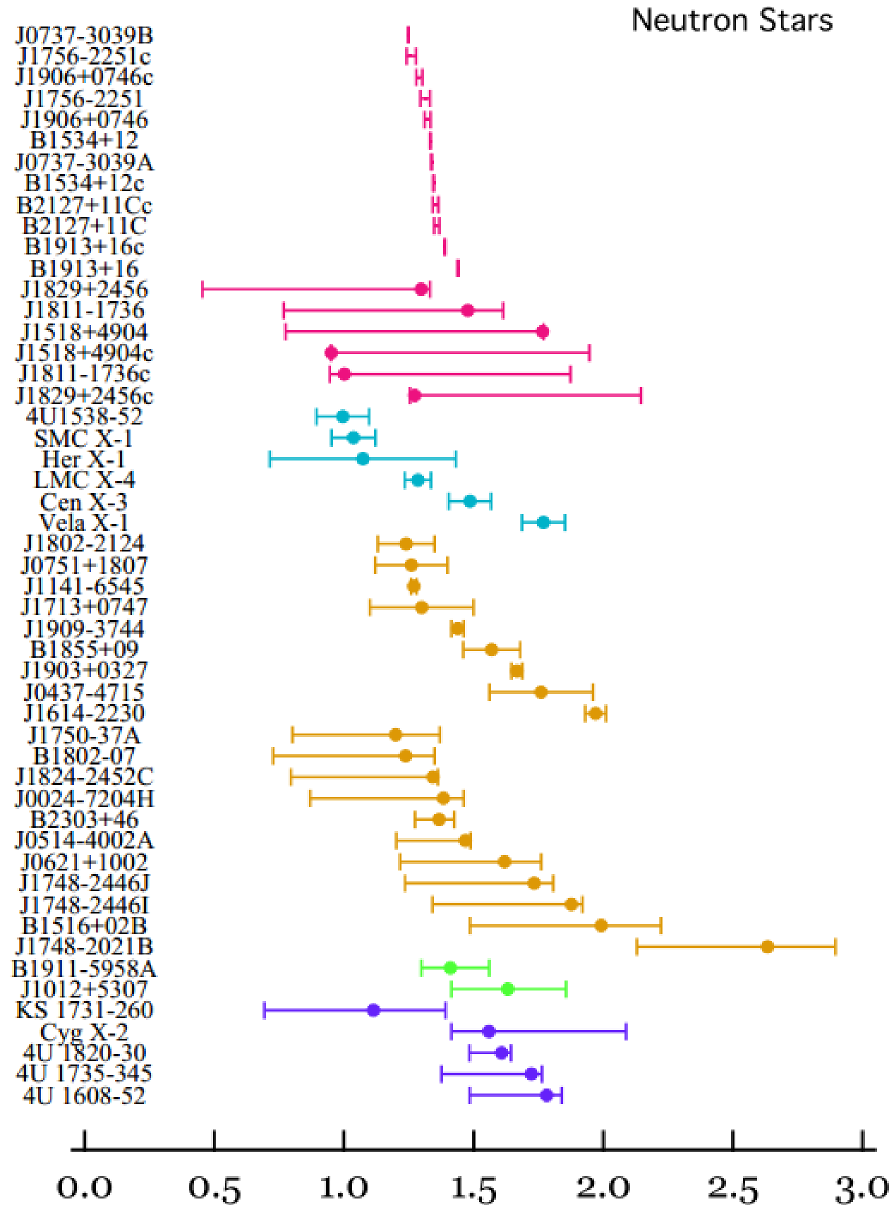
Follow updates at https://stellarcollapse.org/nsmasses

arXiv: 1012.3208

# Update - 2012

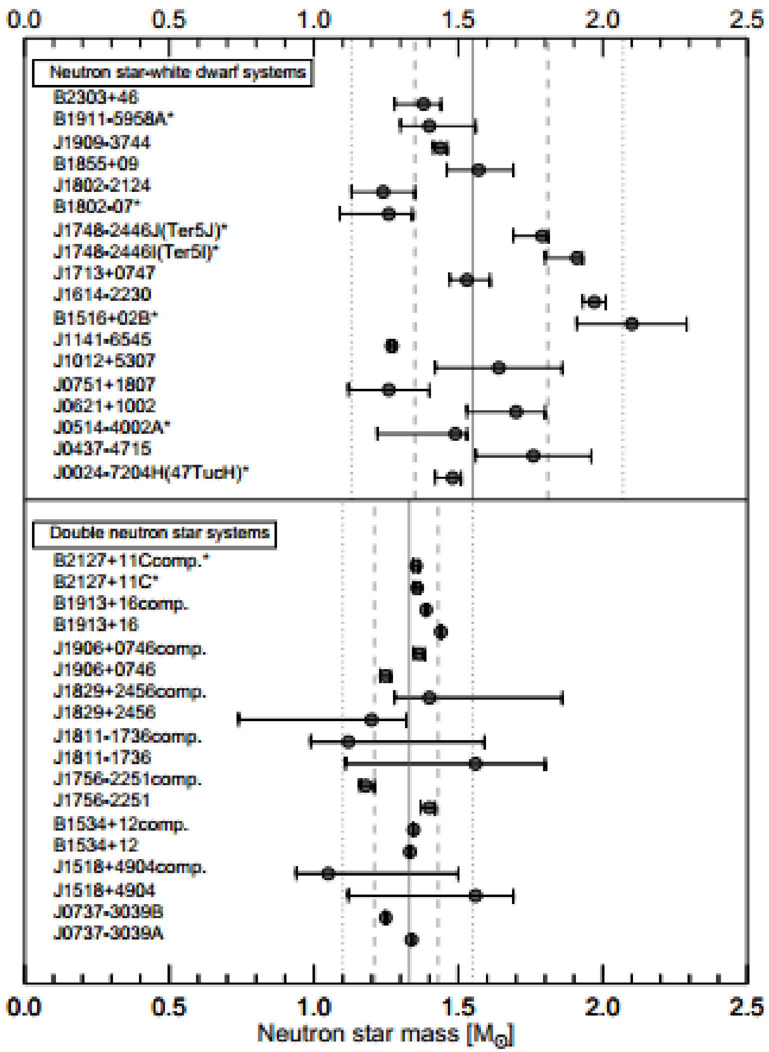


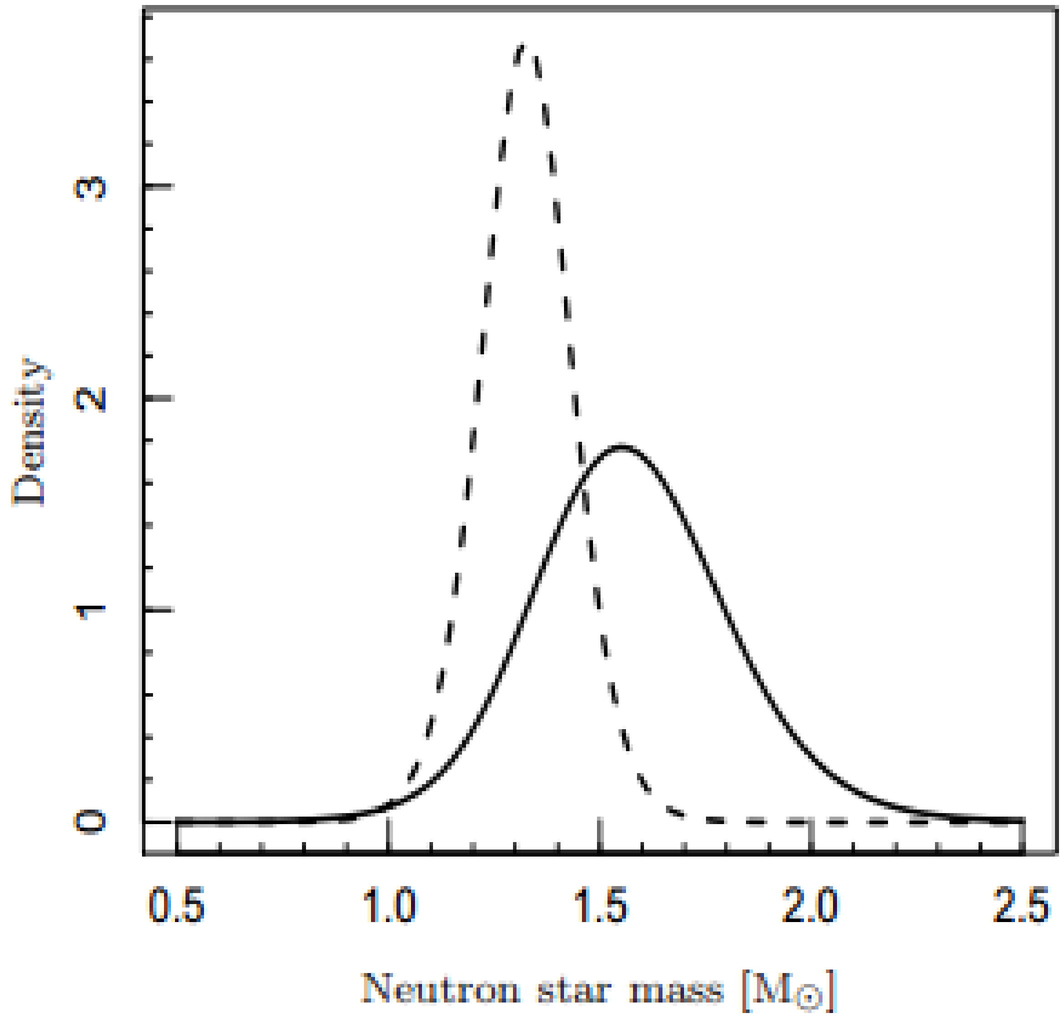




1201.1006

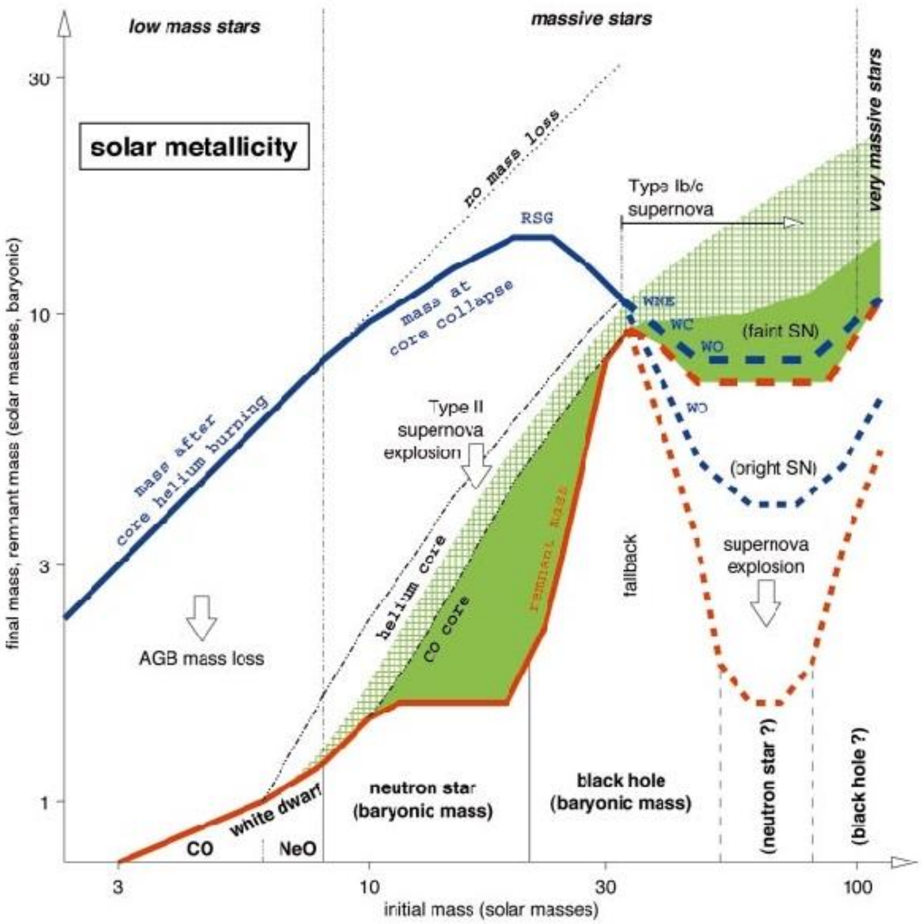
# Update - 2013




1309.6635

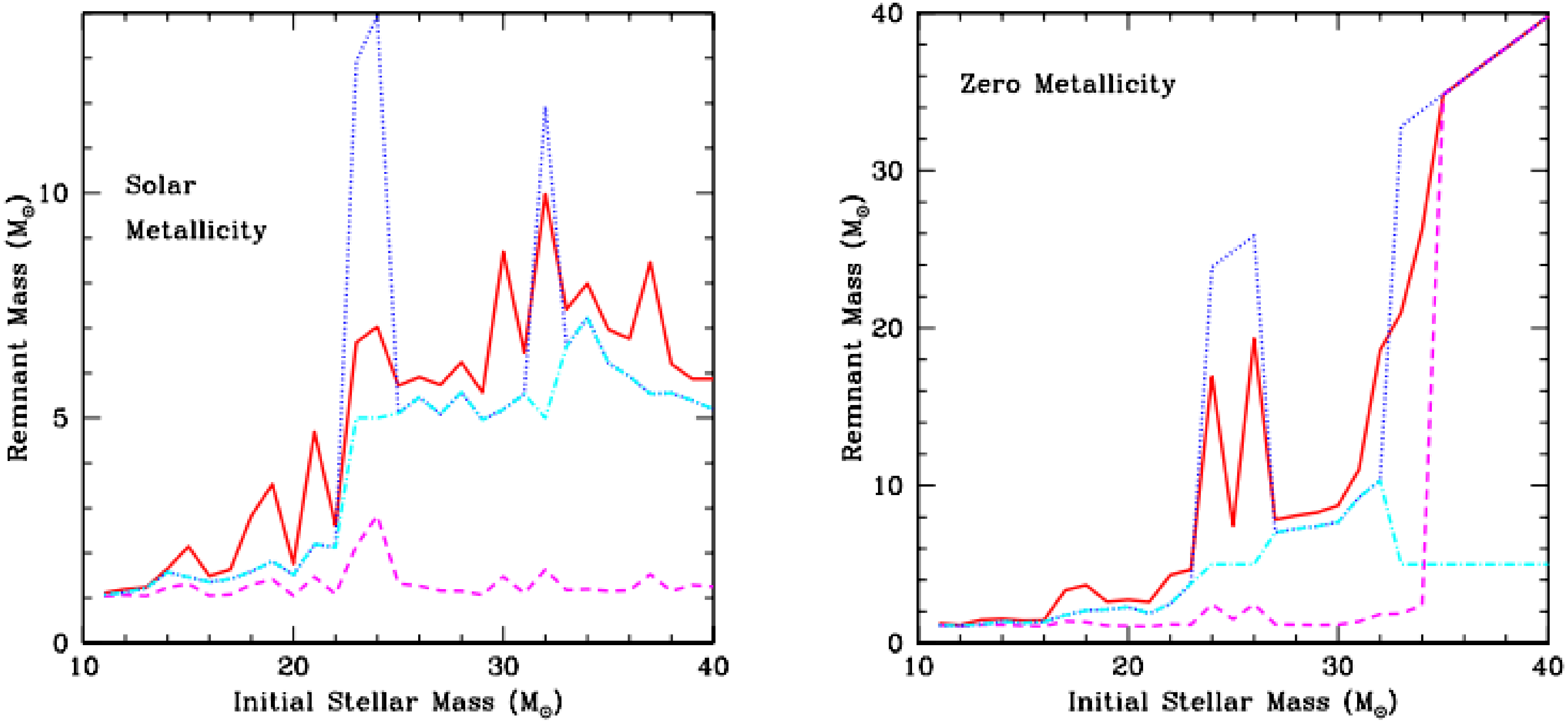
# Compact objects and progenitors. Solar metallicity.




There can be a range of progenitor masses in which NSs are formed, however, for smaller and larger progenitors masses BHs appear.

Woosley et al. 2002

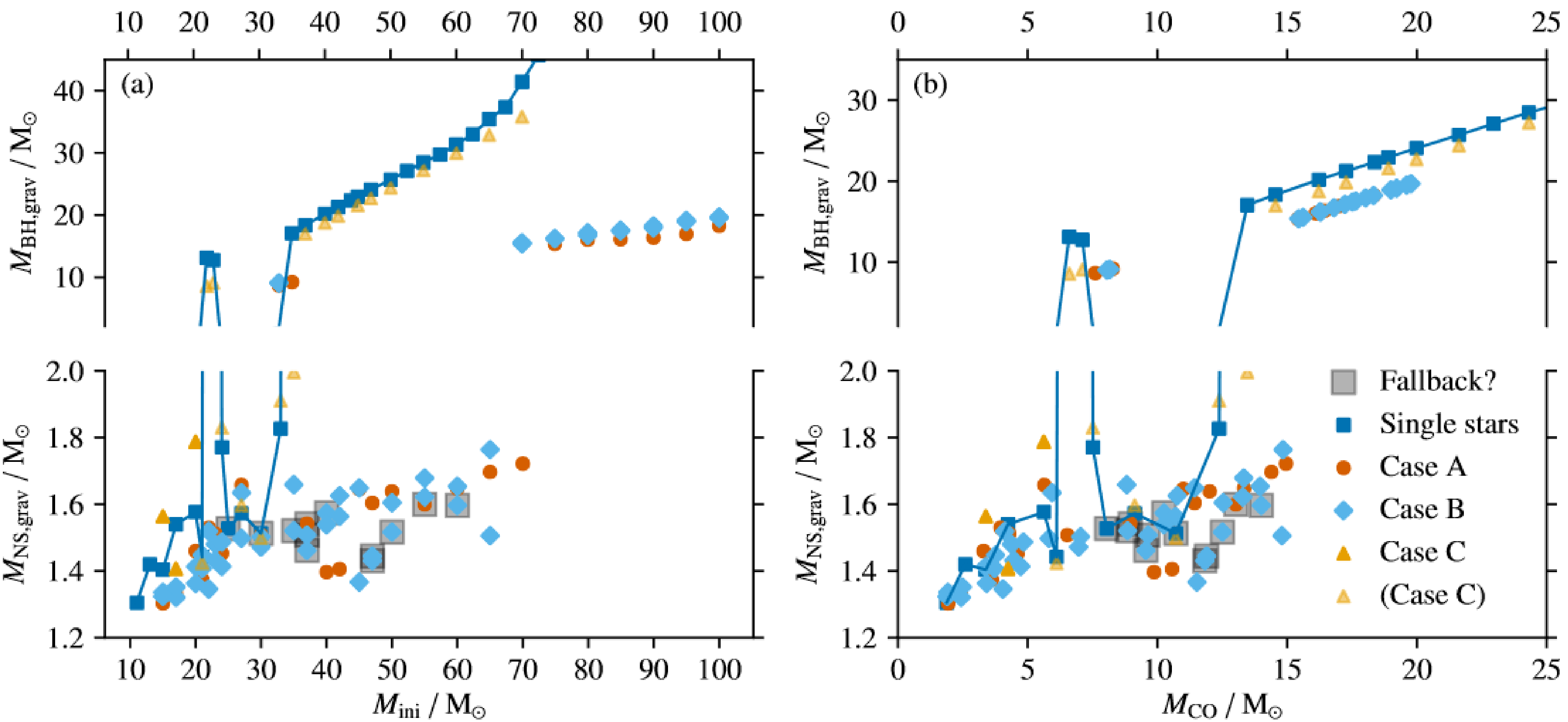
# Calculations of the mass spectrum



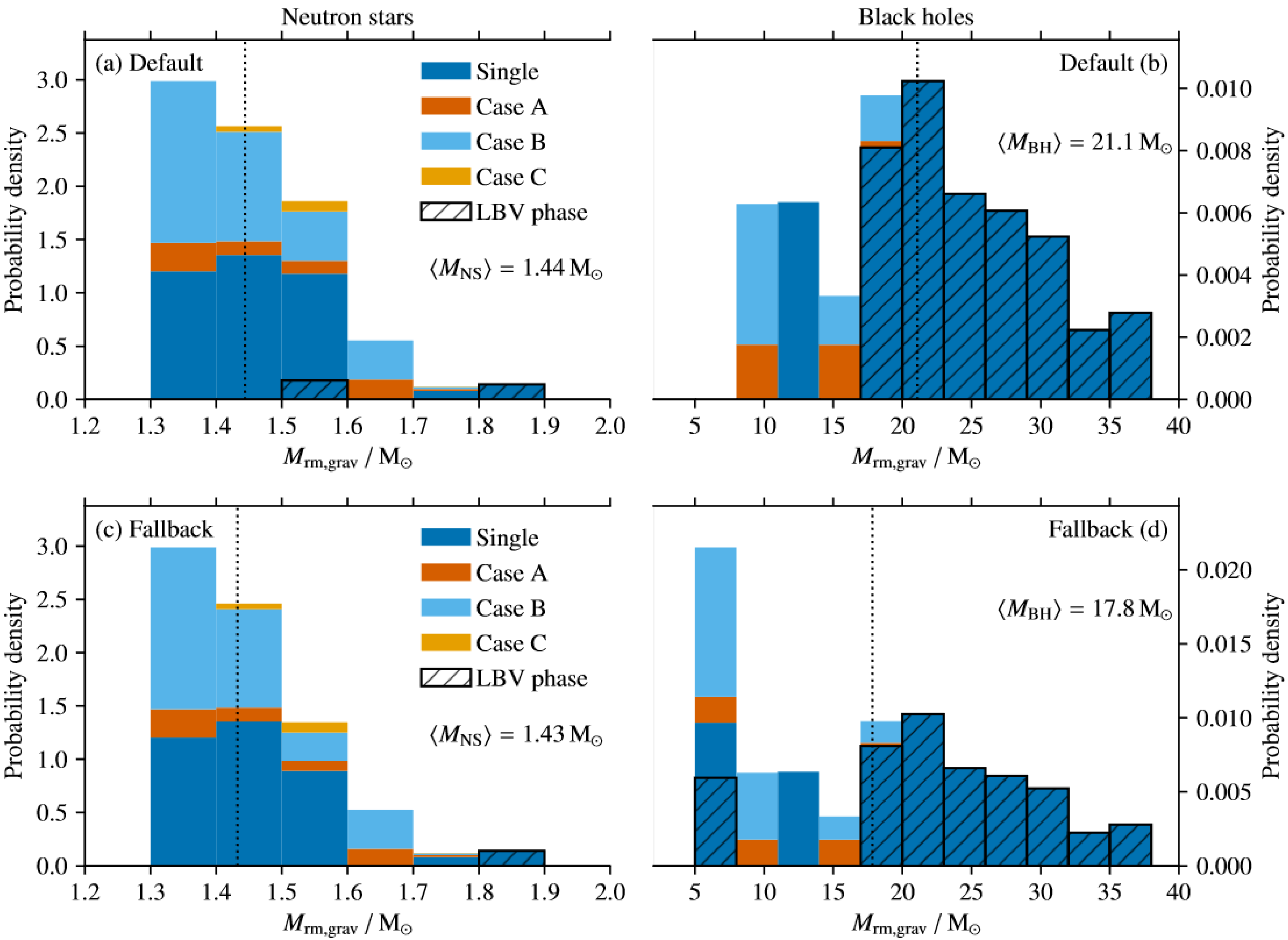

Different curves are plotted for different models of explosion:
dashed – with a magnetar

1110.1726

# Role of binaries




2008.08599



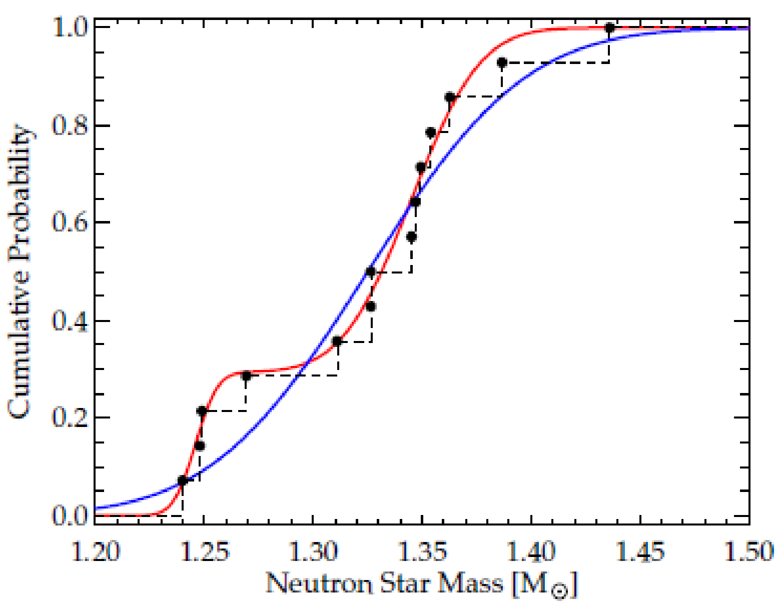
Neutron stars
Black holes
(a) Default
Default (b)
(c) Fallback
Fallback (d)
Single
Case A
Case B
Case C
LBV phase
⟨$M_{NS}$⟩ = 1.44 $M_\odot$
⟨$M_{BH}$⟩ = 21.1 $M_\odot$
⟨$M_{NS}$⟩ = 1.43 $M_\odot$
⟨$M_{BH}$⟩ = 17.8 $M_\odot$
Probability density
$M_{rm,grav}$ / $M_\odot$

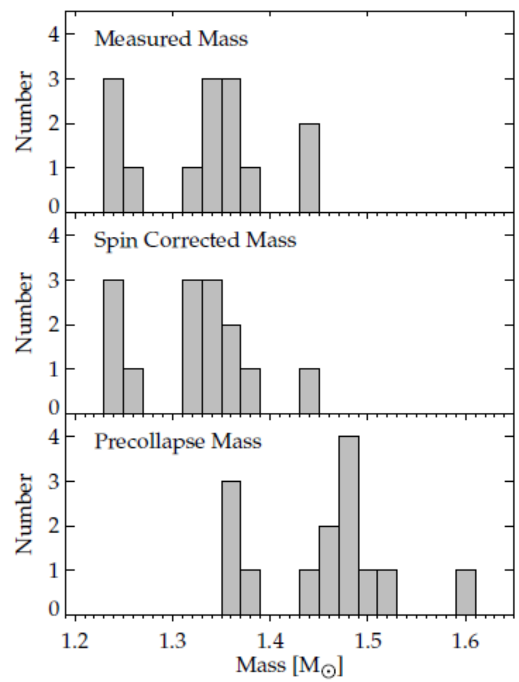

2008.08599

# Bi-modal mass spectrum?

| Pulsar Name | Mass of Recycled Neutron Star ($M_\odot$) | Mass of Young Neutron Star ($M_\odot$) | $P_{orb}$ (hours) | Eccentricity | Pulse Period (ms) | Reference |
|---|---|---|---|---|---|---|
| J0737-3039A/B | 1.3381 ± 0.0007 | 1.2489 ± 0.0007 | 2.4 | 0.088 | 23 | Kramer et al. (2006) |
| B1534+12 | 1.3332 ± 0.0010 | 1.3452 ± 0.0010 | 10.1 | 0.273 | 38 | Stairs et al. (2002) |
| J1756-2251 | 1.32 ± 0.02 | 1.24 ± 0.02 | 7.67 | 0.18 | 28 | Stairs (2008) |
| J1906+0746 | 1.365 ± 0.018 | 1.248 ± 0.018 | 3.98 | 0.085 | 144$^{\dagger}$ | Kasian (2008) |
| B1913+16 | 1.4414 ± 0.0002 | 1.3867 ± 0.0002 | 7.92 | 0.617 | 59 | Weisberg & Taylor (2005) |
| B2127+11C | 1.358 ± 0.010 | 1.354 ± 0.010 | 8.05 | 0.681 | 30 | Jacoby et al. (2006) |
| J1909-3744 | 1.438 ± 0.024 | white dwarf | 36.7 | $\lesssim 10^{-6}$ | 2.9 | Jacoby et al. (2005) |
| J1141-6545 | white dwarf | 1.27 ± 0.01 | 4.74 | 0.172 | 393$^{\dagger}$ | Bhat et al. (2008) |

The low-mass peak the authors relate to e⁻-capture SN.



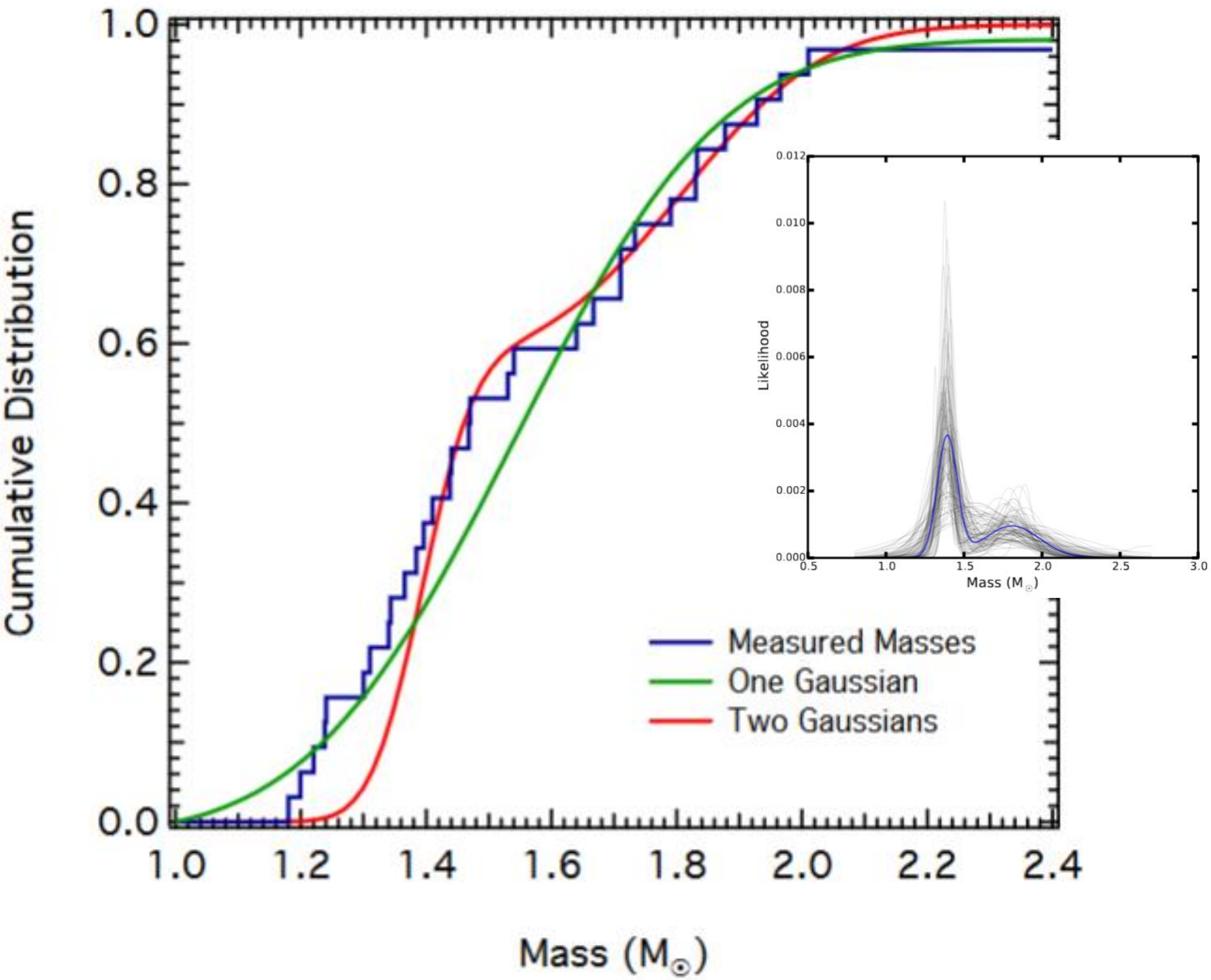

Based on 14 observed systems

1006.4584

# Bimodality in mPSR mass distribution

| # | PSR Name | Mass [$M_\odot$] |
|---|---|---|
| 1 | J0337+1715 | 1.4378(13) |
| 2 | J0348+0432 | 2.01(4) |
| 3 | J0437−4715 | 1.44(7) |
| 4 | J0751+1807 | 1.64(15) |
| 5 | J1012+0507 | 1.83(11) |
| 6 | J1023+0038 | 1.71(16) |
| 7 | J1614−2230 | 1.928(17) |
| 8 | J1713+0747 | 1.31(11) |
| 9 | J1738+0333 | 1.47(7) |
| 10 | J1802−2124 | 1.24(11) |
| 11 | J1807−2500B | 1.3655(21) |
| 12 | B1855+09 | 1.30(11) |
| 13 | J1903+0327 | 1.667(7) |
| 14 | J1909−3744 | 1.540(27) |
| 15 | J1910−5959A | 1.34(8) |
| 16 | J1918−0642 | 1.18(11) |
| 17 | J1946+3417 | 1.832(13) |
| 18 | J2234+0611 | 1.396(11) |

+ 14 PSR with less precisely determined masses




The bimodality reflects birth properties?

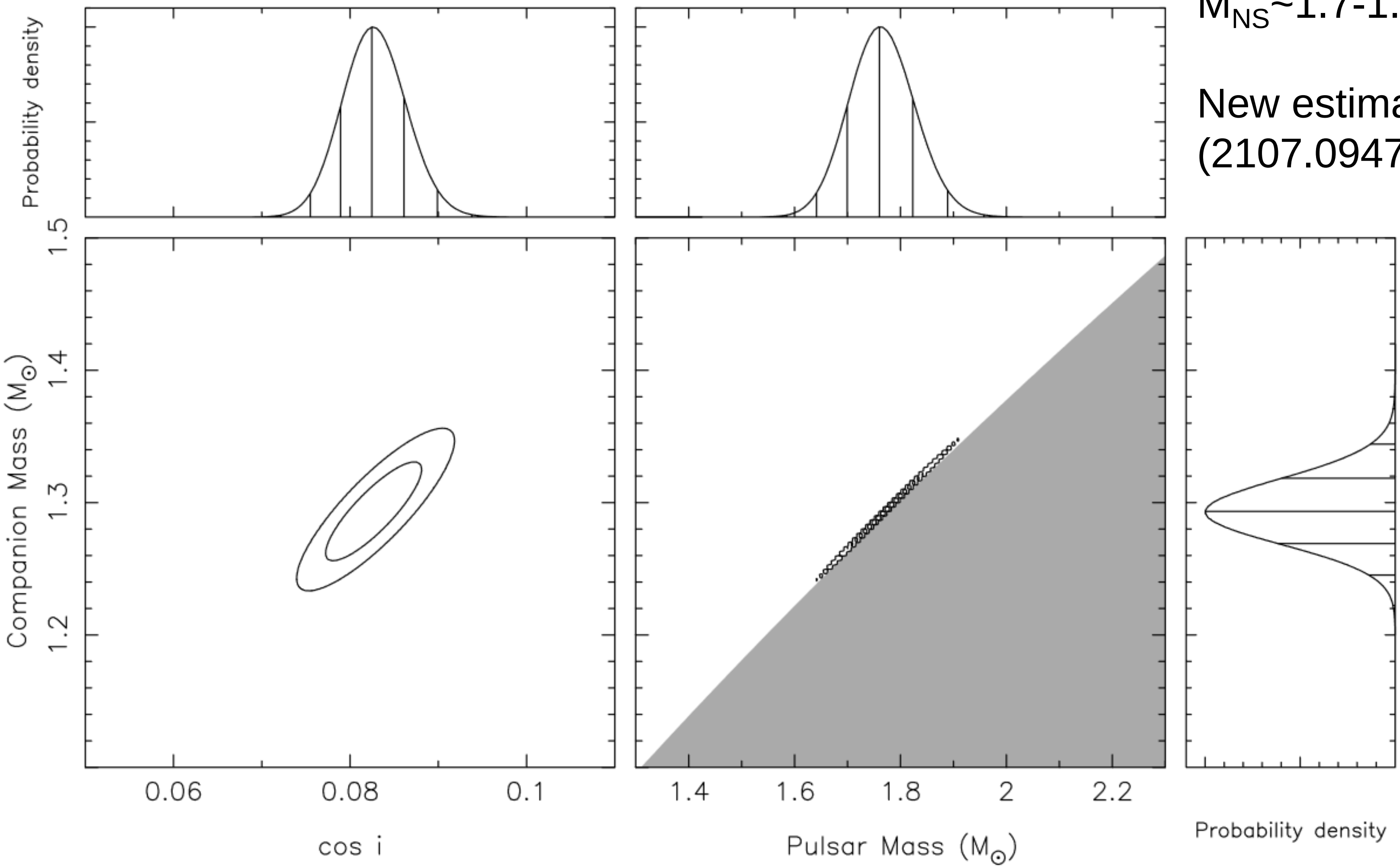
1605.01665

# Massive born NS

PSR J2222−0137
WD companion
P=32.8 msec
$M_{NS}$~1.7-1.8 Msun

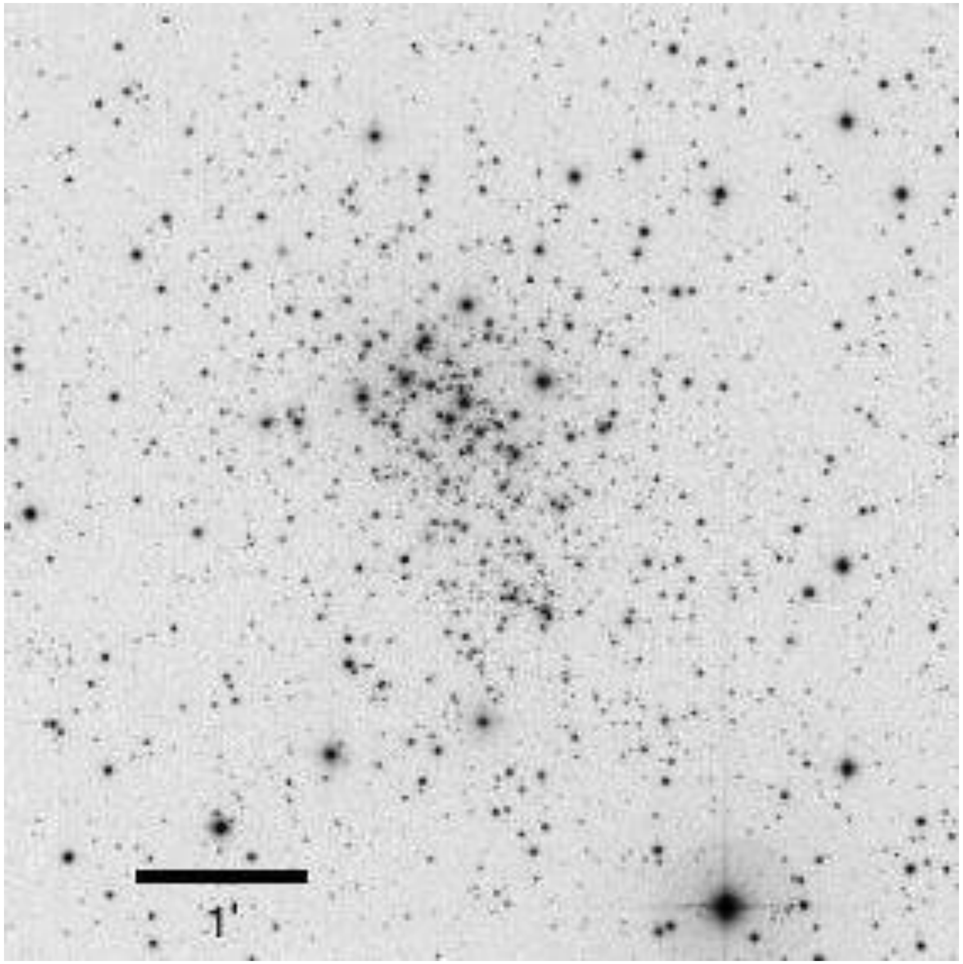
New estimate: 1.81-1.85
(2107.09474)

WD born first!!!!




1706.08060, see also 2002.12583 about PSR J1640+2224

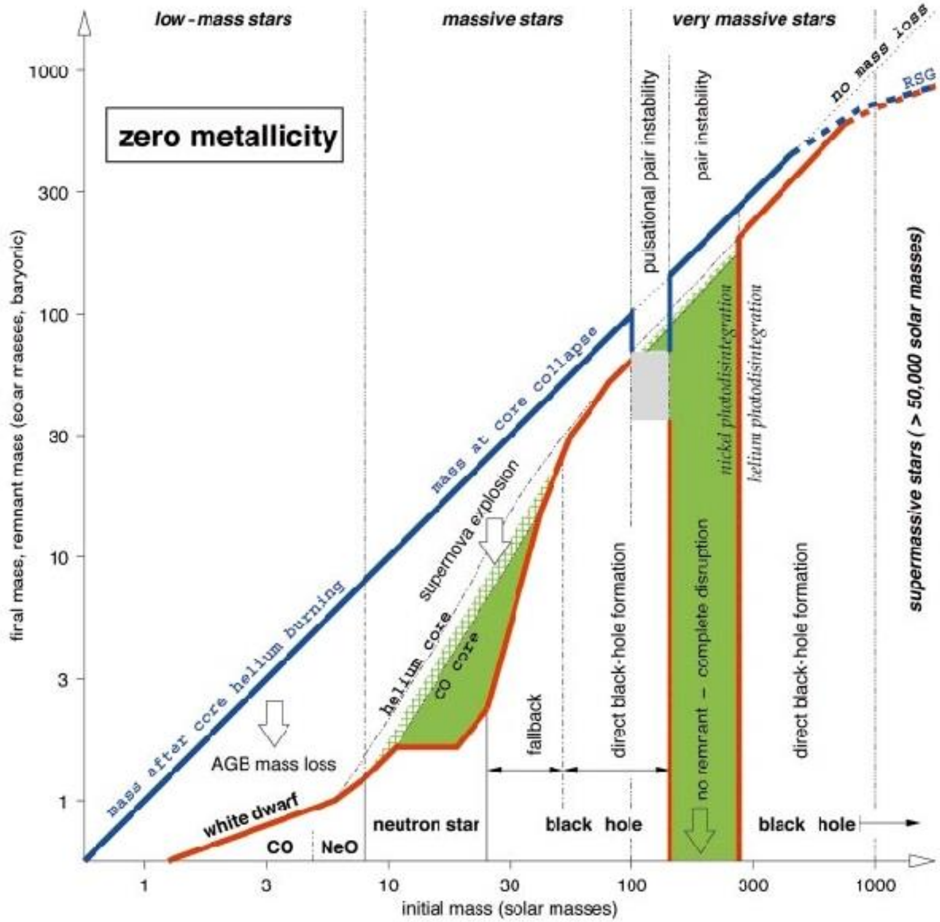
# A NS from a massive progenitor




Anomalous X-ray pulsar
in the association
Westerlund1 most probably has
a very massive progenitor, >40 $M_{\odot}$.

astro-ph/0611589

# The case of zero metallicity



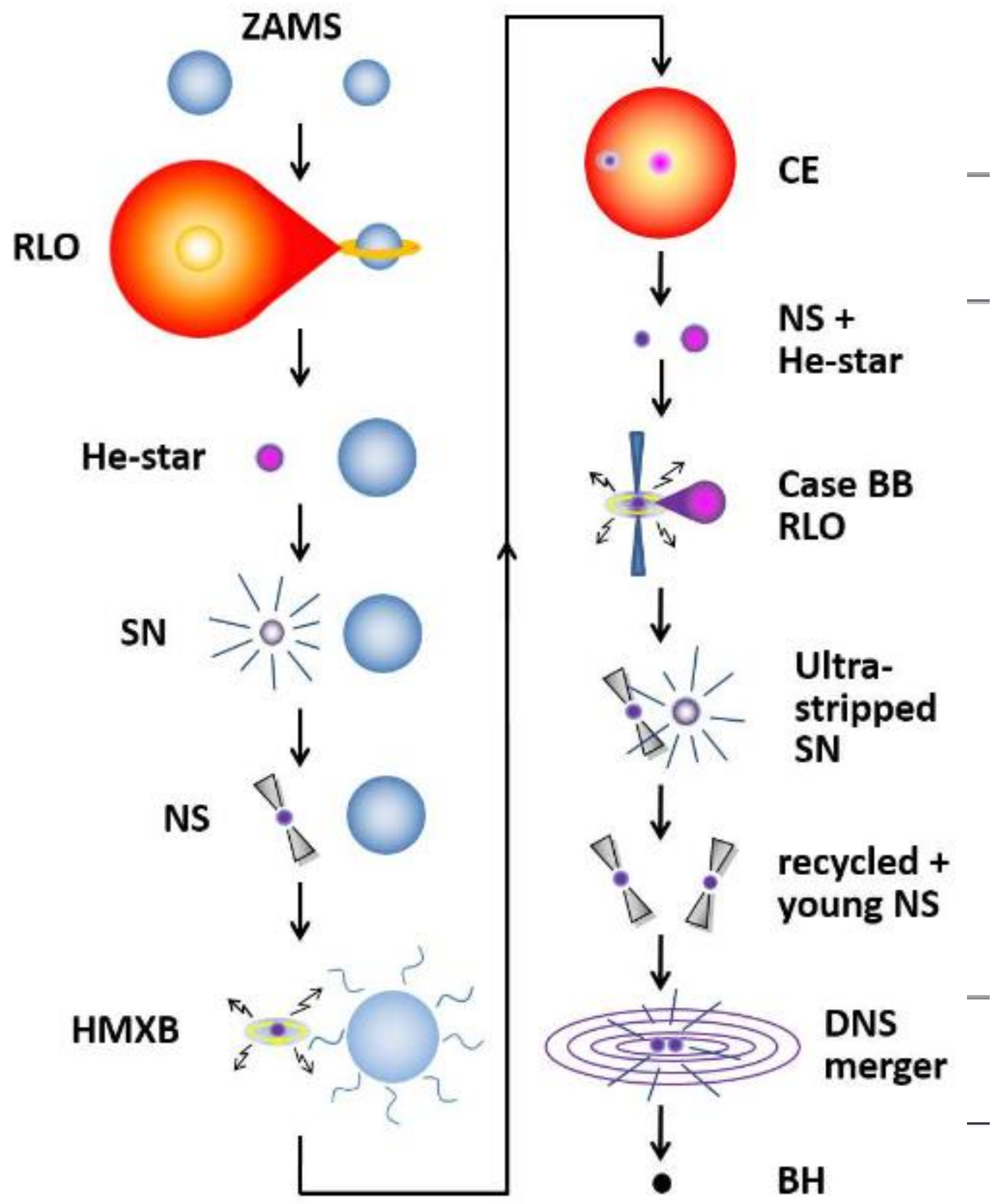

No intermediate mass range for NS formation.

Woosley et al. 2002

# DNS

| Radio Pulsar | Type | $P$ (ms) |
|---|---|---|
| J0453+1559[a] | recycled | 45.8 |
| J0737−3039A[b] | recycled | 22.7 |
| J0737−3039B[b] | young | 2773.5 |
| J1518+4904[c] | recycled | 40.9 |
| B1534+12[d] | recycled | 37.9 |
| J1753−2240[e] | recycled | 95.1 |
| J1755−2550[f]* | young | 315.2 |
| J1756−2251[g] | recycled | 28.5 |
| J1811−1736[h] | recycled | 104.2 |
| J1829+2456[i] | recycled | 41.0 |
| J1906+0746[j]* | young | 144.1 |
| J1913+1102[k] | recycled | 27.3 |
| B1913+16[l] | recycled | 59.0 |
| J1930−1852[m] | recycled | 185.5 |
| J1807−2500B[n]* | GC | 4.2 |
| B2127+11C[p] | GC | 30.5 |

1706.09438

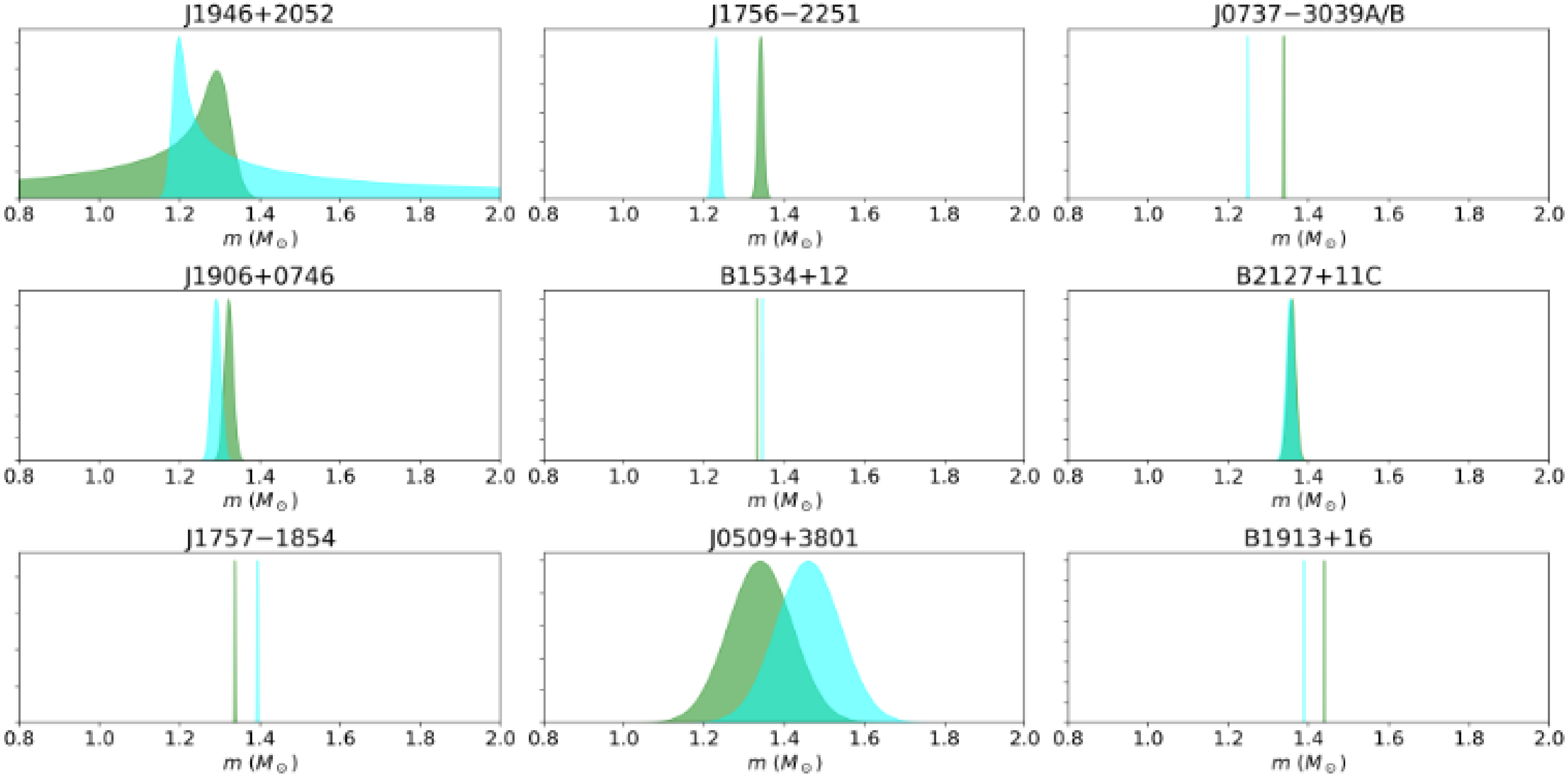


# DNS parameters

| Pulsar Name | $M_T$ ($M_\odot$) | $m_r$ ($M_\odot$) | $m_s$ ($M_\odot$) | $\mathcal{M}_c$ ($M_\odot$) | $q$ | $P_b$ (day) | $T_c$ (Gyr) |
|---|---|---|---|---|---|---|---|
| Systems will merge within a Hubble time | | | | | | | |
| J1946+2052 | 2.50(4) | $< 1.35$ | $> 1.17$ | (1.05, 1.11) | (0.68, 1) | 0.078 | 0.046 |
| J1756−2251 | 2.56999(6) | 1.341(7) | 1.230(7) | 1.1178(3) | 0.92(1) | 0.320 | 1.656 |
| J0737−3039A/B | 2.58708(16) | 1.3381(7) | 1.2489(7) | 1.1253(1) | 0.933(1) | 0.102 | 0.086 |
| J1906+0746 | 2.6134(3) | 1.322(11) | 1.291(11) | 1.1372(2) | (0.956, 1) | 0.166 | 0.308 |
| B1534+12 | 2.678463(4) | 1.3330(2) | 1.3455(2) | 1.165870(2) | 0.9907(3) | 0.421 | 2.734 |
| B2127+11C | 2.71279(13) | 1.358(10) | 1.354(10) | 1.18043(8) | (0.975, 1) | 0.335 | 0.217 |
| J1757−1854 | 2.73295(9) | 1.3384(9) | 1.3946(9) | 1.18930(4) | 0.960(1) | 0.184 | 0.076 |
| J0509+3801 | 2.805(3) | 1.34(8) | 1.46(8) | 1.215(5) | (0.793, 1) | 0.380 | 0.574 |
| B1913+16 | 2.828378(7) | 1.4398(2) | 1.3886(2) | 1.230891(5) | 0.9644(3) | 0.323 | 0.301 |
| J1913+1102 | 2.886(1) | 1.65(5) | 1.24(5) | 1.242(8) | 0.75(5) | 0.206 | 0.473 |
| Systems will not merge within a Hubble time | | | | | | | |
| J1807−2500B | 2.57190(73) | 1.3655(21) | 1.2064(21) | 1.1169(3) | 0.883(3) | 9.957 | 1044 |
| J1518+4904 | 2.7183(7) | 1.41(8) | 1.31(8) | 1.181(5) | (0.794, 1) | 8.634 | 8832 |
| J0453+1559 | 2.733(4) | 1.559(5) | 1.174(4) | 1.175(2) | 0.753(5) | 4.072 | 1453 |
| J1411+2551 | 2.538(22) | $< 1.64$ | $> 0.92$ | (1.05, 1.11) | (0.57, 0.95) | 2.616 | 466 |
| J1811−1736 | 2.57(10) | $< 1.75$ | $> 0.91$ | (1.02, 1.17) | (0.58, 0.95) | 18.78 | 1794 |
| J1829+2456 | 2.59(2) | $< 1.36$ | $> 1.25$ | (1.08, 1.14) | (0.65, 1) | 1.176 | 55 |
| J1930−1852 | 2.59(4) | $< 1.32$ | $> 1.30$ | (1.07, 1.15) | (0.58, 0.96) | 45.06 | $\sim 10^5$ |

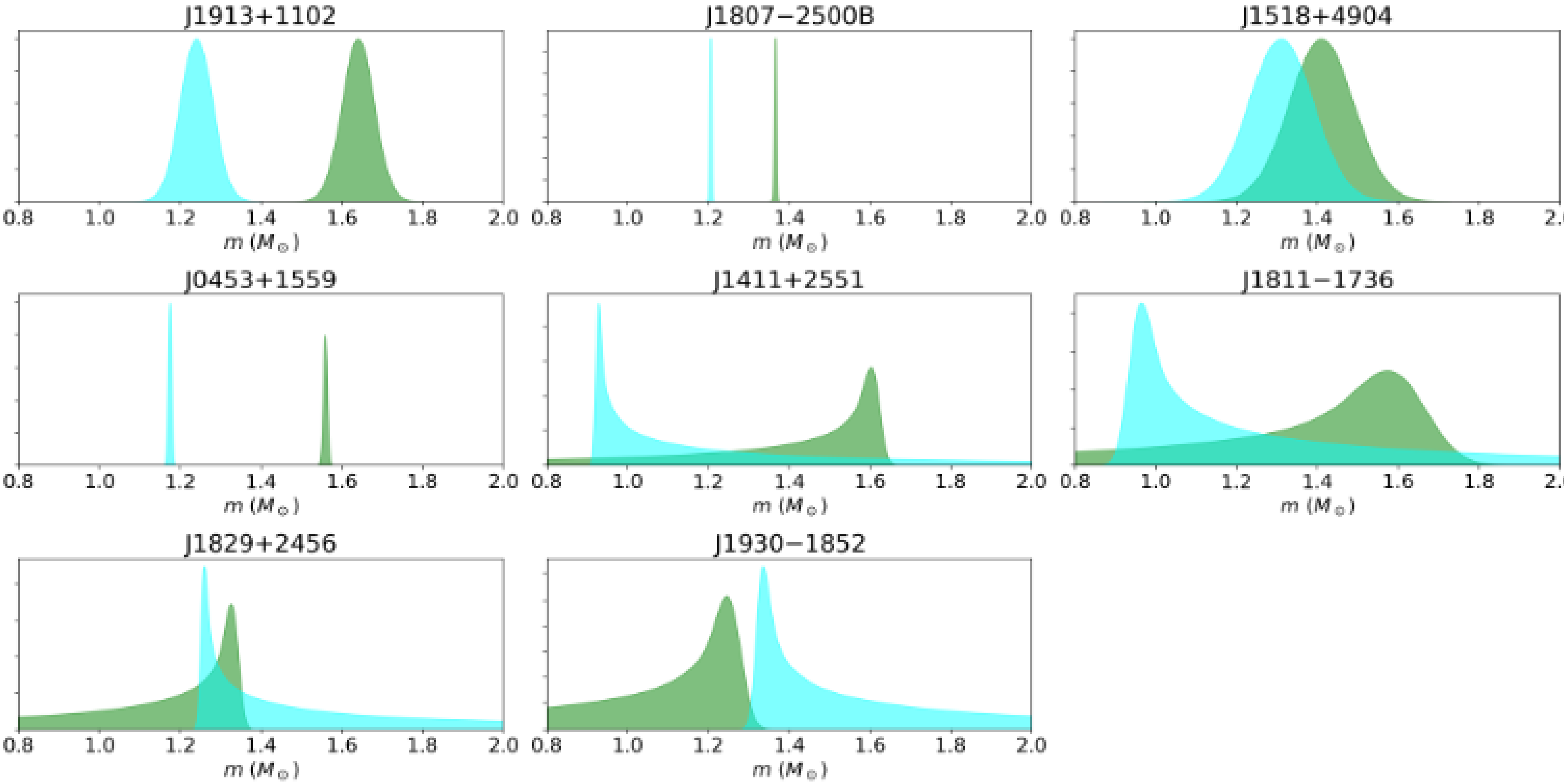
1902.03300

# Individual masses of DNS



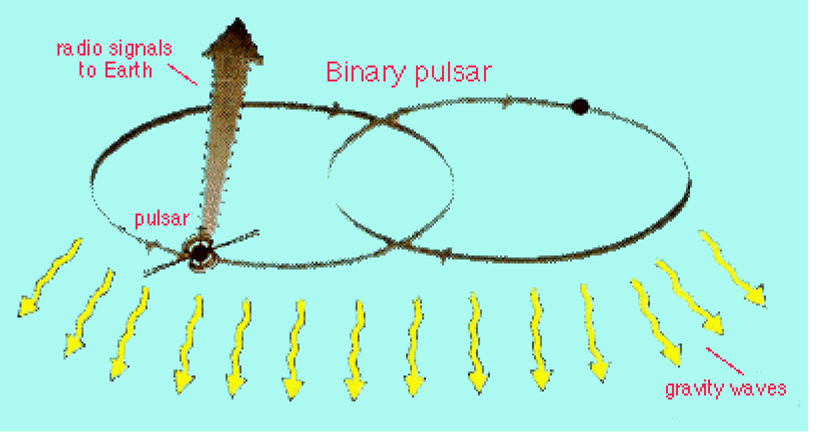

1902.03300

# Individual masses of DNS




1902.03300

# Binary pulsars




$$\frac{d\Delta_{E\odot}}{dt} = \sum_i \frac{Gm_i}{c^2 r_i} + \frac{v_\oplus^2}{2c^2} - \text{constant}\ .$$

$$\Delta_{S\odot} = -\frac{2GM_\odot}{c^3}\log(1+\cos\theta)\ ,$$

$$T = t_{\text{obs}} - t_0 + \Delta_C - D/f^2 + \Delta_{R\odot}(\alpha,\delta,\mu_\alpha,\mu_\delta,\pi)$$
$$+\Delta_{E\odot} - \Delta_{S\odot}(\alpha,\delta)$$
$$-\Delta_R(x,e,P_b,T_0,\omega,\dot{\omega},\dot{P}_b) - \Delta_E(\gamma) - \Delta_S(r,s)$$

See 1502.05474 for a recent detailed review

# Relativistic corrections and measurable parameters

$$\dot{\omega}=3\left[\frac{P_b}{2\pi}\right]^{-5/3}(T_\odot M)^{2/3}(1-e^2)^{-1} ,$$

$$\gamma=e\left[\frac{P_b}{2\pi}\right]^{1/3}T_\odot^{2/3}M^{-4/3}m_2(m_1+2m_2) ,$$

$$\dot{P}_b=-\frac{192\pi}{5}\left[\frac{P_b}{2\pi}\right]^{-5/3}\left[1+\frac{73}{24}e^2+\frac{37}{96}e^4\right]$$
$$\times(1-e^2)^{-7/2}T_\odot^{5/3}m_1m_2M^{-1/3} ,$$

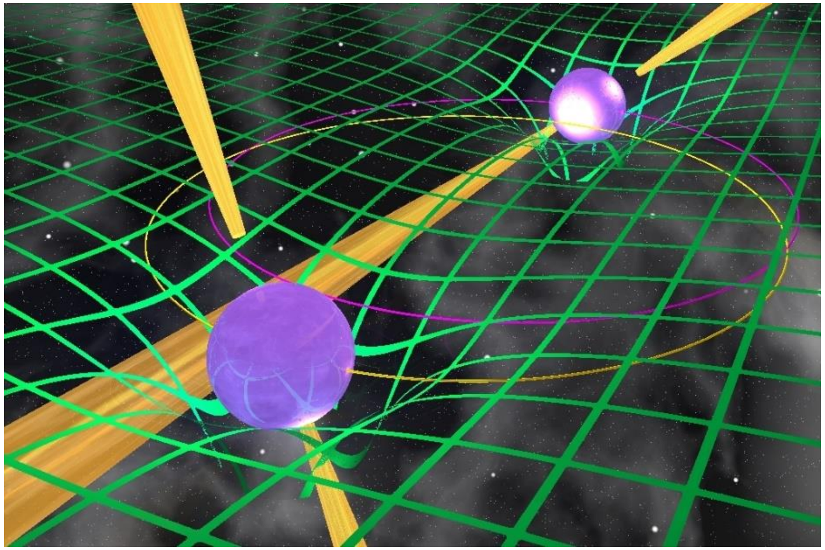
$$r=T_\odot m_2 ,$$

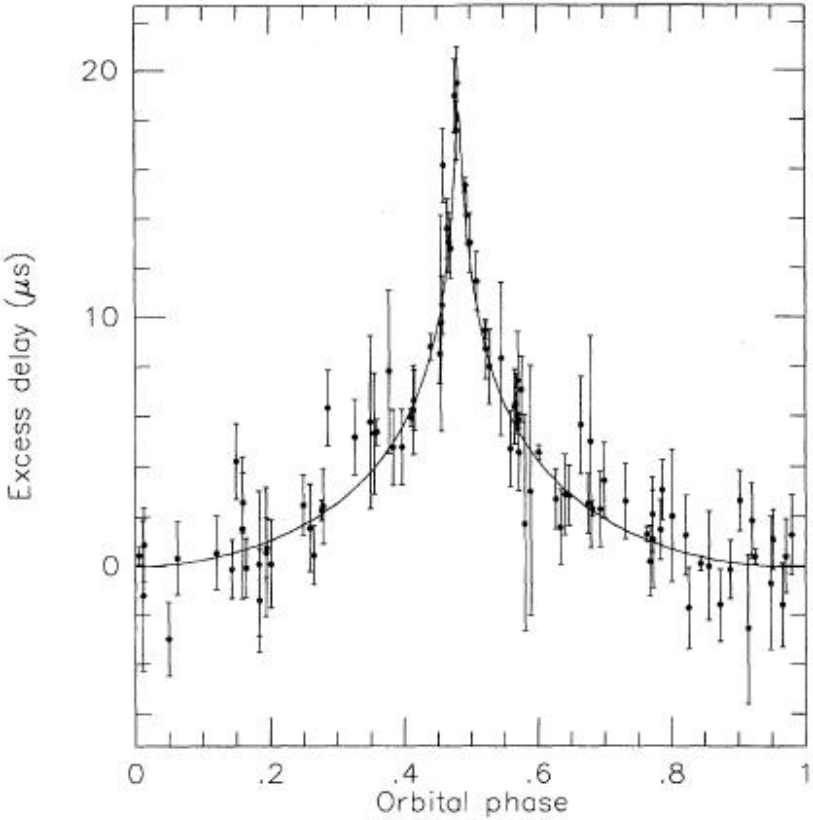
$$s=x\left[\frac{P_b}{2\pi}\right]^{-2/3}T_\odot^{-1/3}M^{2/3}m_2^{-1} .$$

For details see
Taylor, Weisberg 1989
ApJ 345, 434

# Shapiro delay

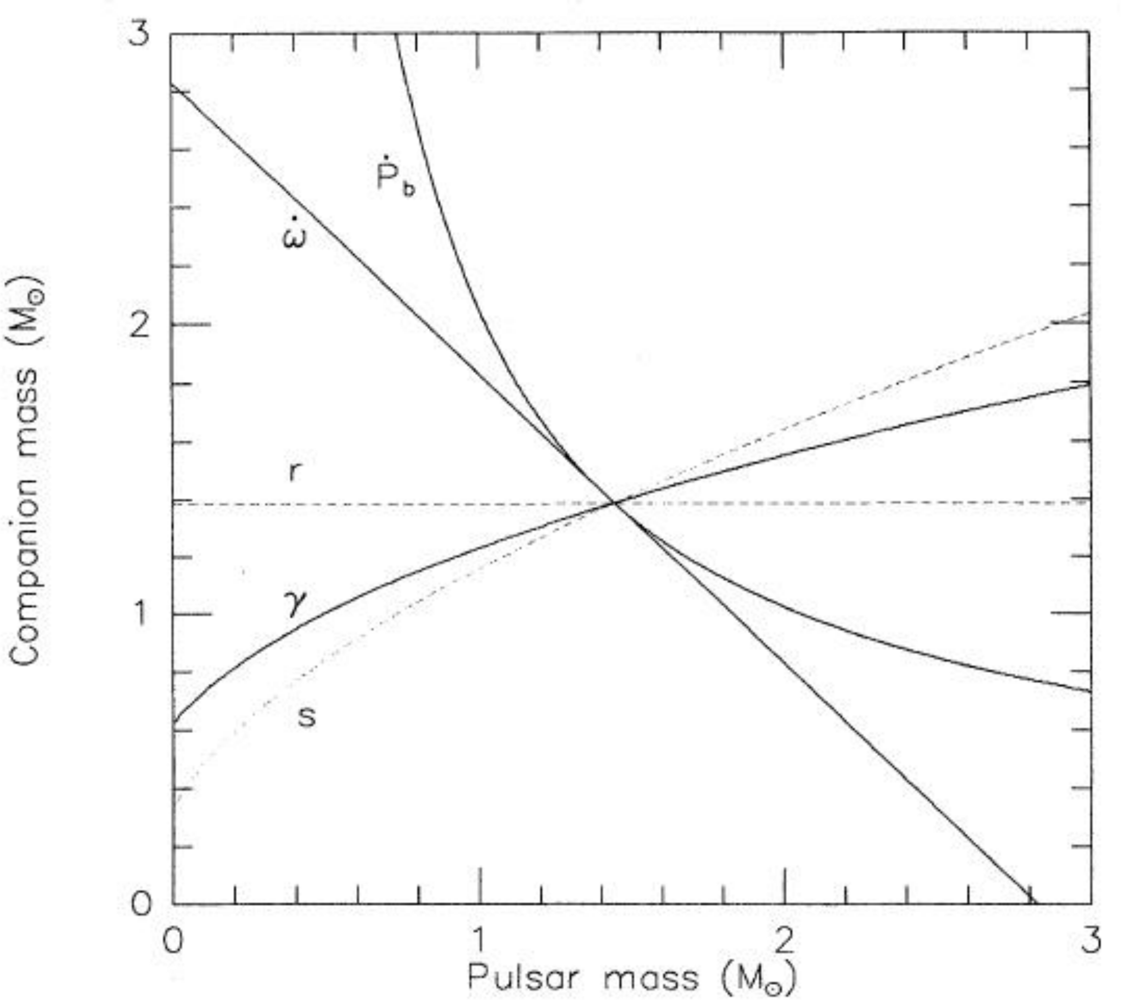
$$\Delta_S = -2r\log(1 - s\cos[2\pi(\phi - \phi_0)])$$






PSR 1855+09 (Taylor, Nobel lecture)

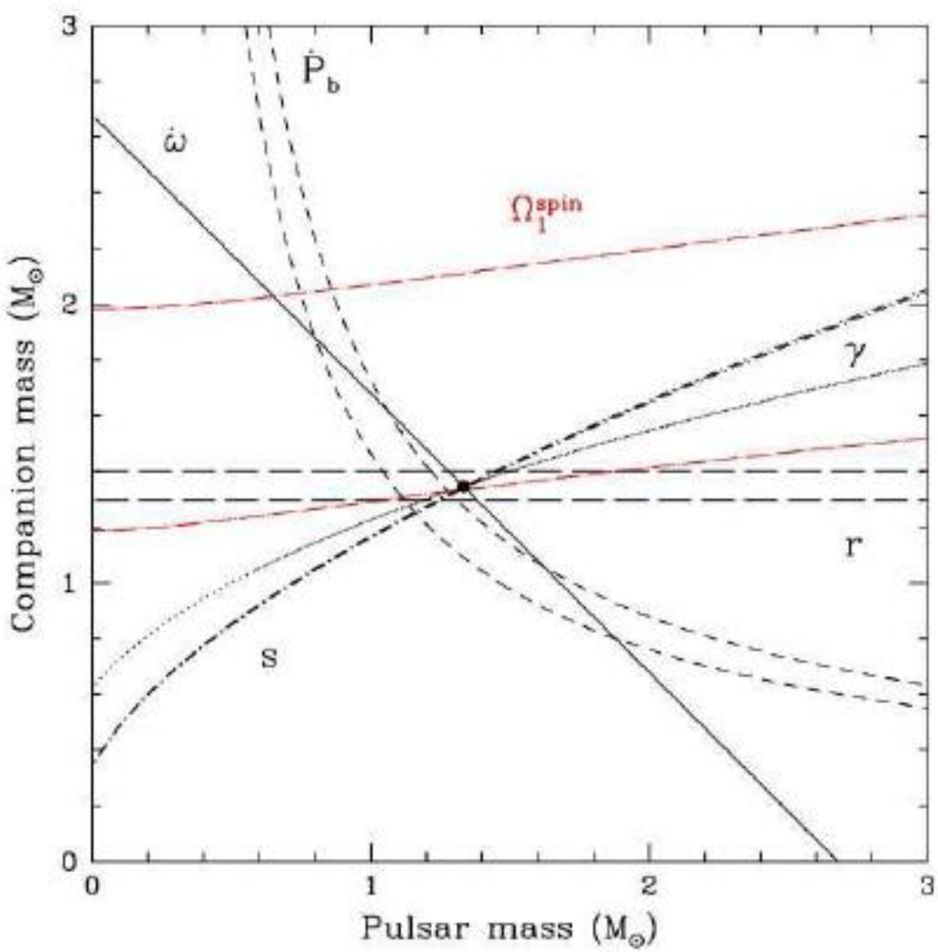
# Mass measurements




PSR 1913+16

Taylor

# Uncertainties and inverse problems



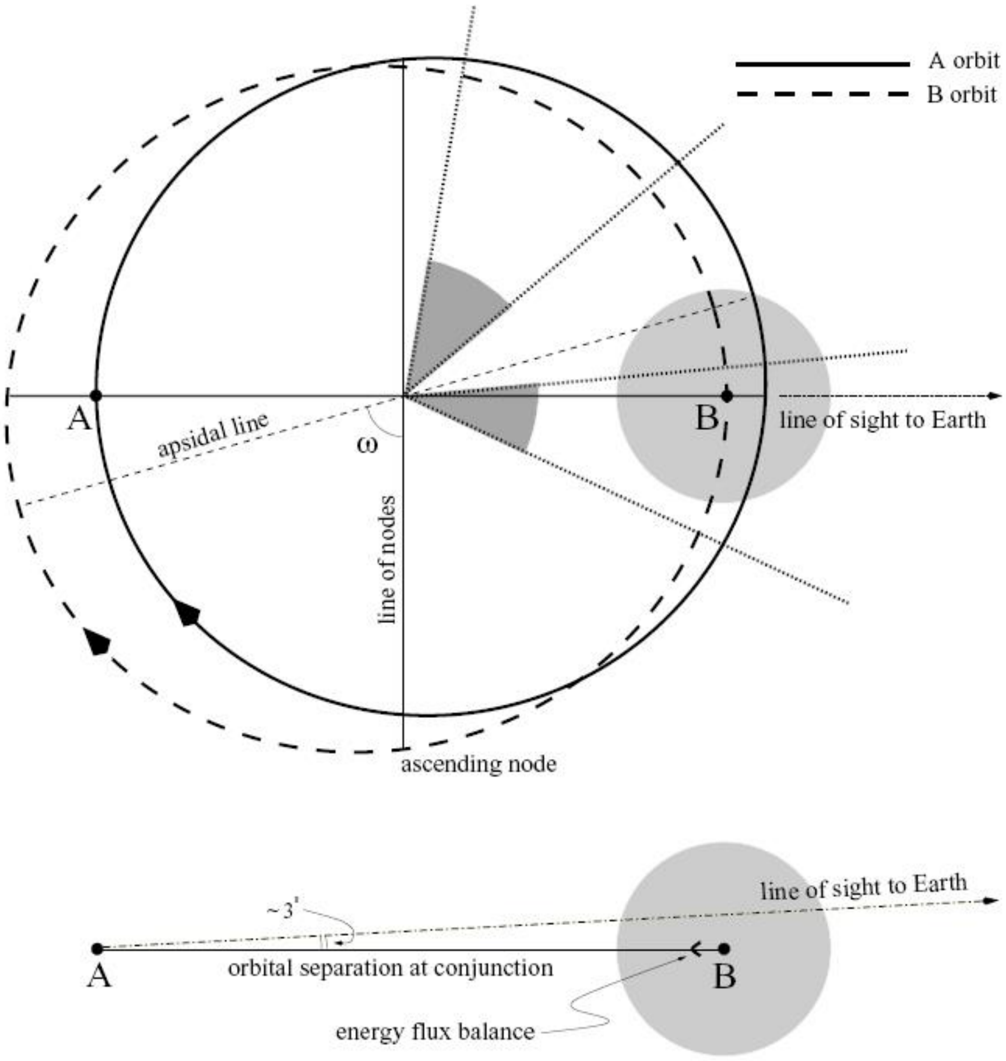

$P_b$dot depends on the Shklovskii effect. So, if distance is not certain, it is difficult to have a good measurement of this parameter.

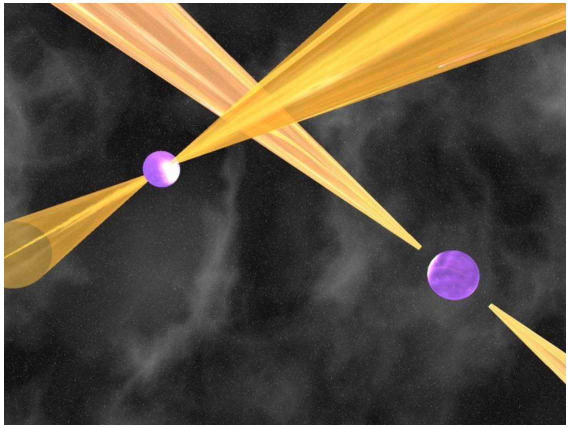
It is possible to invert the problem. Assuming that GR is correct, one can improve the distance estimate for the given source.

PSR B1534+12.

1502.05474

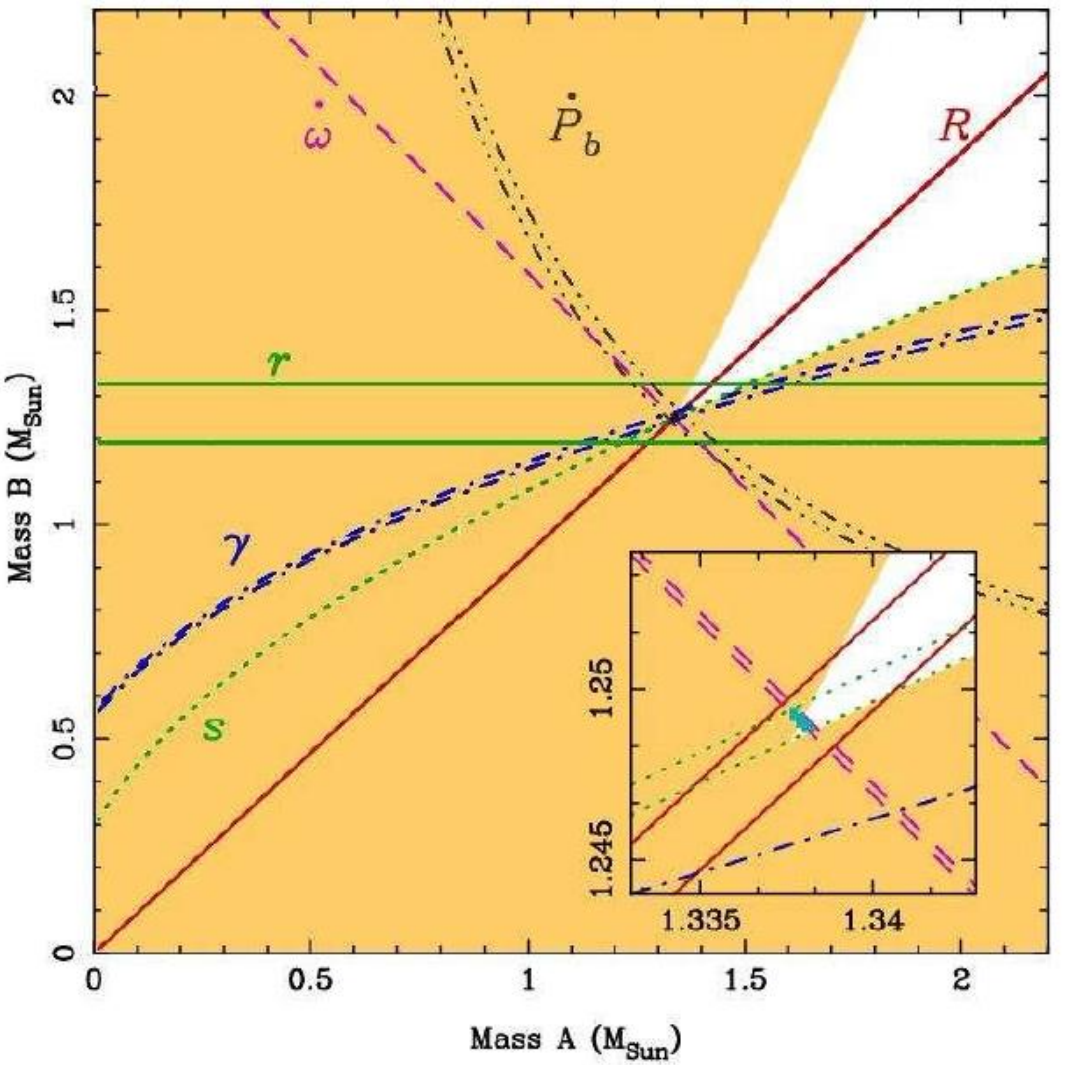
# Double pulsar J0737-3039



Lyne et al. astro-ph/0401086

# Masses for PSR J0737-3039

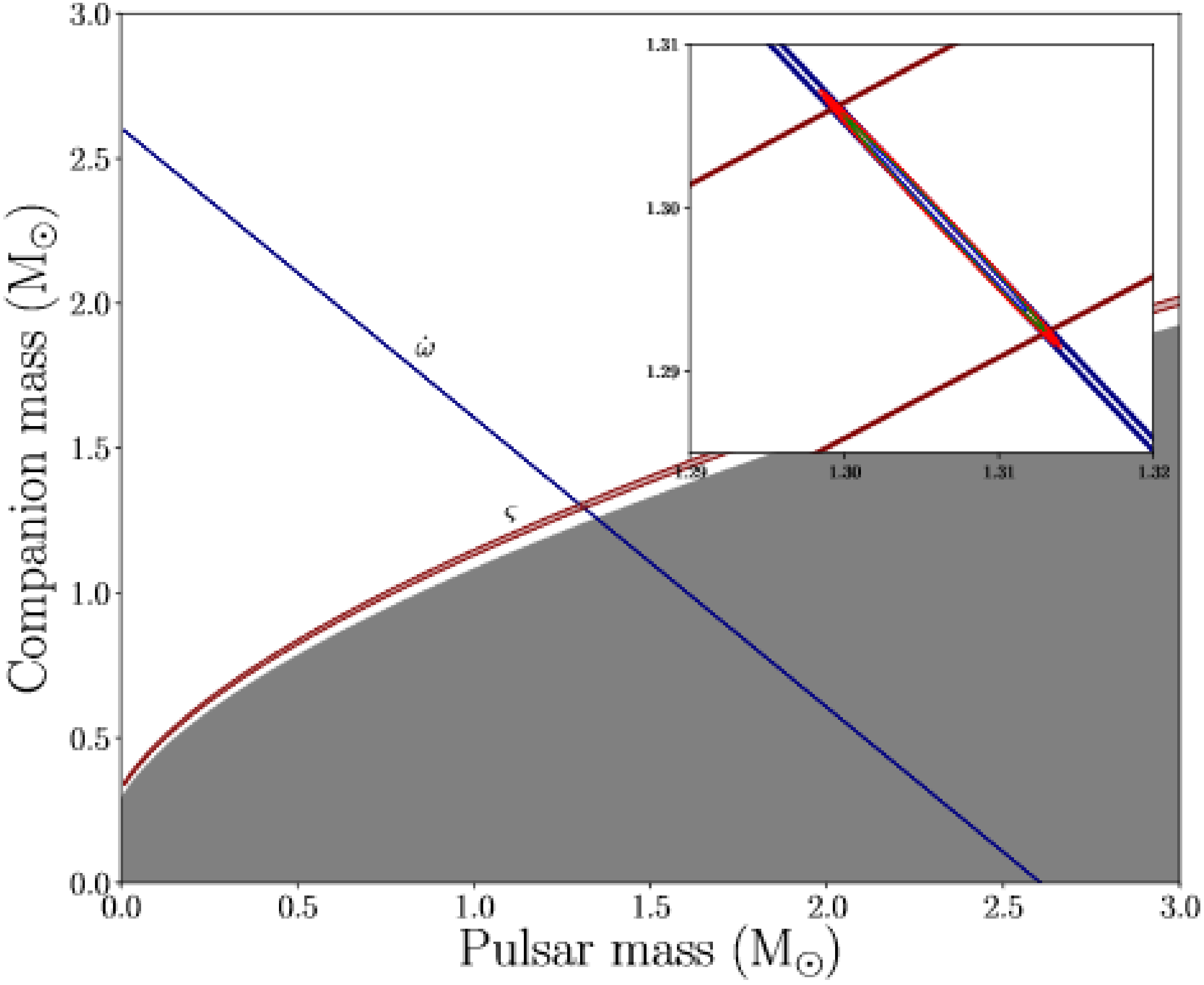



The most precise values.

New mass estimates have uncertainties <0.001

Kramer et al. astro-ph/0609417

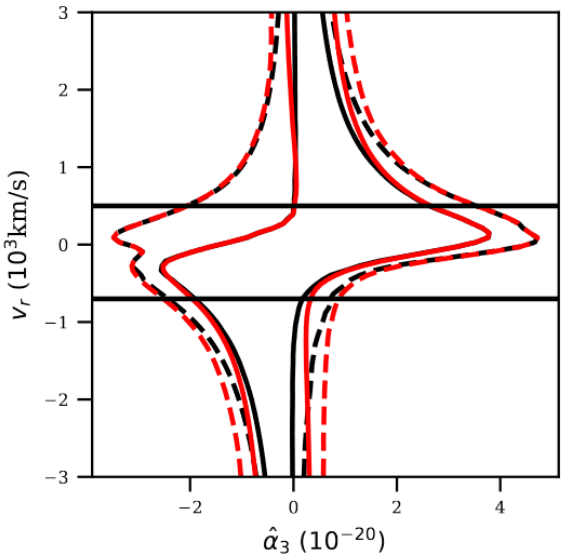
# DNS J1829+2456 mass measurements

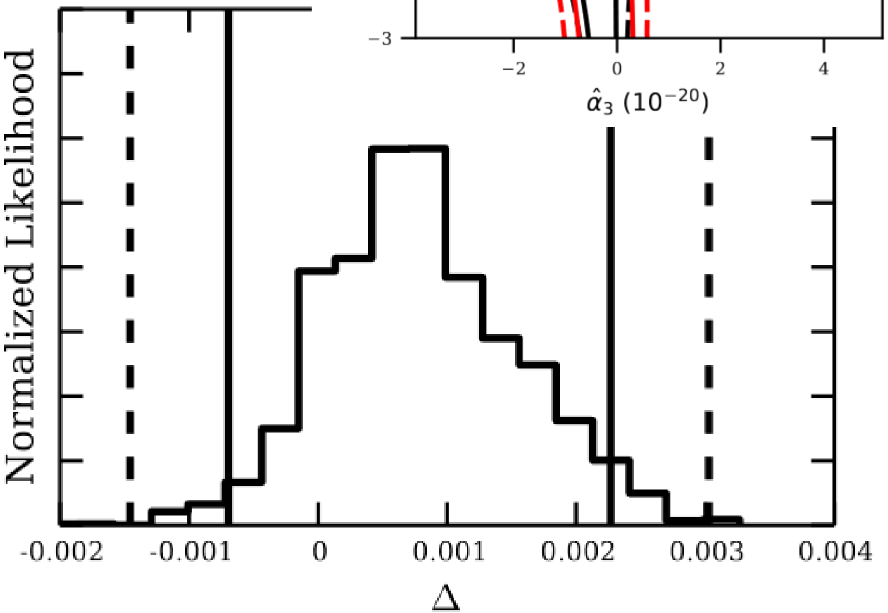


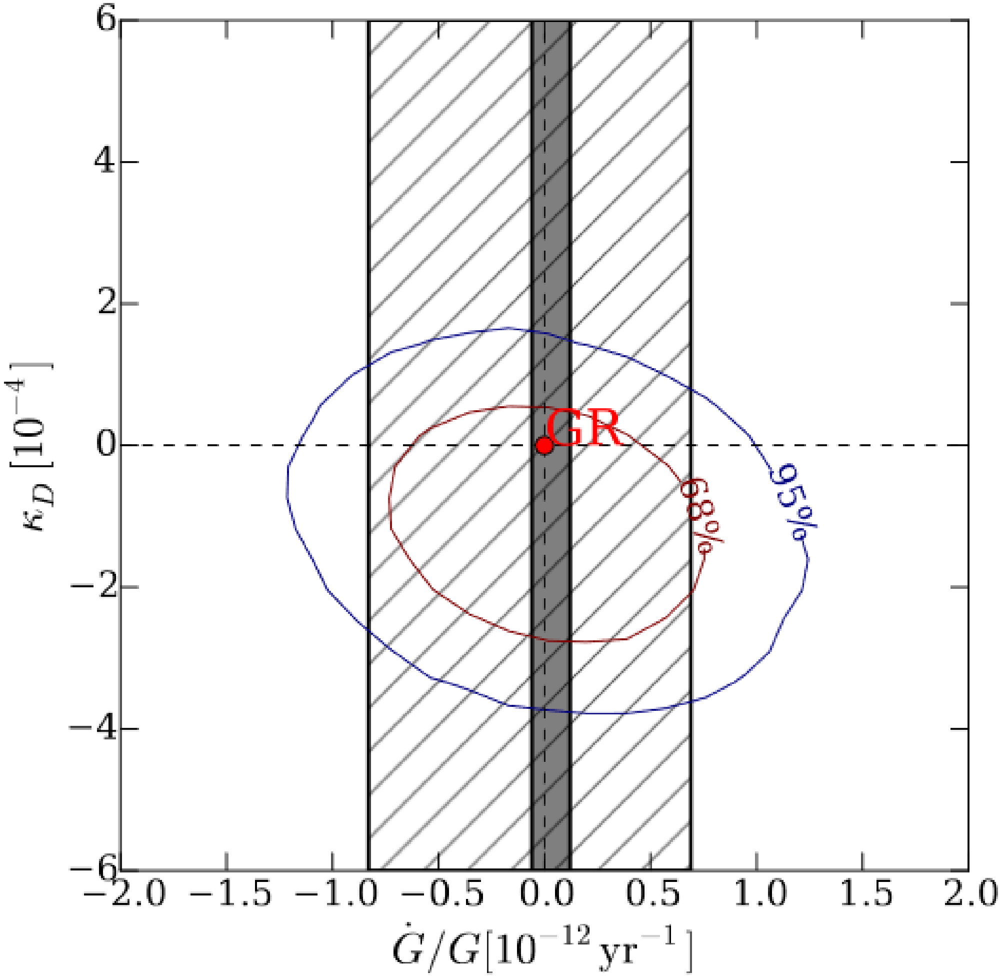

2007.07565

# Tests of theories of gravity

J1713+0747

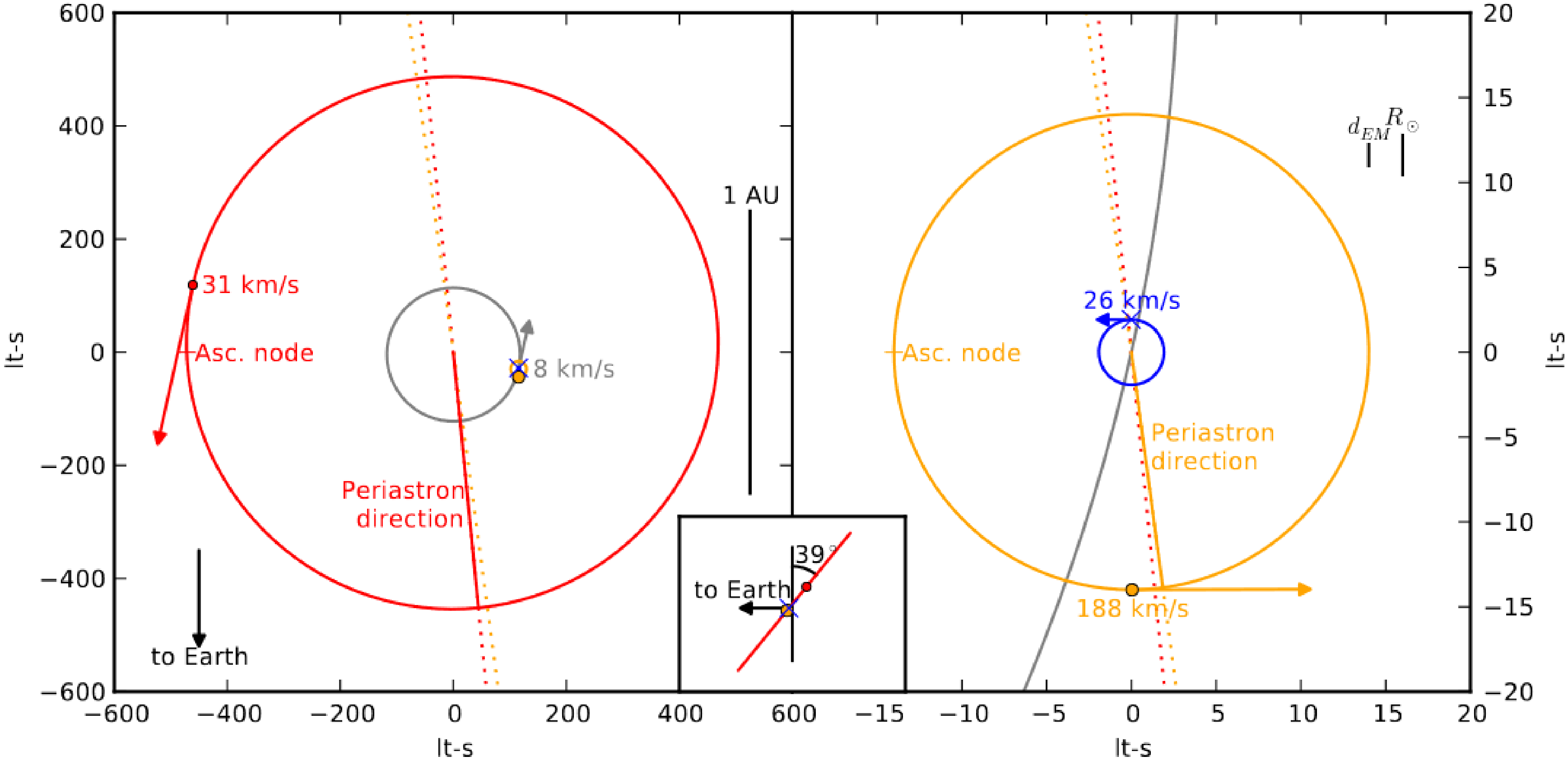



1802.09206

# Testing strong equivalence principle with triple pulsar PSR J0337+1715




NS+WD+WD

1401.0535

# NS+WD binaries

**<u>Some examples</u>**

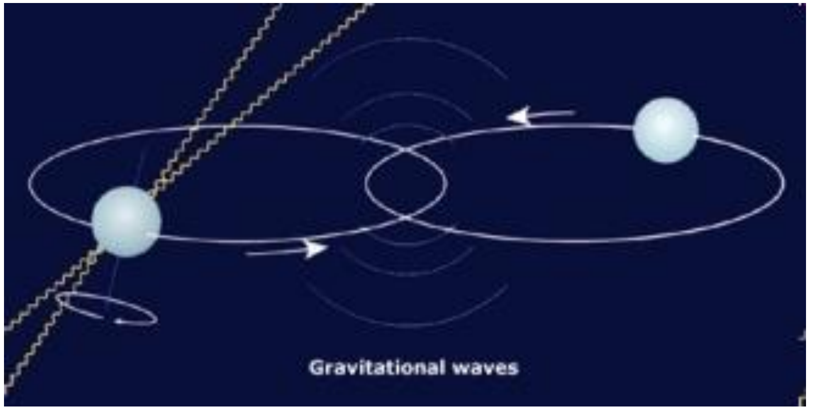
**PSR J0437-4715. WD companion [0801.2589, 0808.1594 ].**
**The closest millisecond PSR. $M_{NS}$=1.76+/-0.2 solar.**

**The case of PSR J0751+1807.**
**Initially, it was announced that it has a mass ~2.1 solar [astro-ph/0508050].**
**However, then in 2007 at a conference the authors announced that the result was incorrect. Actually, the initial value was 2.1+/-0.2 (1 sigma error).**
**New result: 1.26 +/- 0.14 solar**
**[Nice et al. 2008, Proc. of the conf. "40 Years of pulsars"]**

**It is expected that most massive NSs get their additional "kilos" due to accretion from WD companions [astro-ph/0412327 ].**

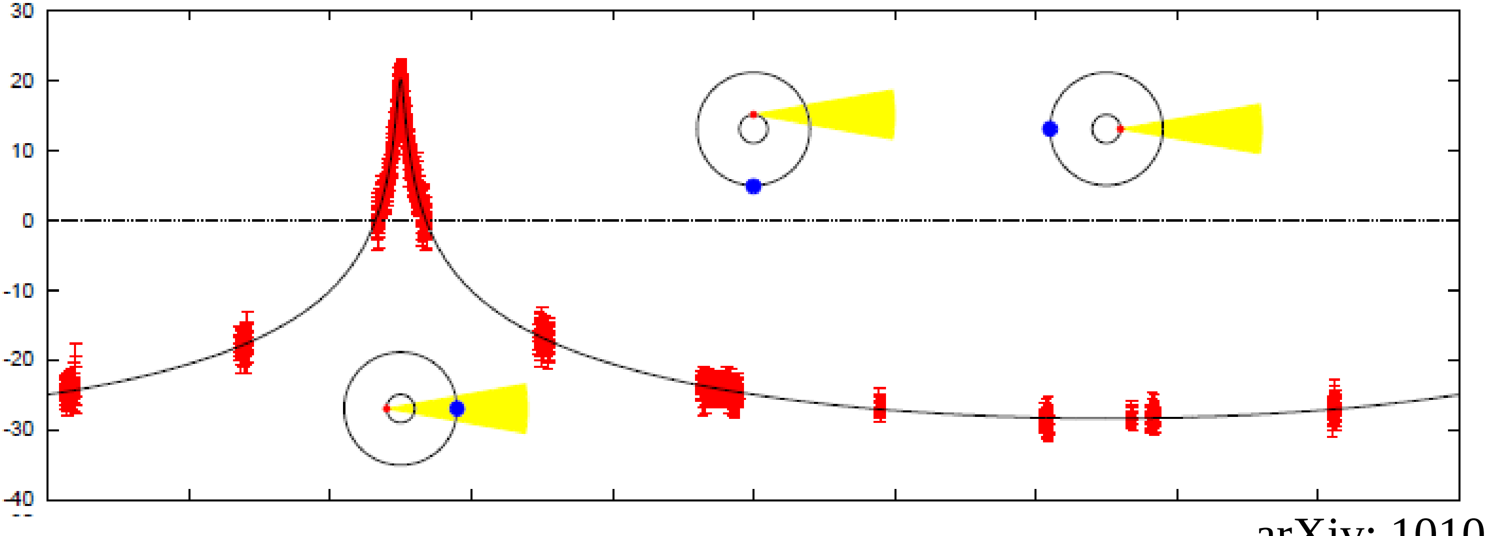
# Very massive neutron star

Binary system: pulsar + white dwarf
PSR 1614-2230




Mass ~ 2 solar

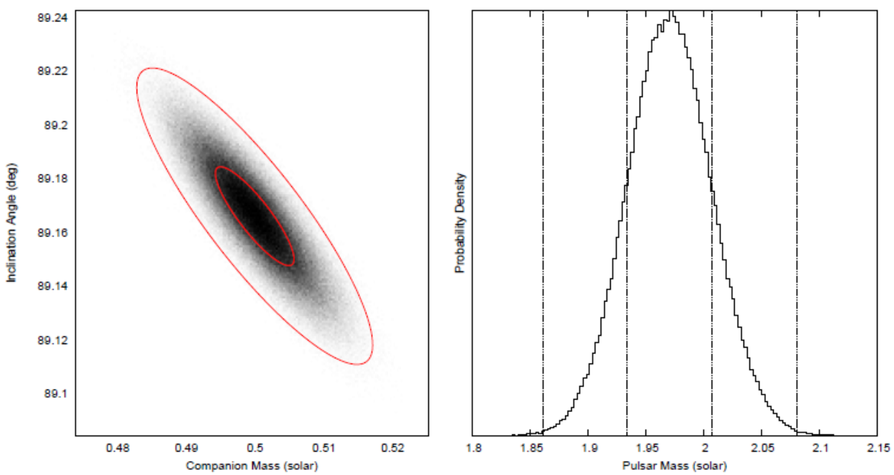
About the WD see 1106.5497.
The object was identified in optics.



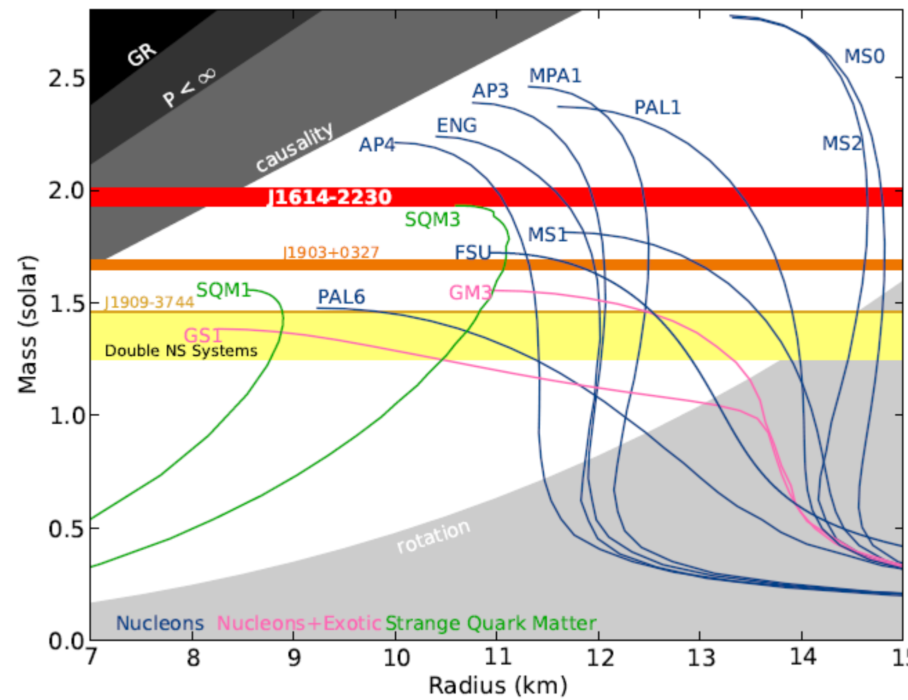

arXiv: 1010.5788

About formation of this objects see 1103.4996

# Why is it so important?




The maximum mass is a crucial property of a given EoS



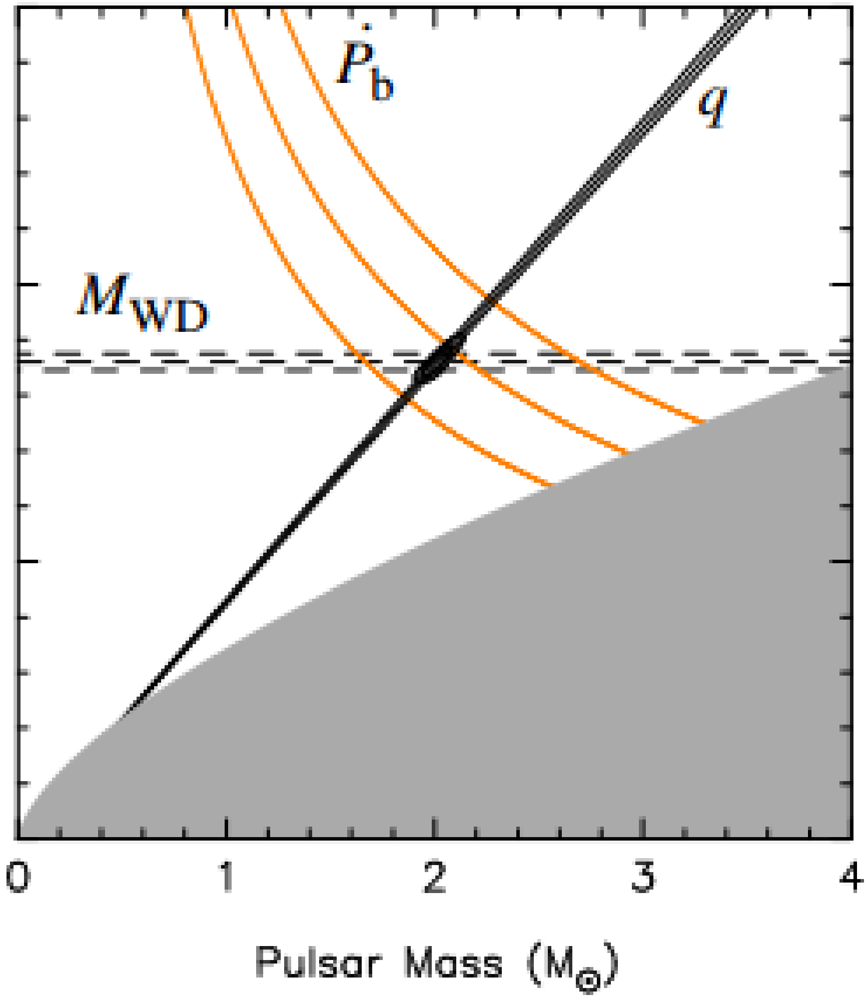

Collapse happens earlier for
softer EoSs, see however, 1111.6929
about quark and hybrid stars
to explain these data.

Interestingly, it was suggested that just
<0.1 solar masses was accreted (1210.8331)

arXiv: 1010.5788

In the future specific X-ray sources (eclipsing msec PSR like SWIFT J1749.4−2807)
can show Shapiro delay and help to obtain masses for a different kind of systems,
see 1005.3527 , 1005.3479 .

# 2.01 solar masses NS




PSR J0348+0432
39 ms, 2.46 h orbit
WD companion

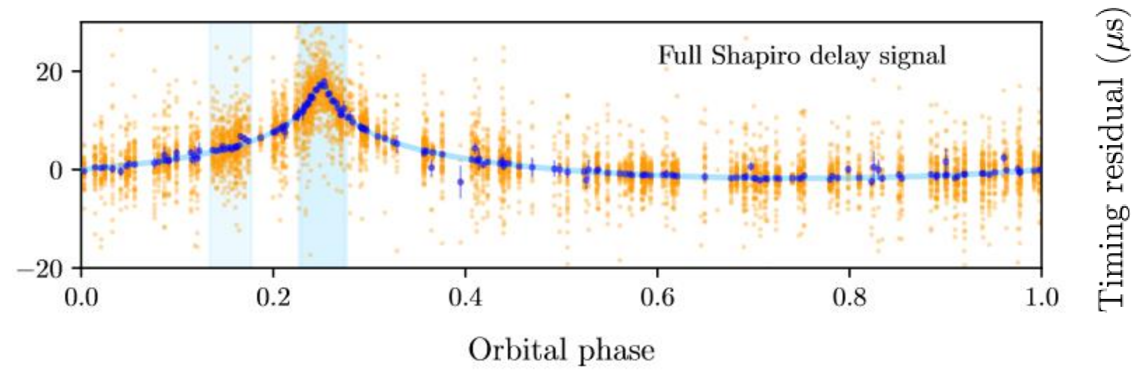
The NS mass is estimated to be:
1.97 – 2.05 solar mass at 68.27%
1.90 – 2.18 solar mass at 99.73%
confidence level.

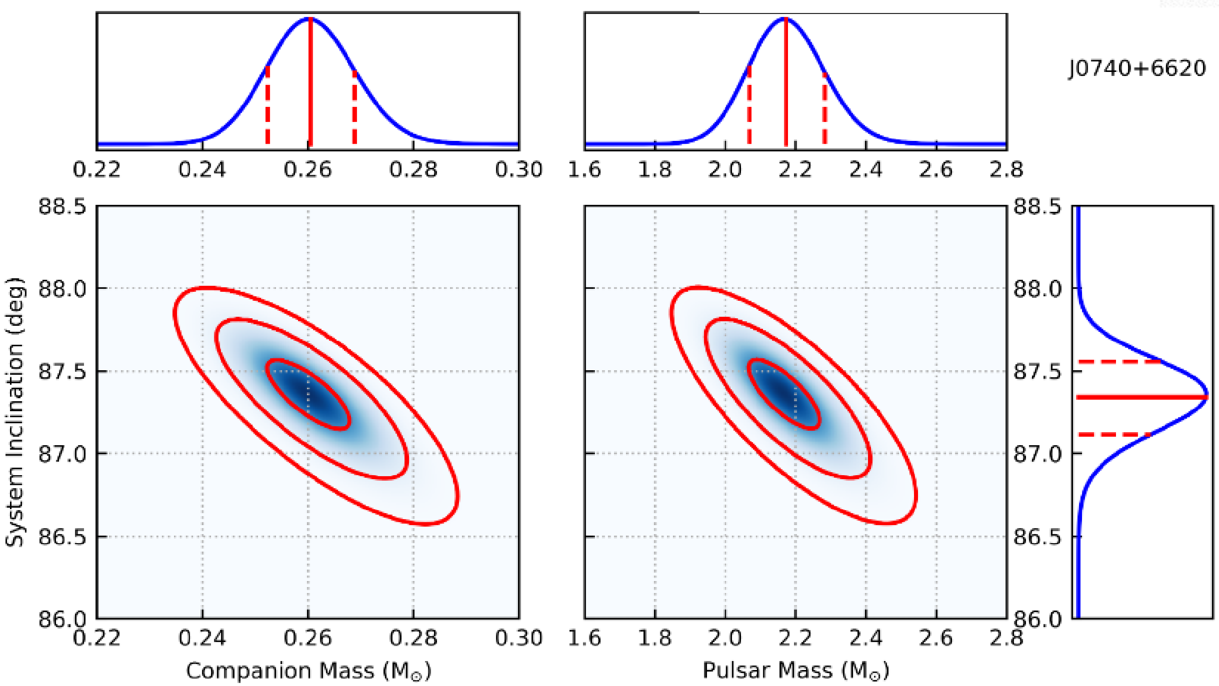
System is perfect for probing
theories of gravity as it is very compact.

1304.6875

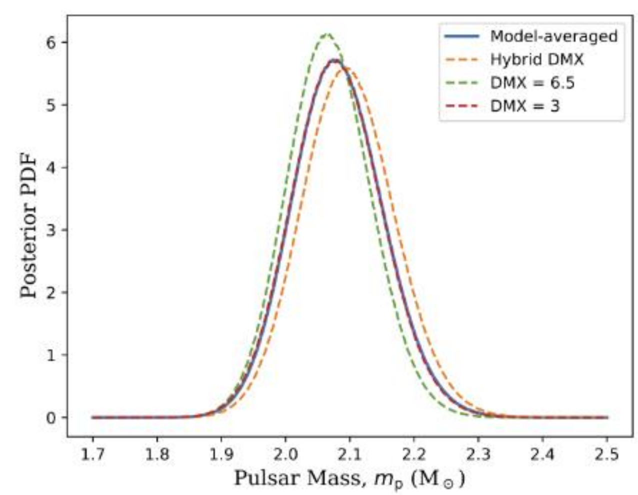
# 2.14 solar mass NS

J0740+6620
2.14 solar masses



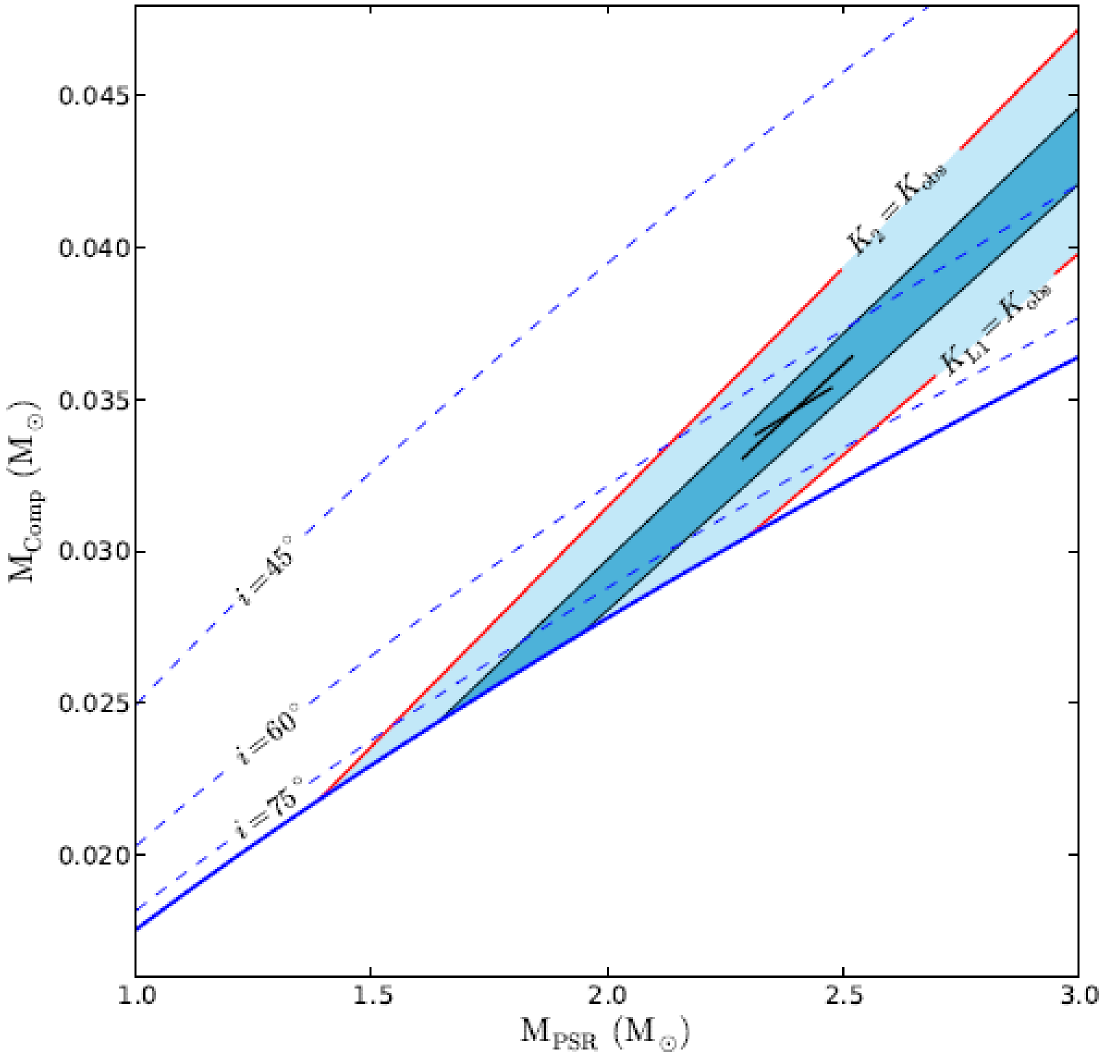

New data confirm it:
2.01-2.17 Msun (1-sigma)




1904.06759

2104.00880

# The most extreme (but unclear) example




BLACK WIDOW PULSAR
PSR B1957+20

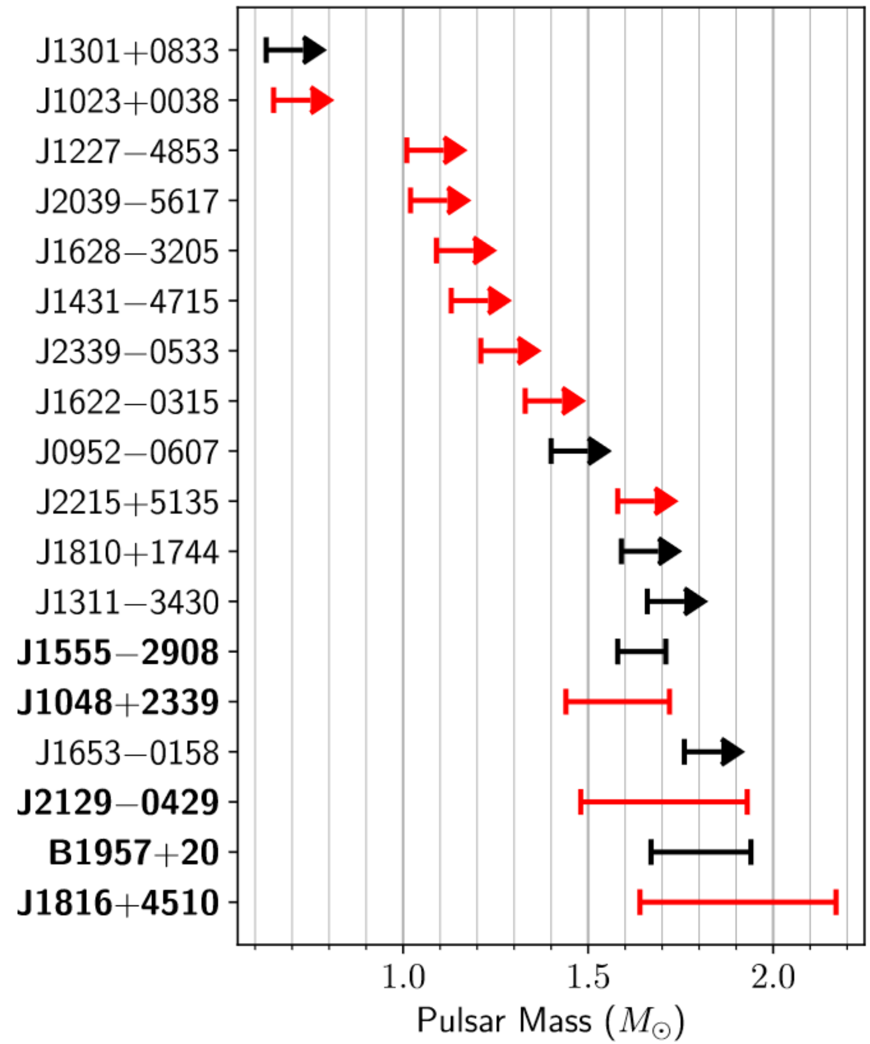
2.4+/-0.12 solar masses

1009.5427

# New estimates from gamma eclipses

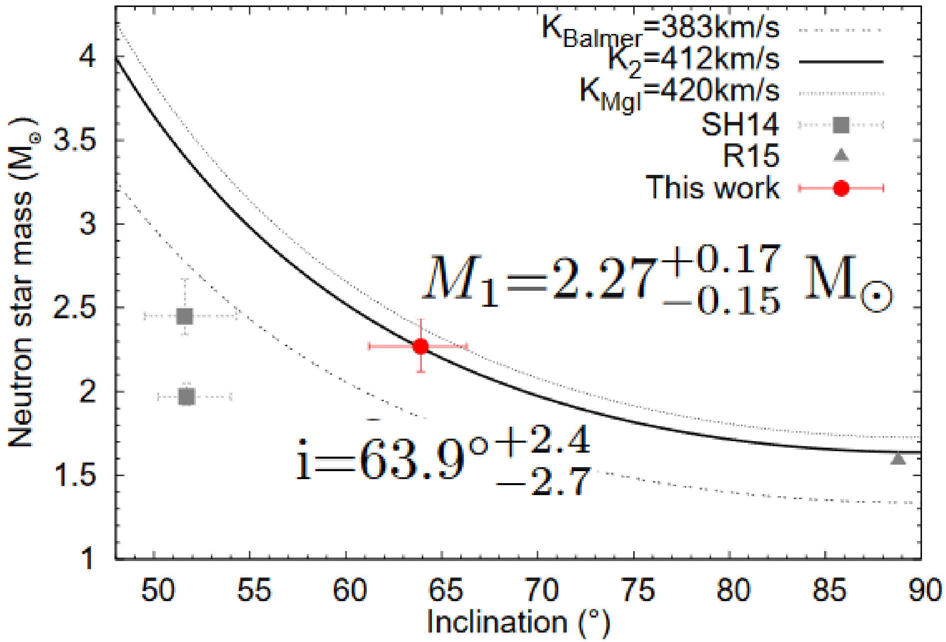
PSR B1957+20
$M_{psr} = 1.81 \pm 0.07 M_{sun}$

Fermi observation
~50 black widow and red backs
For several gamma-ray
eclipses are found.
This allows to obtain
good estimates of inclination.




2301.10995

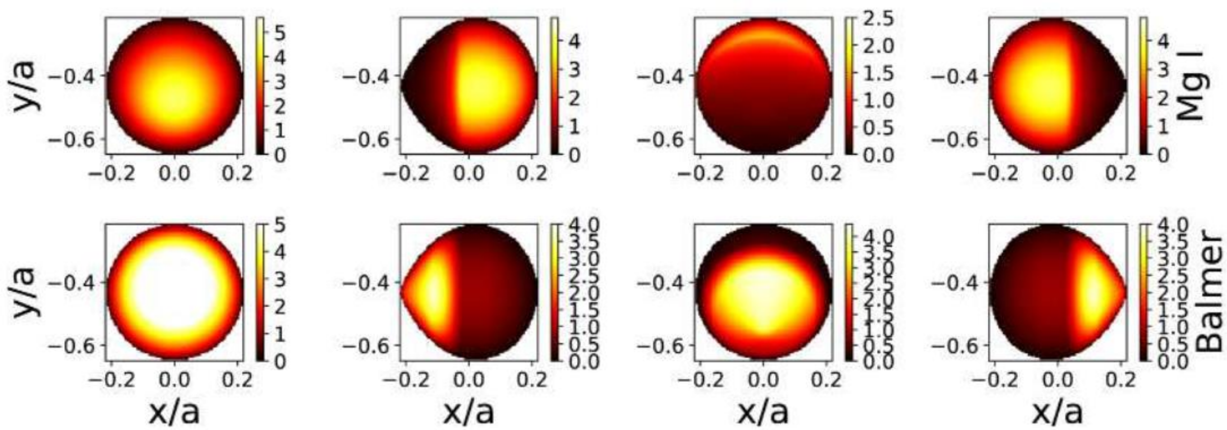
# A massive NS in PSR J2215+5135




Different lines provide different velocity as they are emitted from different sides of the companion.

Different sides of the companion move with different velocity.

Thus, a correct model provides new mass determination.



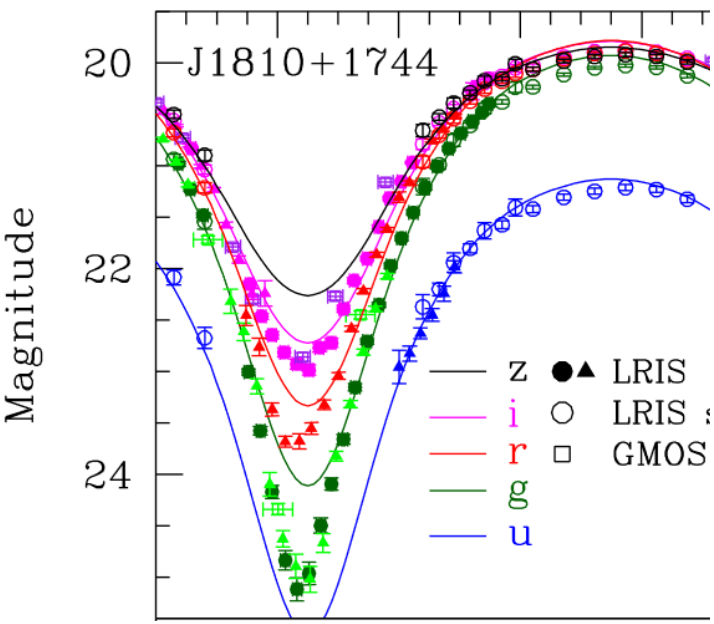

New calculations confirm high mass of the NS (2002.12483).

1805.08799

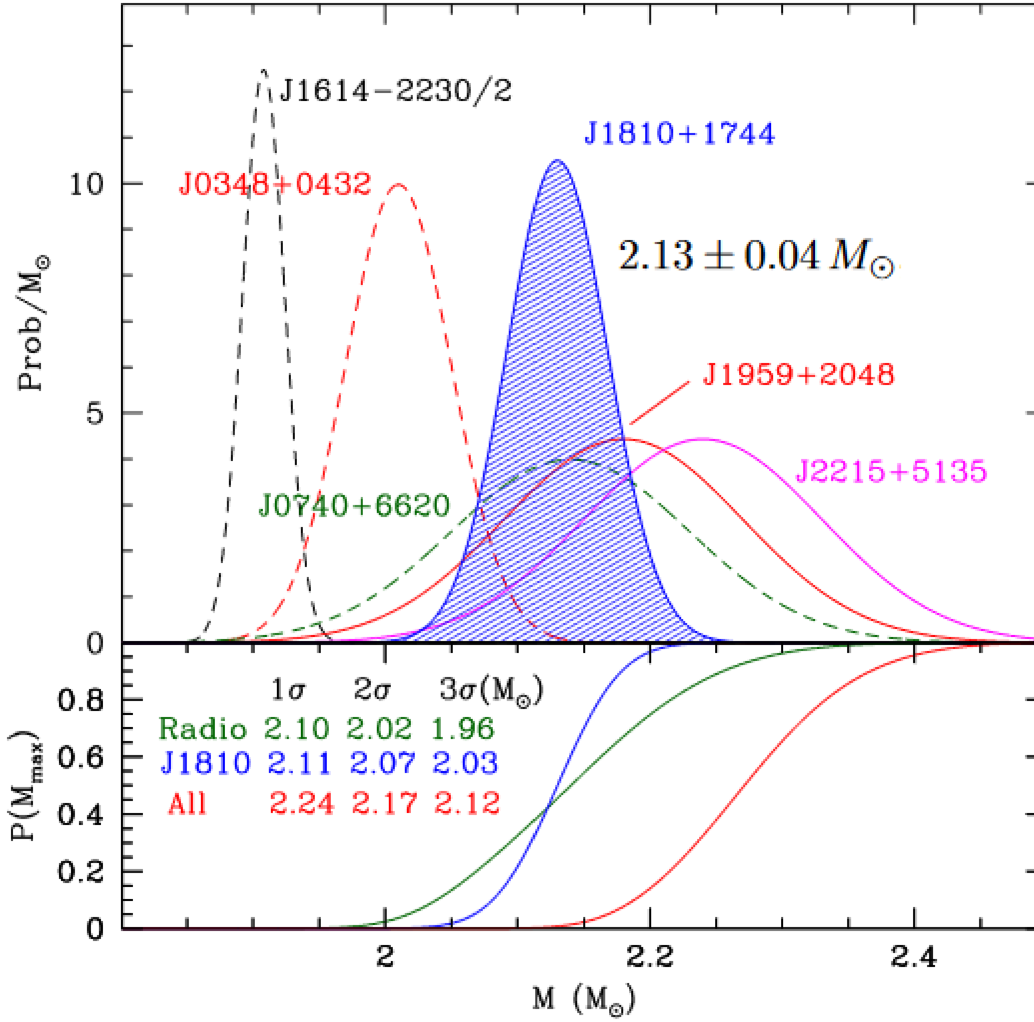
# High mass of PSR J1810+1744

Black widow – like system

Detailed studies of companion are necessary to measure mass.




Keck light curves

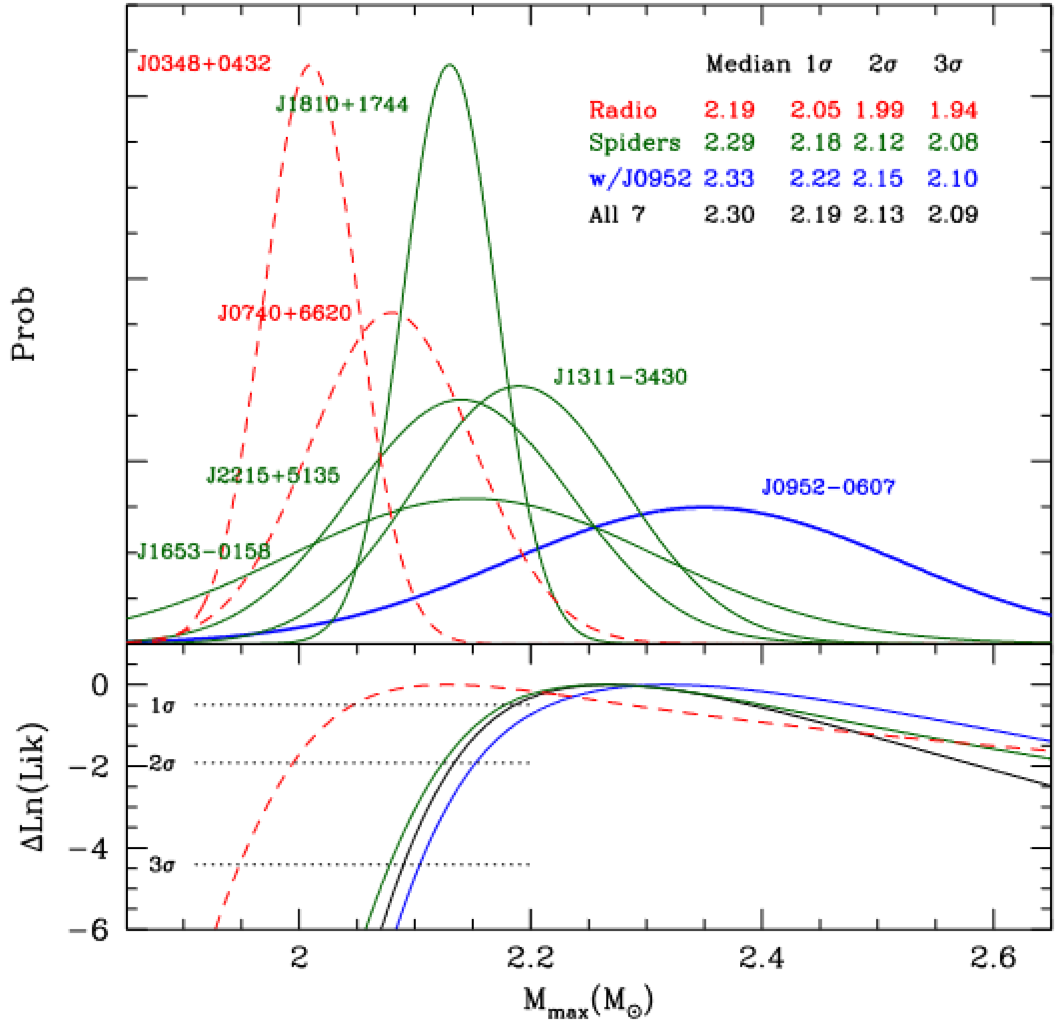



2101.09822

# PSR J0952-0607: the heaviest

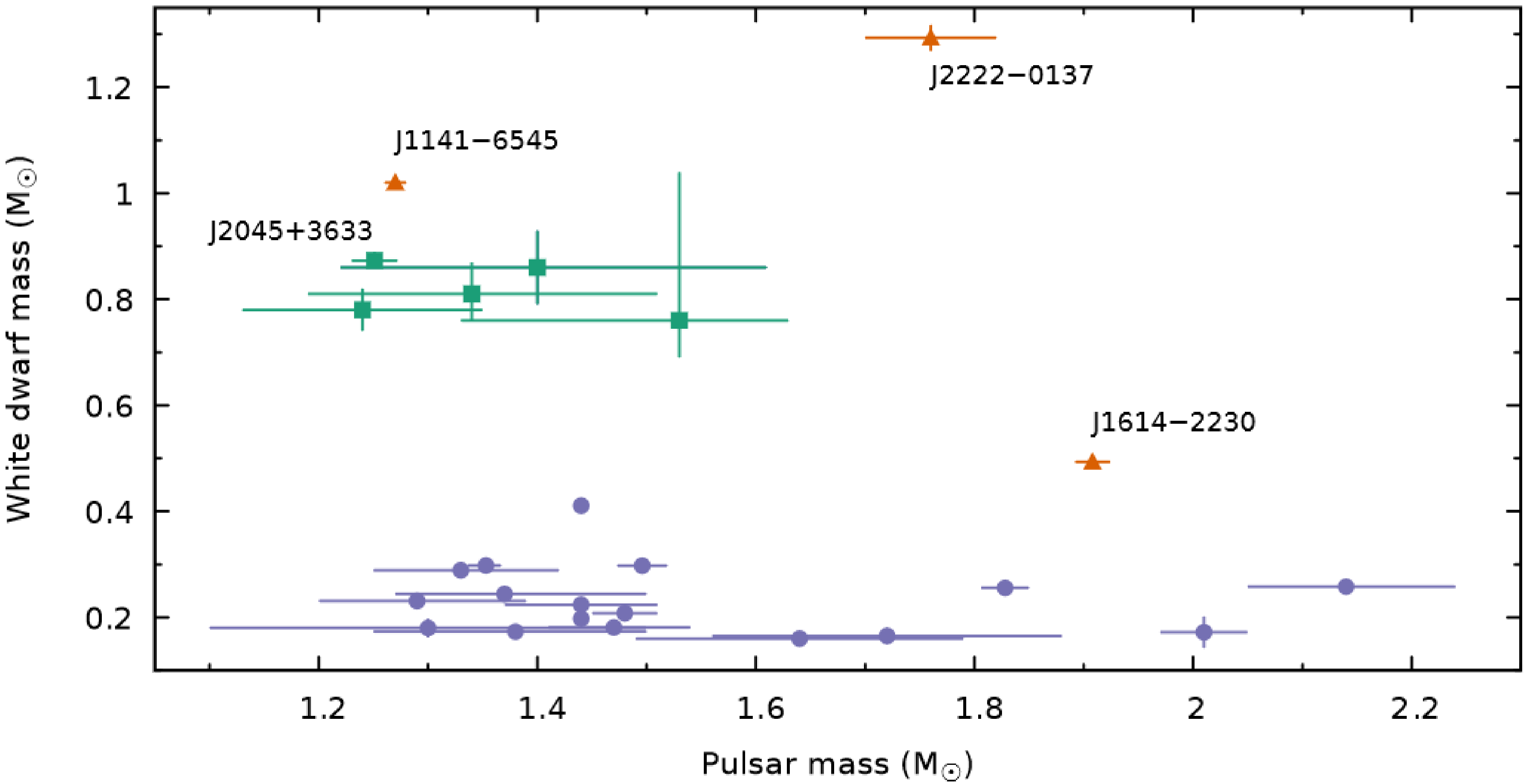
Pspin=1.41 msec
Porb=6.42 hour
low magnetic field ~6 $10^7$ G

| Parameters | Trimmed | All |
|---|---|---|
| $i$ (deg) | $59.8^{+2.0}_{-1.9}$ | $58.5^{+1.9}_{-1.8}$ |
| $f_1$ | $0.79 \pm 0.01$ | $0.77 \pm 0.01$ |
| $L_H$ /$10^{34}$ (erg/s) | $3.81^{+0.46}_{-0.43}$ | $6.22^{+0.88}_{-0.77}$ |
| $T_N$ (K) | $3085^{+85}_{-80}$ | $3206^{+100}_{-95}$ |
| $d_{kpc}$ | $6.26^{+0.36}_{-0.40}$ | $7.60^{+0.74}_{-0.82}$ |
| $\chi^2$/DoF | 286/(298-11)[][1.00] | 451/(314-11)[1.49] |
| $K_{CoM}$ (km/s) | $376.1 \pm 5.1$ | $379.1 \pm 6.8$ |
| $M_{NS}$ ($M_\odot$) | $2.35 \pm 0.17$ | $2.50 \pm 0.20$ |
| $M_C$ ($M_\odot$) | $0.032 \pm 0.002$ | $0.034 \pm 0.002$ |
| $\chi^2$/DoF | 55/(40-2)[1.4] | 90/(43-2)[2.2] |




2207.05124

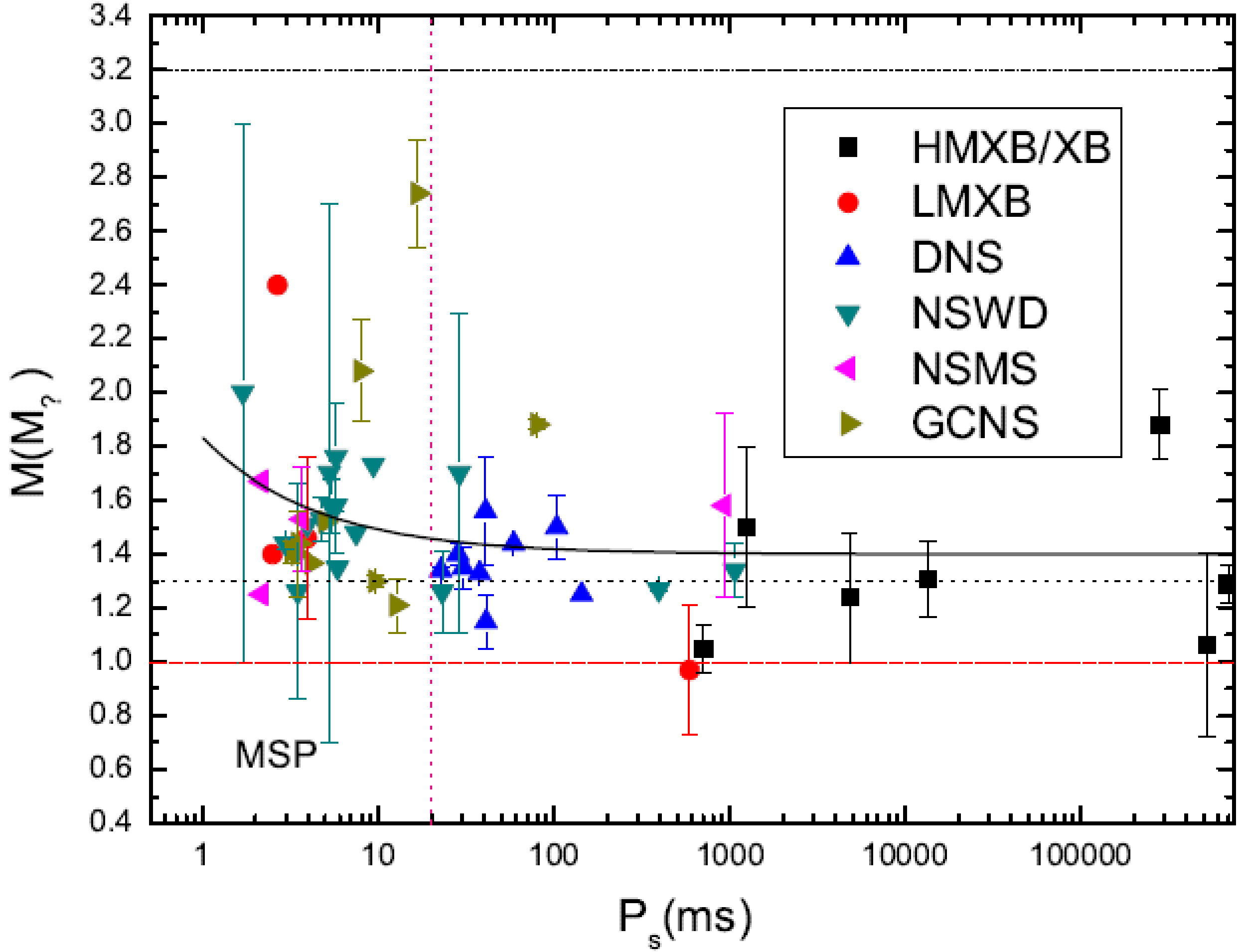
# PSR-WD masses




Light helium white dwarf companions are shown as purple circles, and the systems with massive white dwarf (CO WD) companions are shown as green squares. Triangles – non-recycled PSRs (WD formed first).

2009.12283

# How much do PSRs accrete?

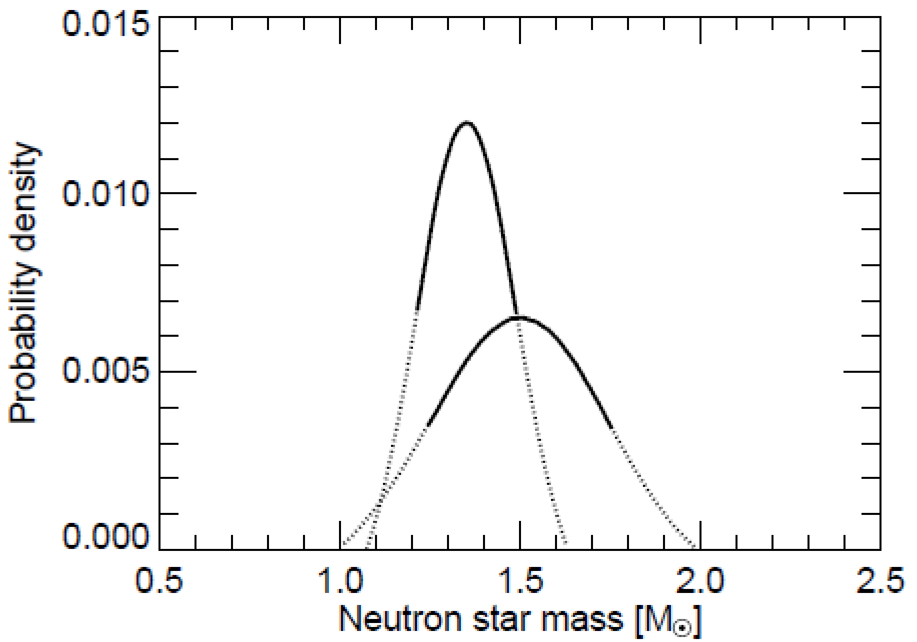



$M=1.4+0.43(P/ms)^{-2/3}$

Millisecond pulsars are ~0.2 solar masses more massive than the rest ones.

1010.5429

# DNS and NS+WD binaries



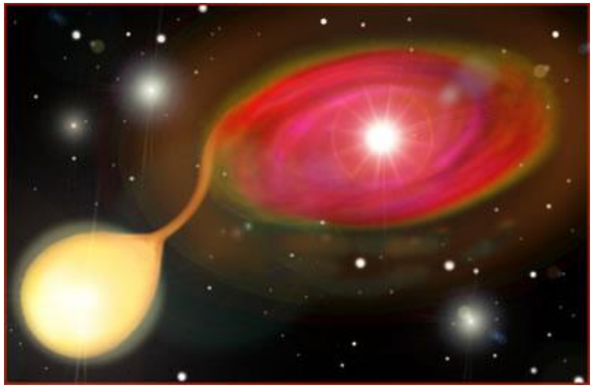

1.35+/-0.13 and 1.5+/-0.25

Cut-off at ~2.1 solar masses can be mainly due to evolution in a binary, not due to nuclear physics (see 1309.6635)

1011.4291

# Neutron stars in binaries

Study of close binary systems gives an opportunity to obtain mass estimate for progenitors of NSs (see for example, Ergma, van den Heuvel 1998 A&A 331, L29).
For example, an interesting estimate was obtained for GX 301-2.
The progenitor mass is >50 solar masses.
On the other hand, for several other systems with both NSs and BHs progenitor masses a smaller: from 20 up to 50.
Finally, for the BH binary LMC X-3 the progenitor mass is estimated as >60 solar.
So, the situation is tricky.
Most probably, in some range of masses, at least in binary systems, stars can produce both types of compact objects: NSs and BHs.



# Mass determination in binaries: mass function

$$f_v(m)\frac{m_x^3 \sin i^3}{(m_x+m_v)^2} = 1,038 \cdot 10^{-7} K_v^3 P (1-e^2)^{3/2}\,,$$

$m_x$, $m_v$ - masses of a compact object and of a normal star (in solar units),
$K_v$ – observed semi-amplitude of line of sight velocity of the normal star (in km/s),
$P$ – orbital period (in days), $e$ – orbital eccentricity, $i$ – orbital inclination
(the angle between the orbital plane and line of sight).

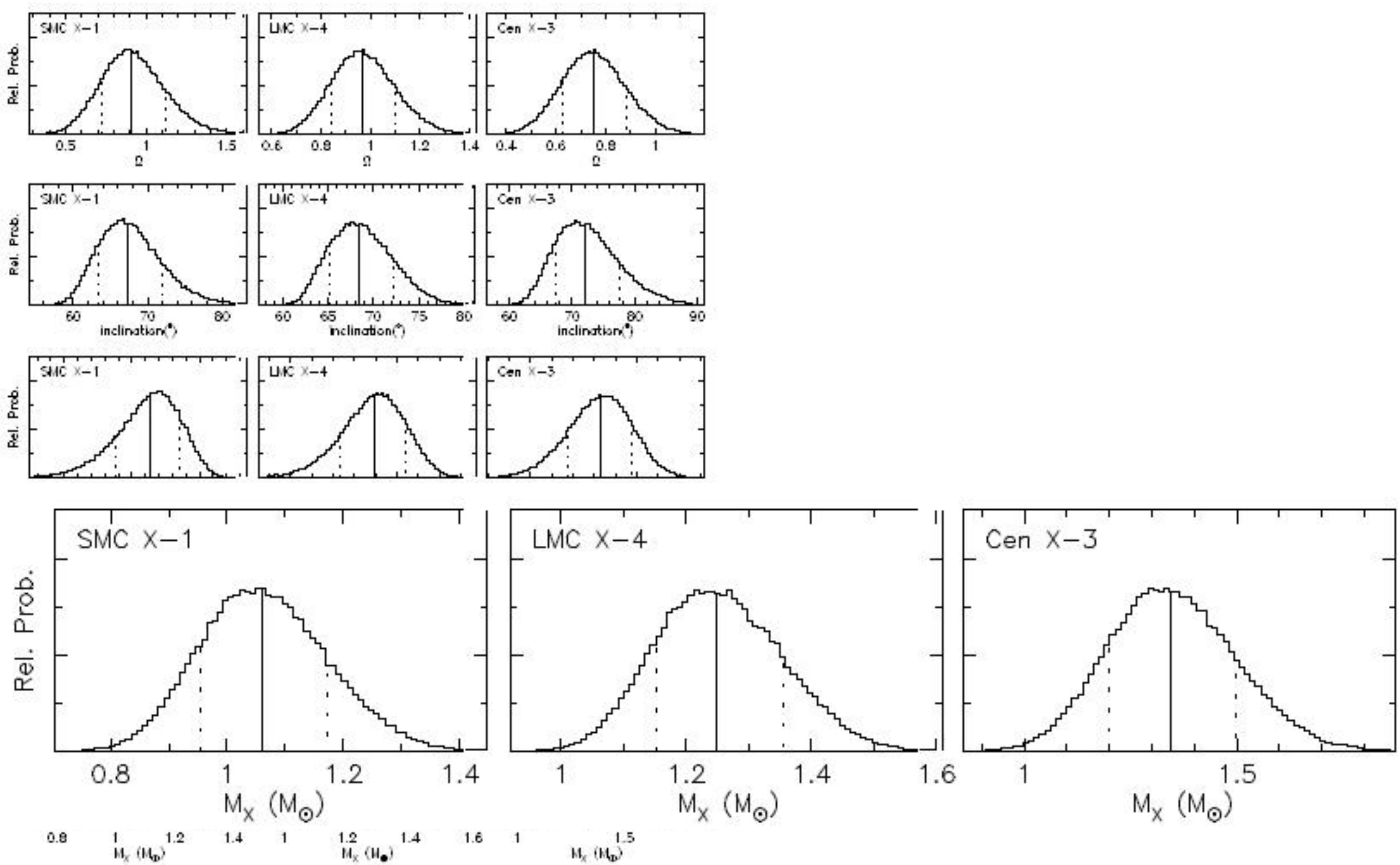
One can see that the mass function is the lower limit for the mass of a compact star.

The mass of a compact object can be calculated as:

$$m_x = f_v(m)\left(1+\frac{m_v}{m_x}\right)^2 \frac{1}{\sin i^3}\,.$$

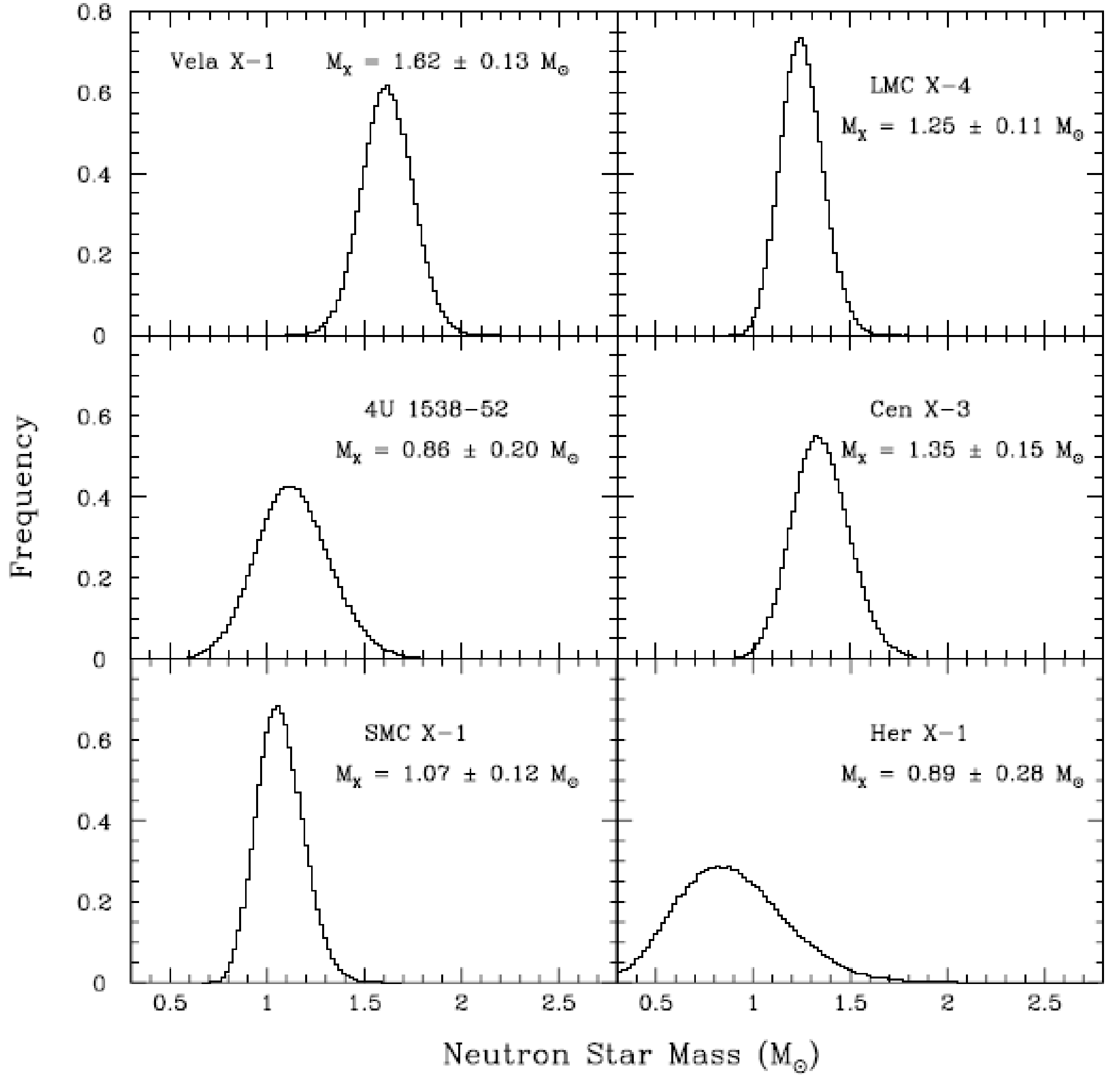
So, to derive the mass it is necessary to know (besides the line of sight velocity) independently two more parameters: mass ration $q=m_x/m_v$, and orbital inclination $i$.

# Some mass estimates




ArXiv: 0707.2802

# More measurements




Six X-ray binary systems.
All are eclipsing pulsars.

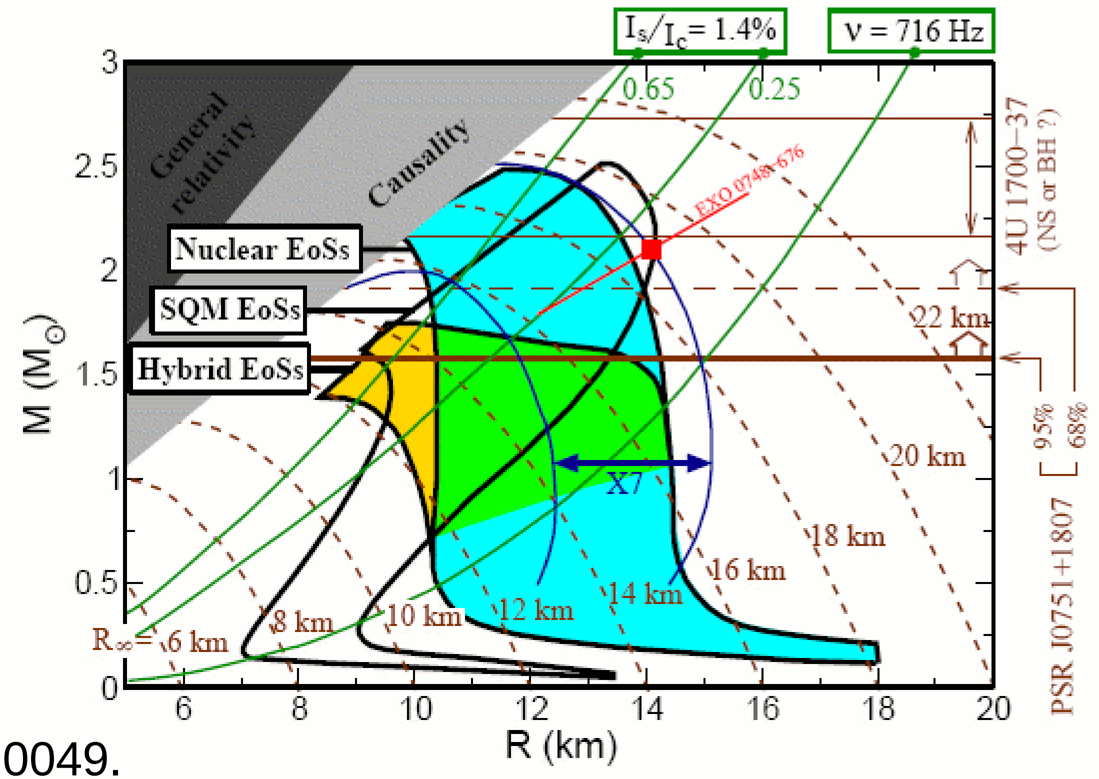
1101.2465

# Mass-radius diagram and constraints

Unfortunately, there are no good data on independent measurements of masses and radii of NSs.

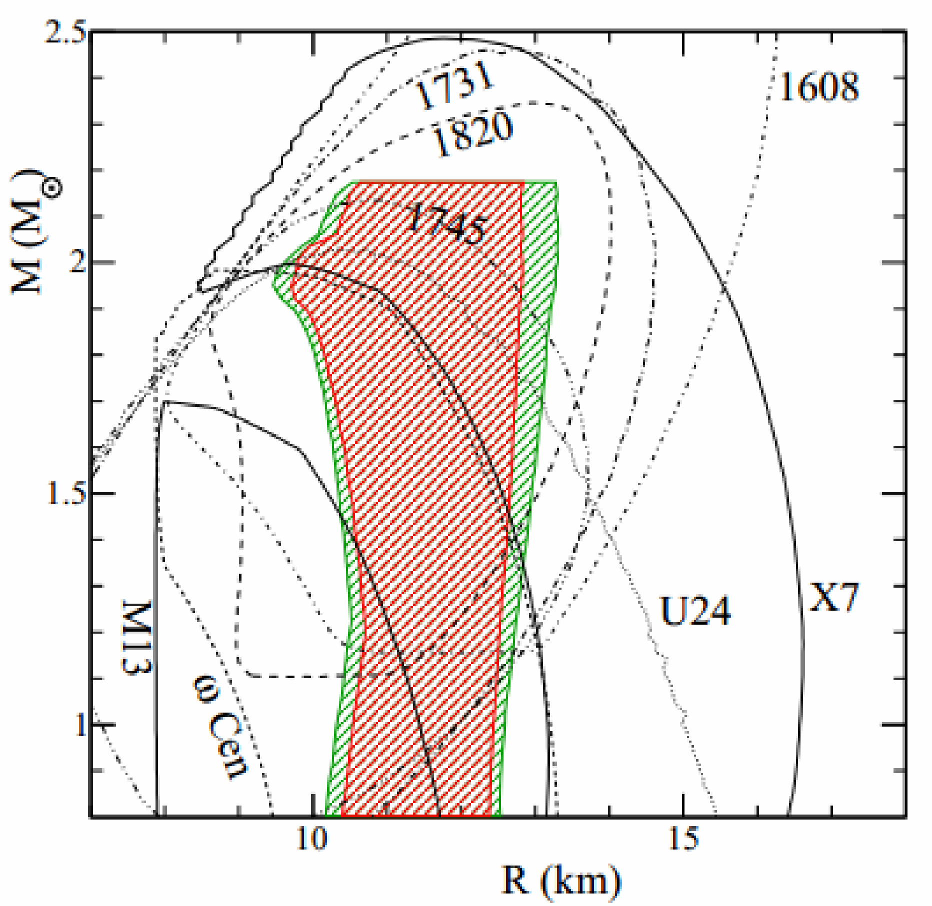
Still, it is possible to put important constraints.
Most of recent observations favour stiff EoS.

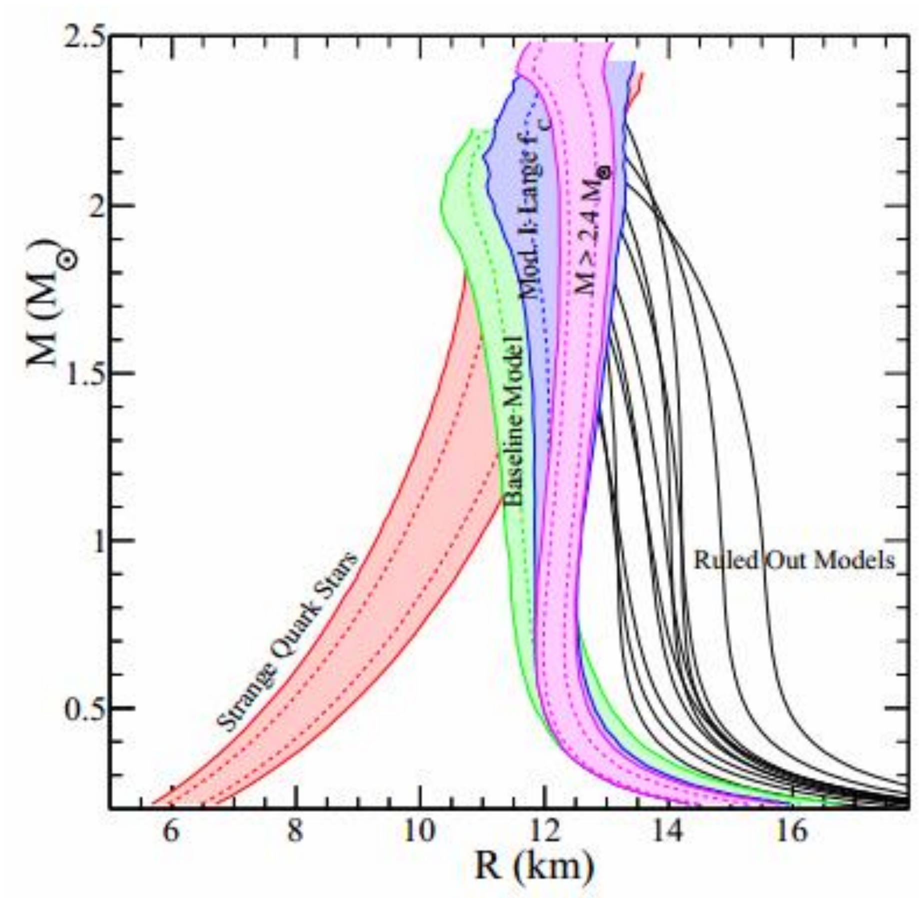
Useful analytical estimates for EoS can be found in 1310.0049.




astro-ph/0608345, 0608360

# Observations vs. data




1205.6871 Some newer results by the same group are presented in 1305.3242

# Mass and radius for a pulsar!

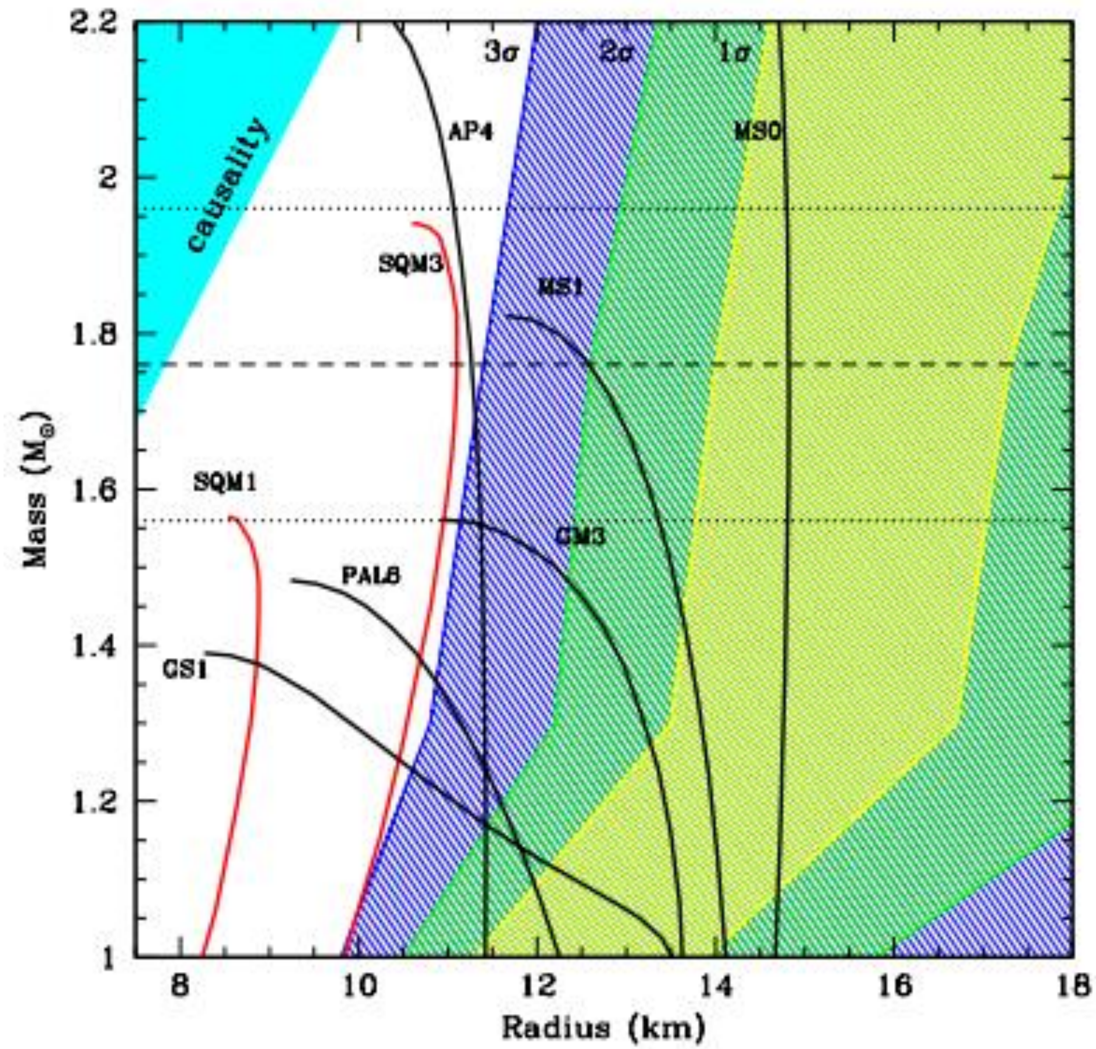
PSR J0437–4715 NS+WD

The nearest known mPSR
155-158 pc

XMM-Newton observations
showed thermal emission.

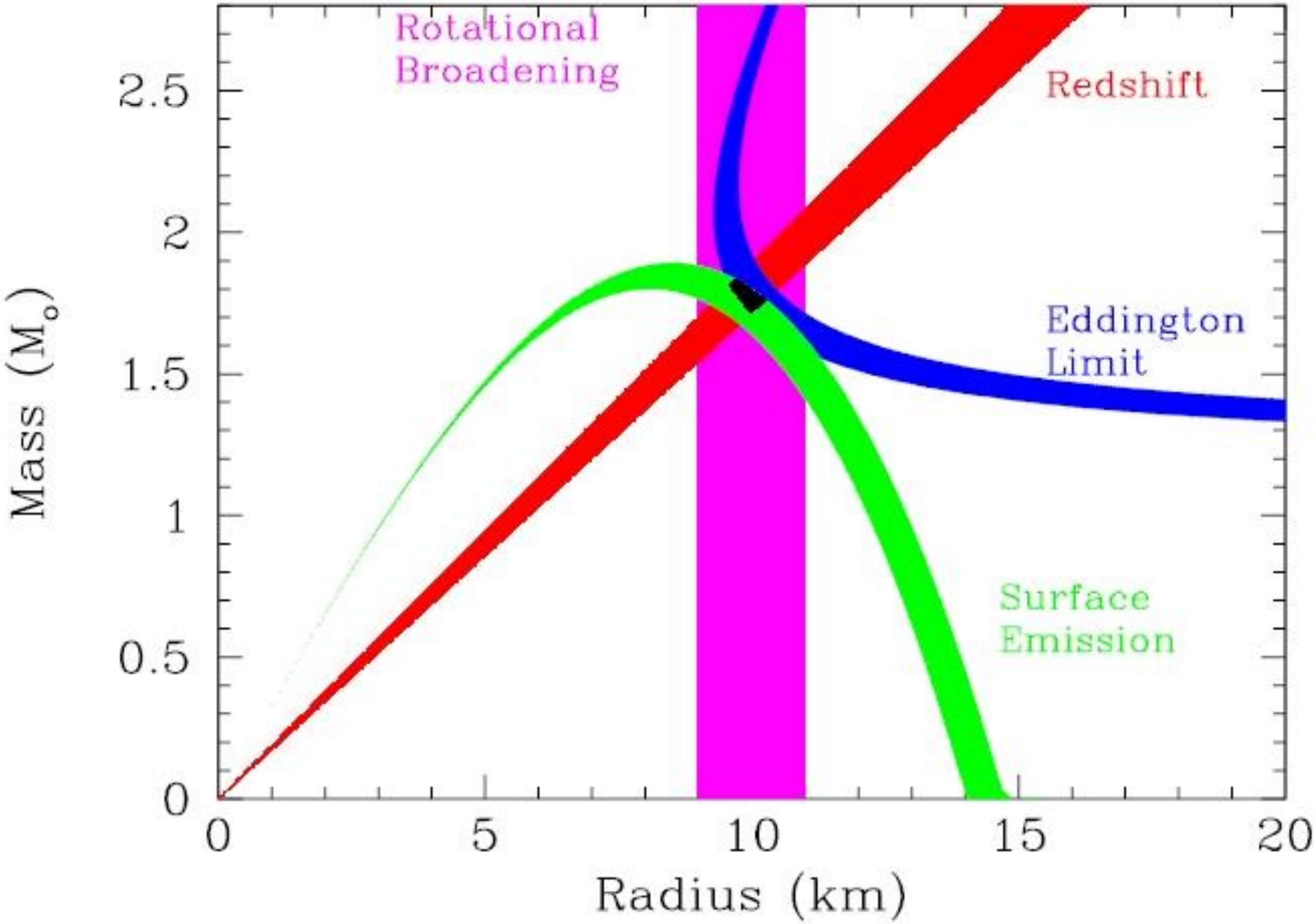
H-atmosphere model fits.

Hot caps are non-antipodal.




1211.6113

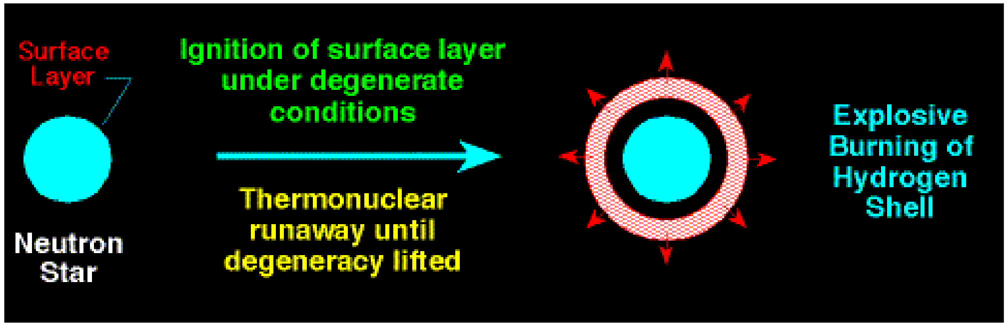
# Combination of different methods




EXO 0748-676

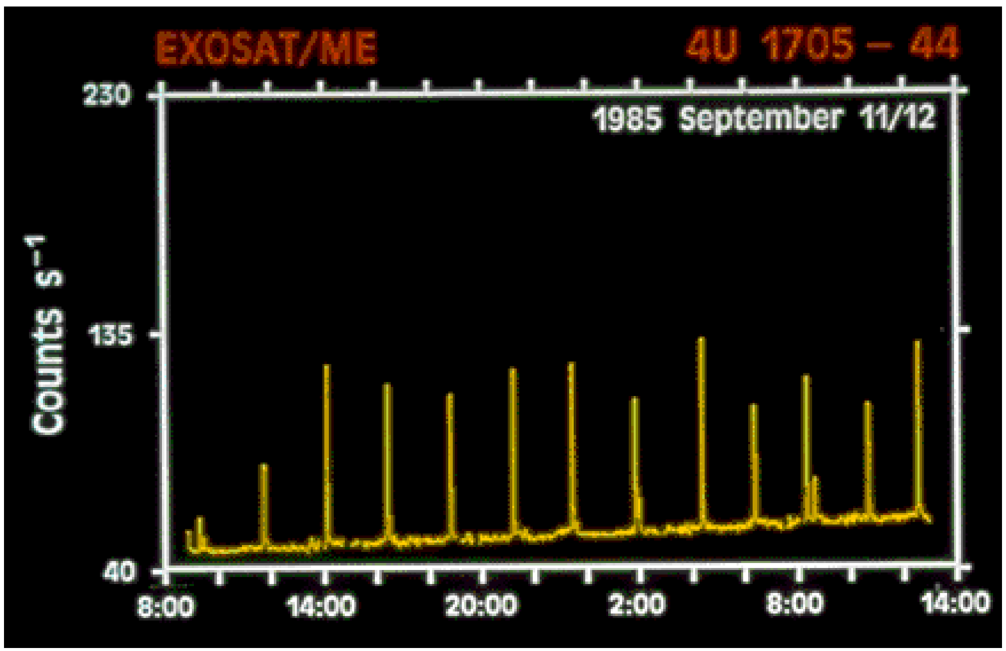
Ozel astro-ph/0605106

# Radius determination in bursters




Explosion with a ~ Eddington liminosity.

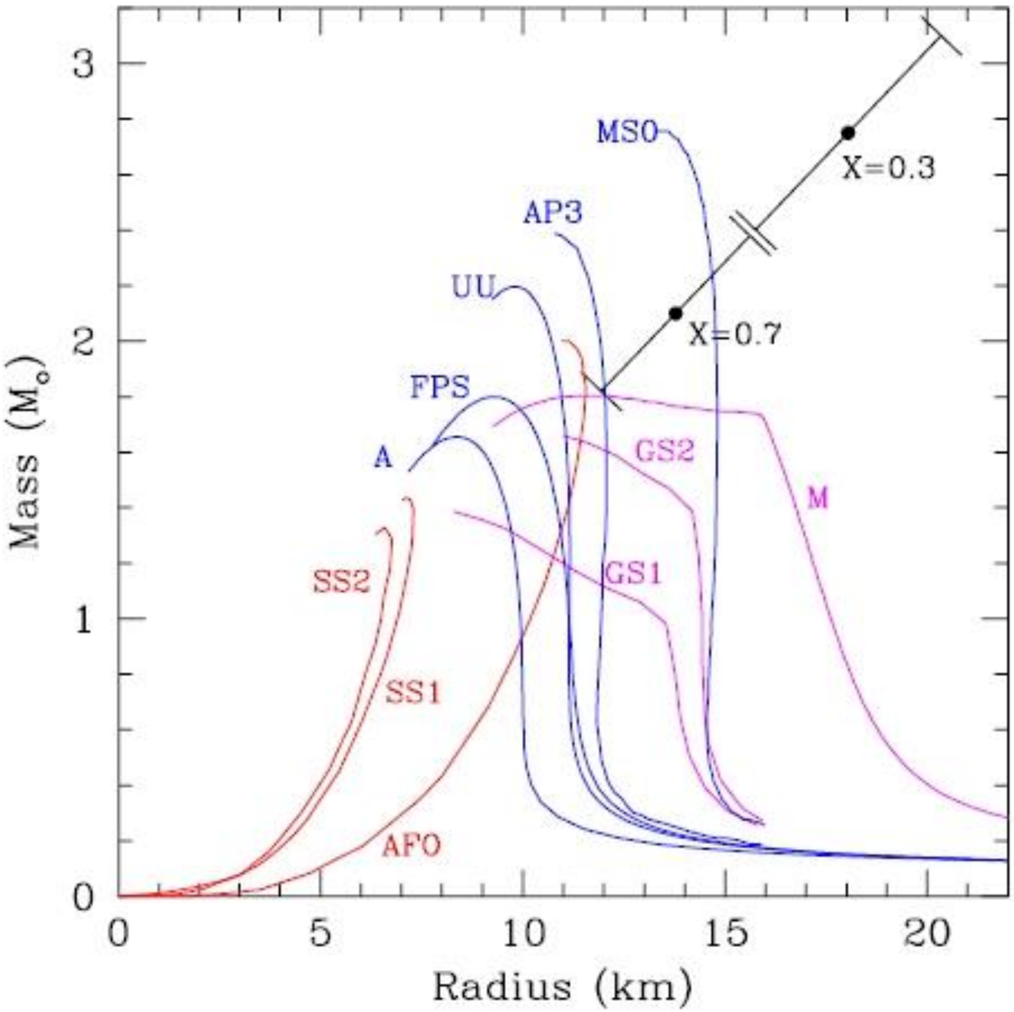
Modeling of the burst spectrum and its evolution.




See, for example,
Joss, Rappaport 1984,
Haberl, Titarchuk 1995

http://www.astro.washington.edu/ben/a510/NSTARS.new.html

# Limits on the EoS from EXO 0748-676

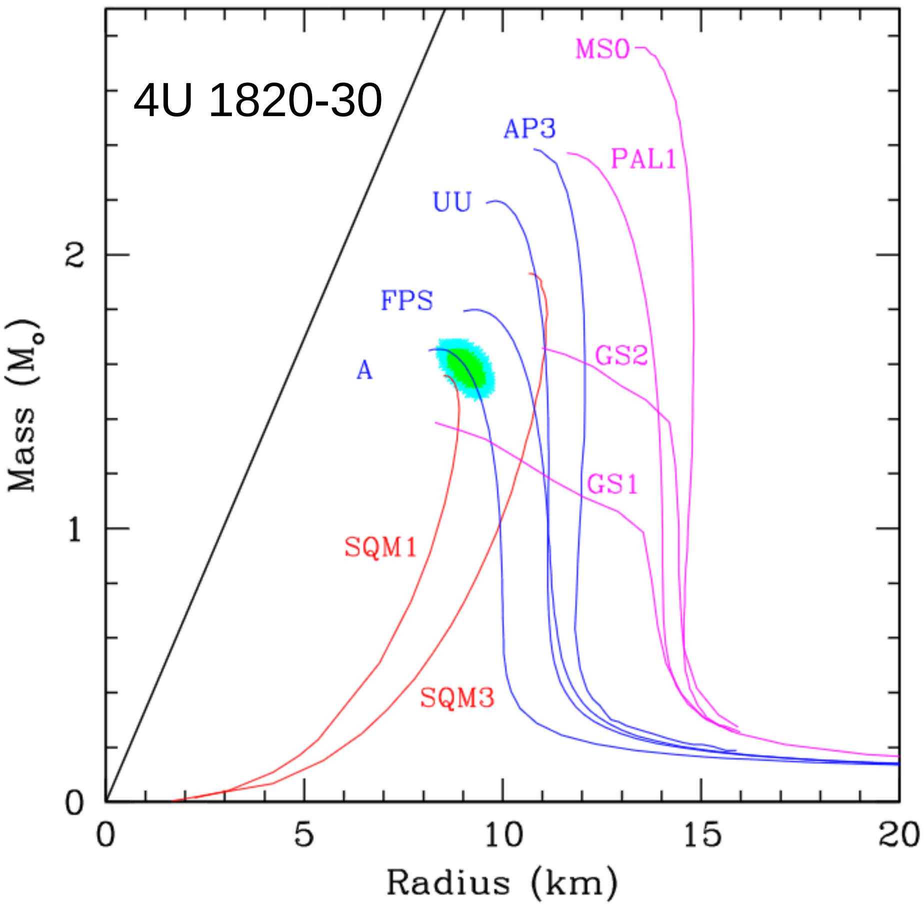


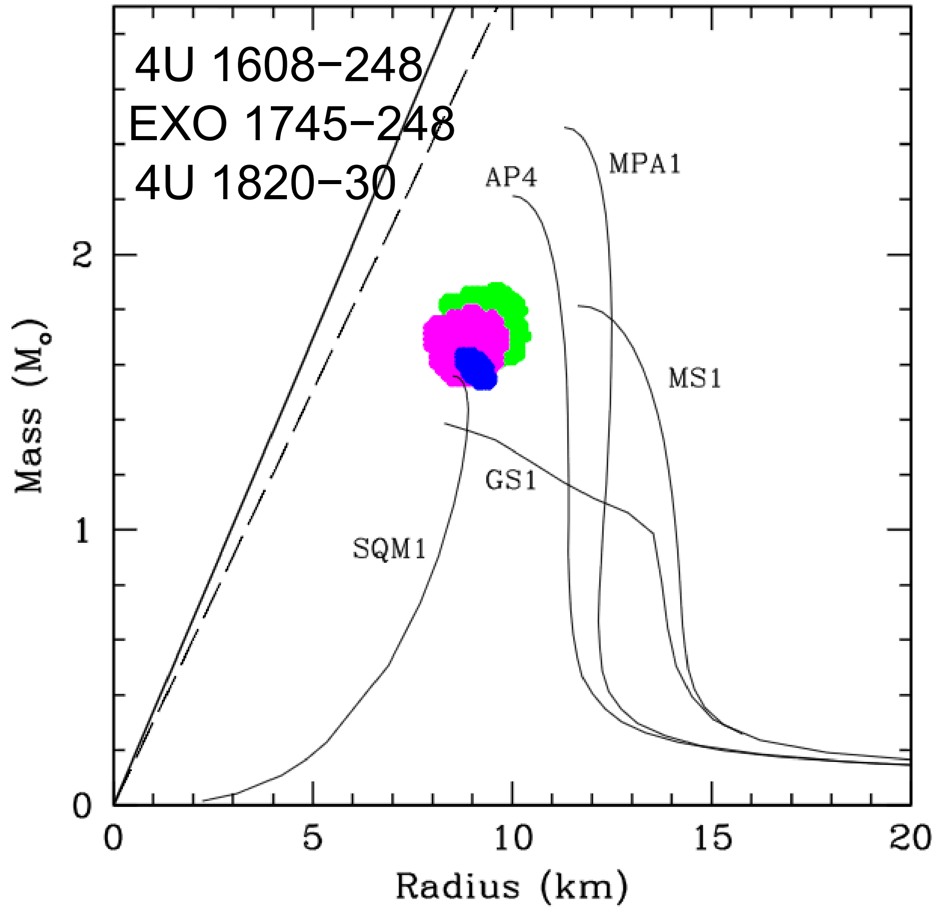

Stiff EoS are better.
Many EoS for strange matter are rejected.
But no all! (see discussion in Nature).

X- hydrogen fraction in the accreted material

Ozel astro-ph/0605106

# Some optimistic estimates

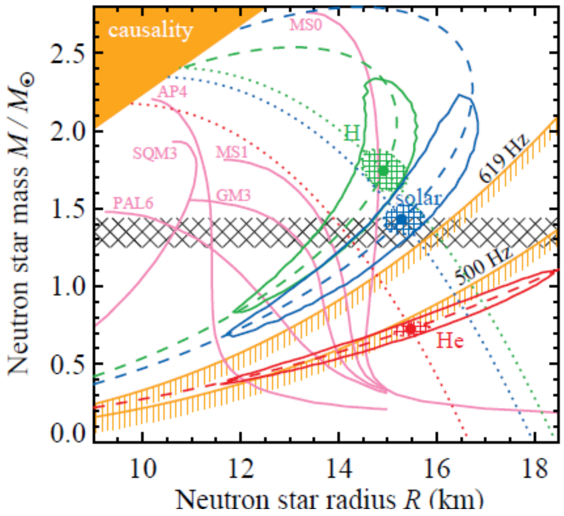



1002.3825

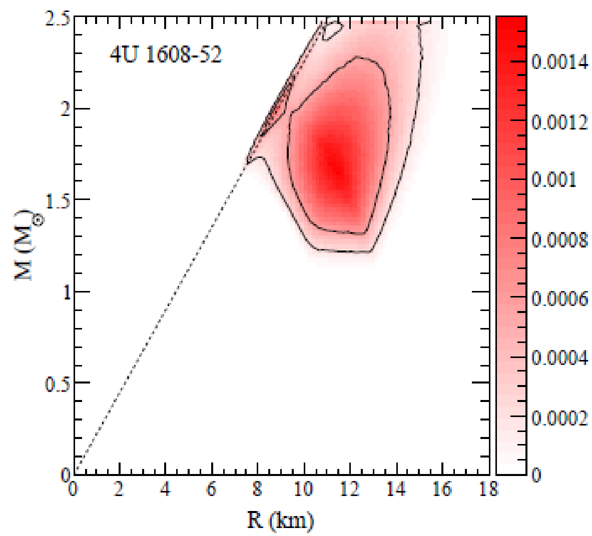
1002.3153

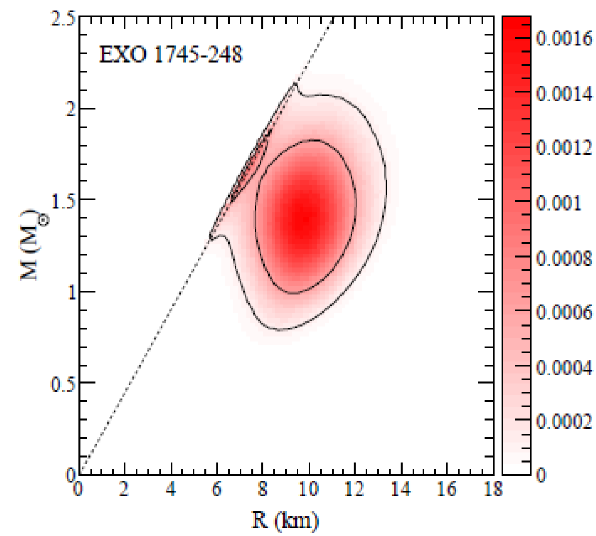
# Pessimistic estimates

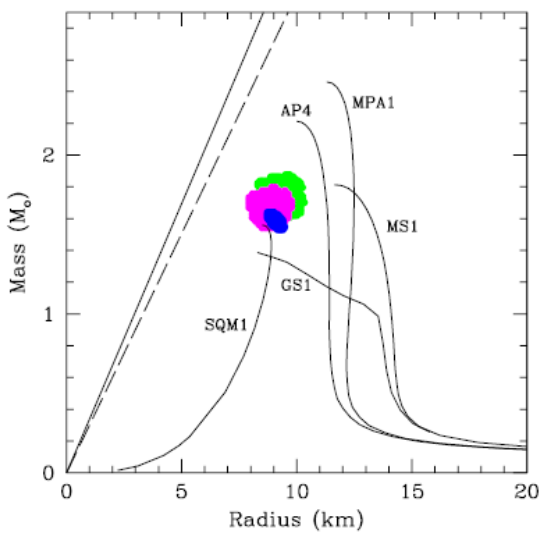



1004.4871

1005.0811



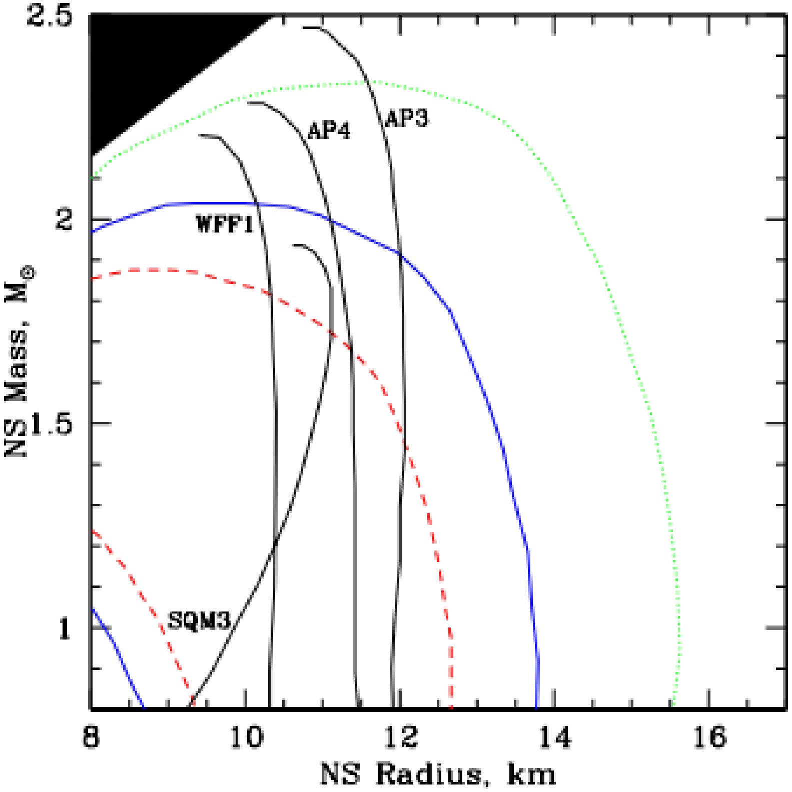




It seems that Ozel et al. underestimate
different uncertainties and make additional assumptions.

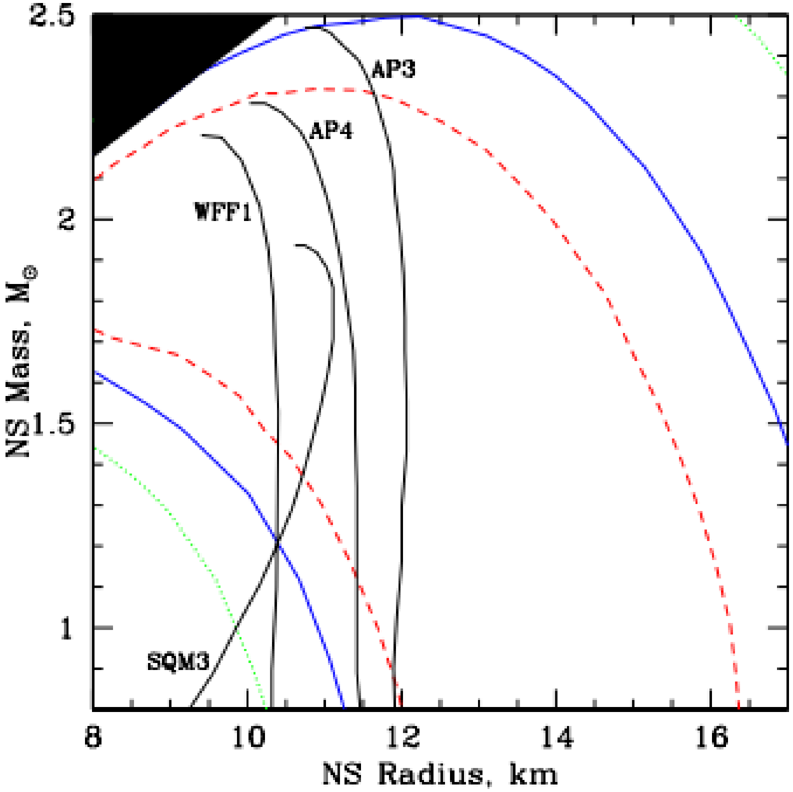
1002.3153

# Atmospheric uncertainties

qLMXB in M13




Hydrogene



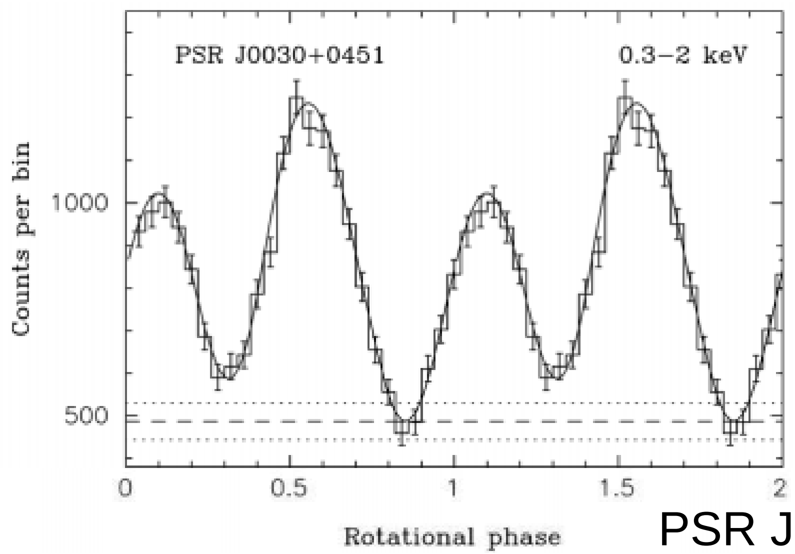

Helium

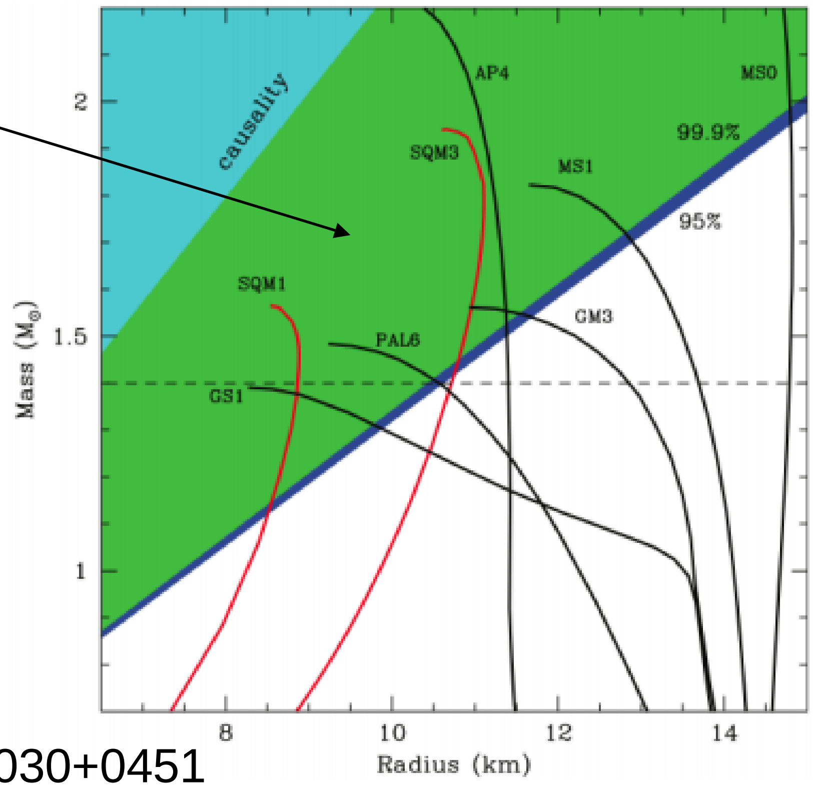
1301.3768

# Pulse profile constraints

The idea is that: sharp pulses are possible only in the case of a large star.

Green – excluded region



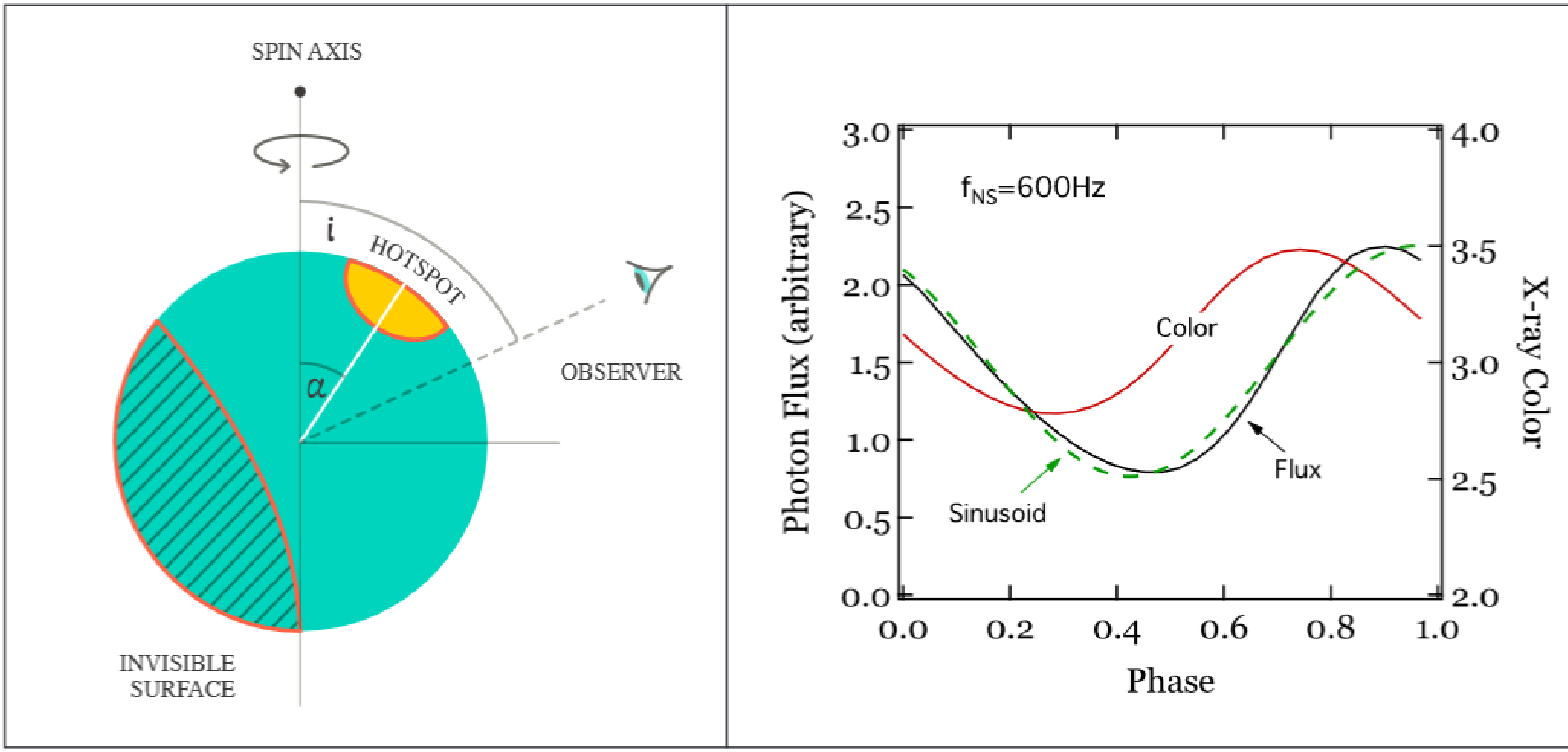




PSR J0030+0451

Based on Bogdanov, Grindlay 2009

1303.0317

# Hot spots and pulse profiles



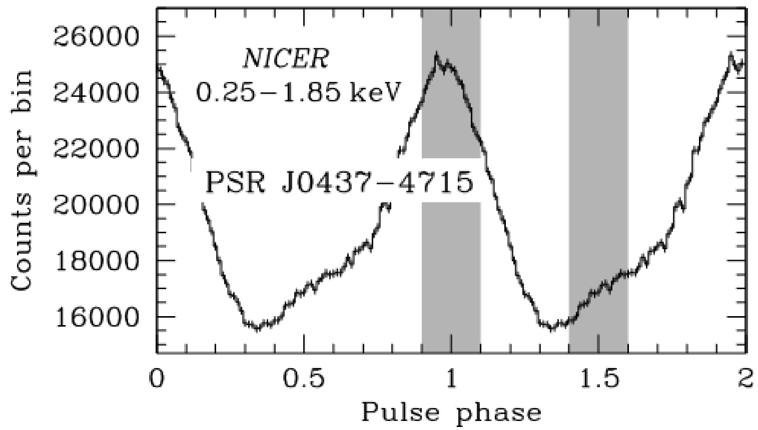

As the neutron star rotates, emission from a surface hotspot generates a pulsation. The figure shows observer inclination i, and hotspot inclination α. The invisible surface is smaller than a hemisphere due to relativistic light-bending.

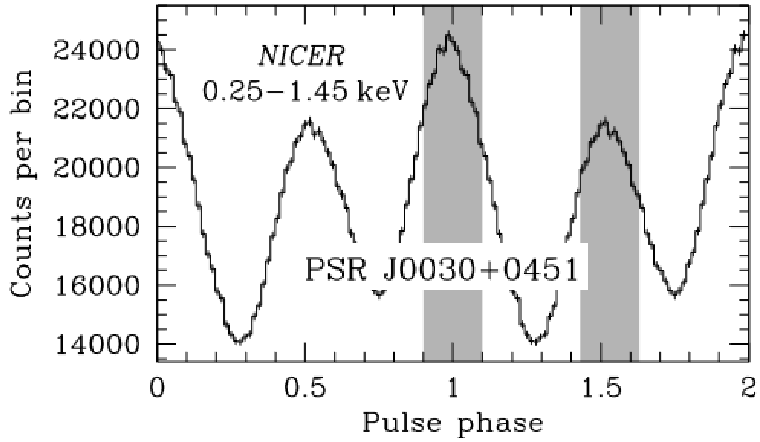
1602.01081

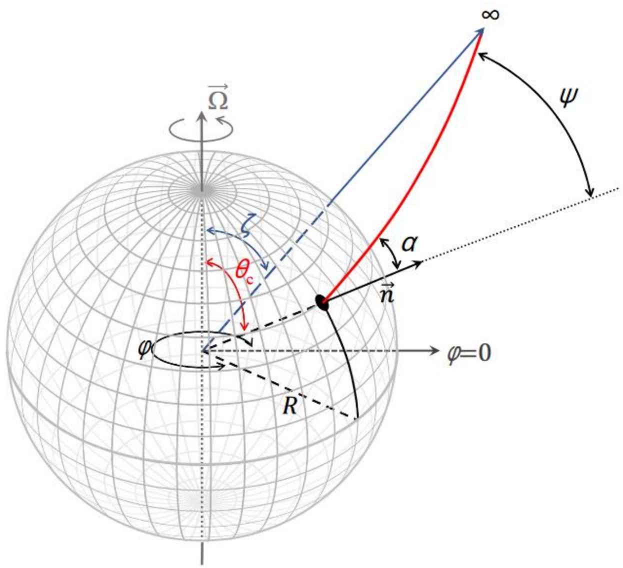
Detailed model description in 2104.06928.

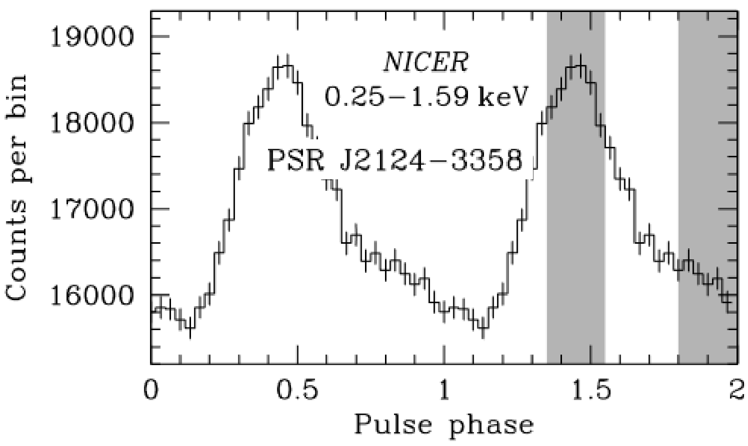
# NICER's mPSRs

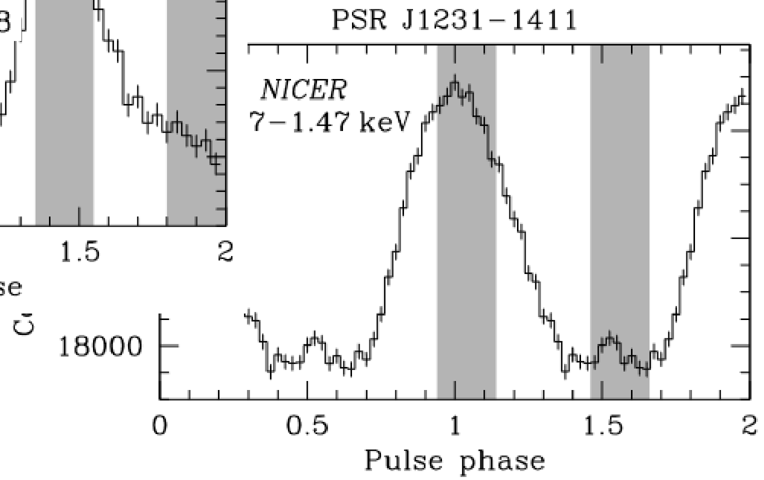
Four near-by millisecond radio pulsars:
J0437−4715
J0030+0451
J1231−1411
J2124−3358




1912.05707, 1912.05706

# Results from NICER. PSR J0030+0451

$1.34^{+0.15}_{-0.16}\ \mathrm{M}_{\odot}$

$12.71^{+1.14}_{-1.19}\ \mathrm{km}$

For the ST-PST model

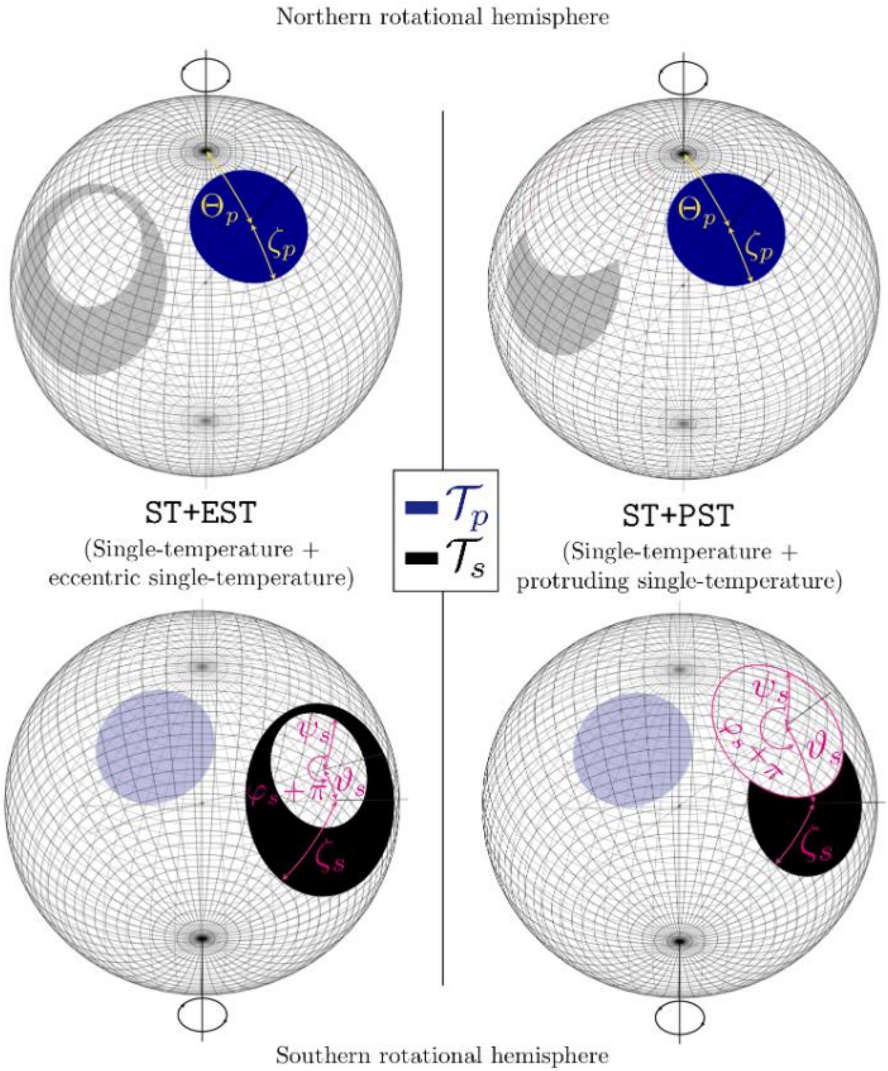
Single temperature+Protruding single temp.
No antipodal symmetry.

But several other tried models are not ruled out.

For example, in the ST-CST model

$$M = 1.44^{+0.18}_{-0.19}\ \mathrm{M}_{\odot}$$

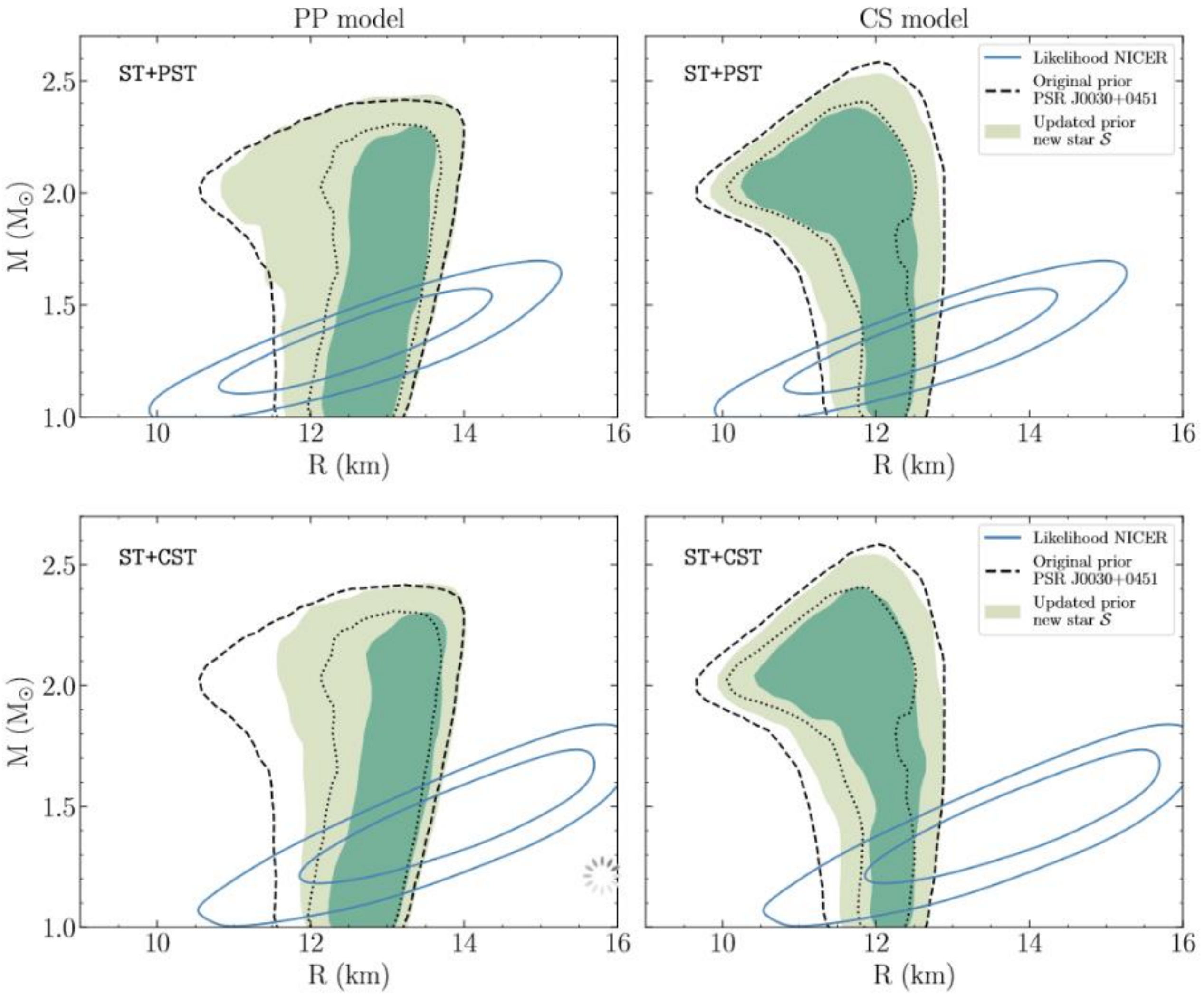
$$R_{\mathrm{eq}} = 13.89^{+1.22}_{-1.39}\ \mathrm{km}$$




1912.05702

# Results from NICER. PSR J0030+0451

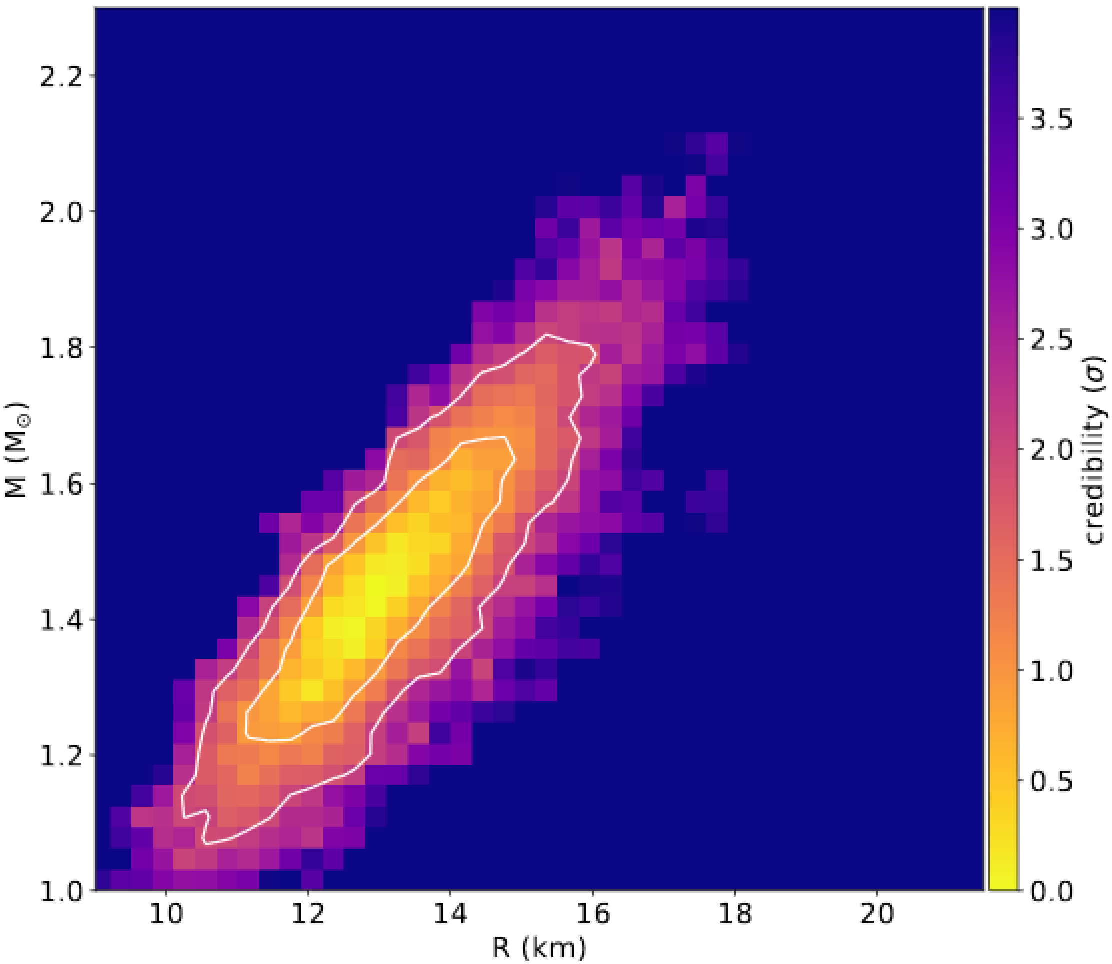
Two types of EoS models are considered:
- PP (piecewise-polytropic);
- CS (speed of sound).

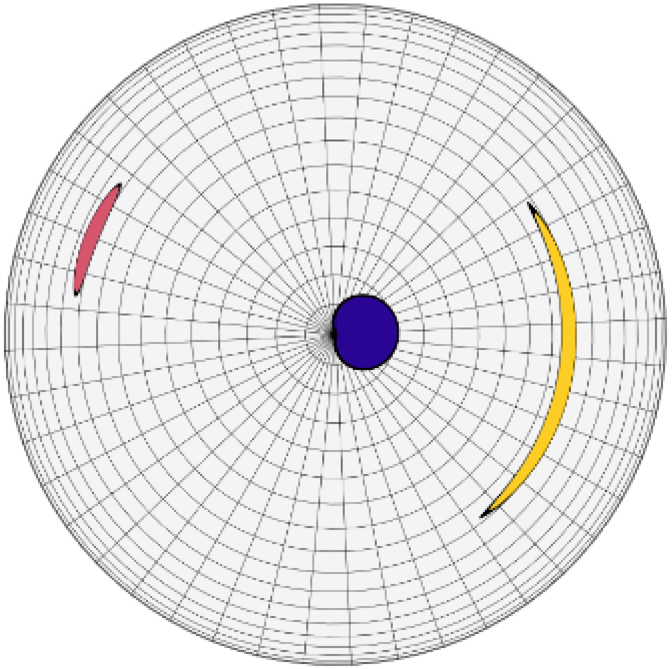



1912.05703

# Results from NICER. PSR J0030+0451

Three oval spots model.
Non-trivial field structure.



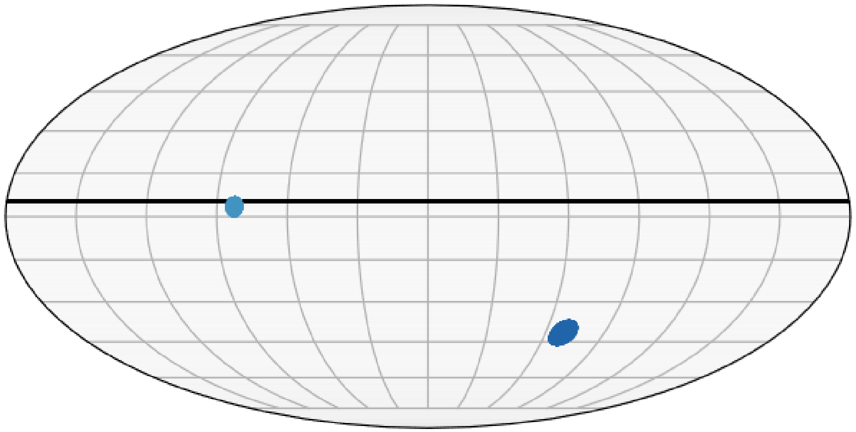



$$R_e = 13.02^{+1.24}_{-1.06} \text{ km}$$

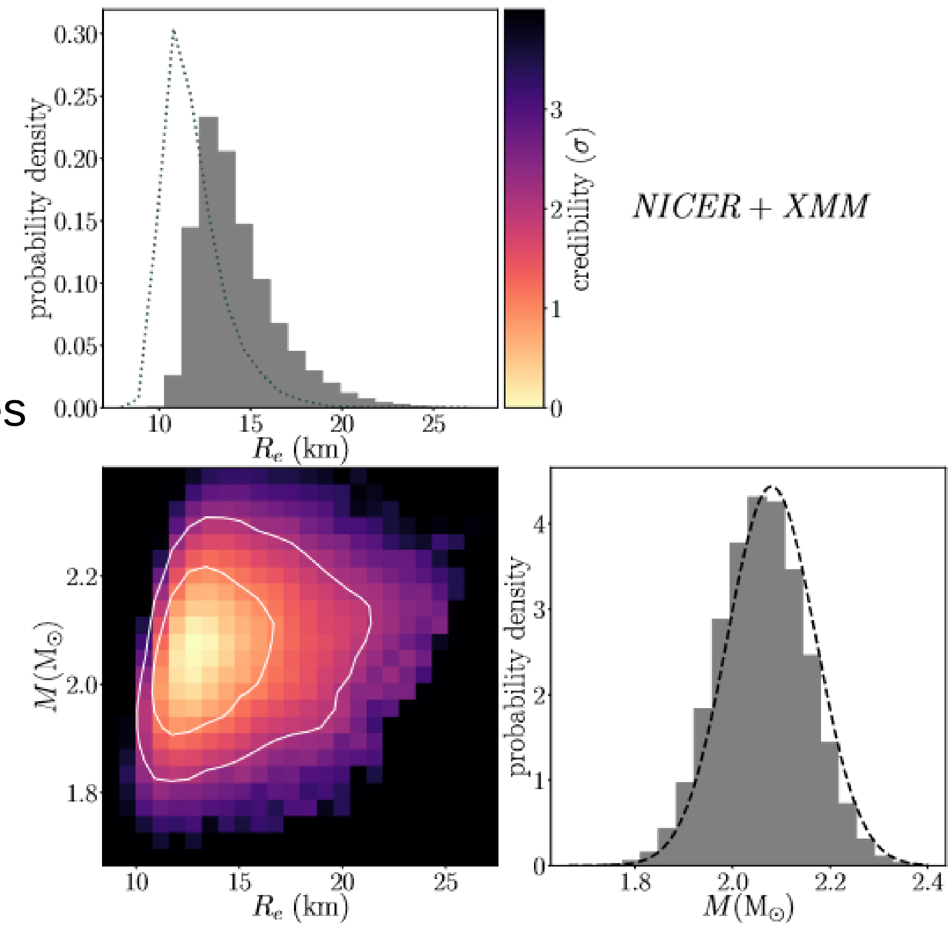
$$M = 1.44^{+0.15}_{-0.14} \ M_\odot$$

1912.05705

# Results from NICER. PSR J0740+6620



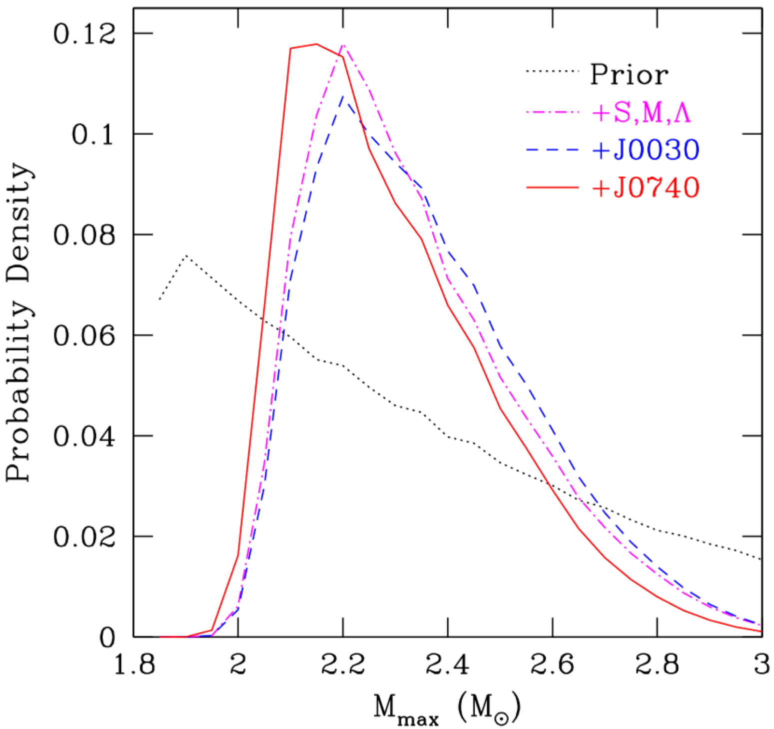
Without XMM data the radius is smaller.

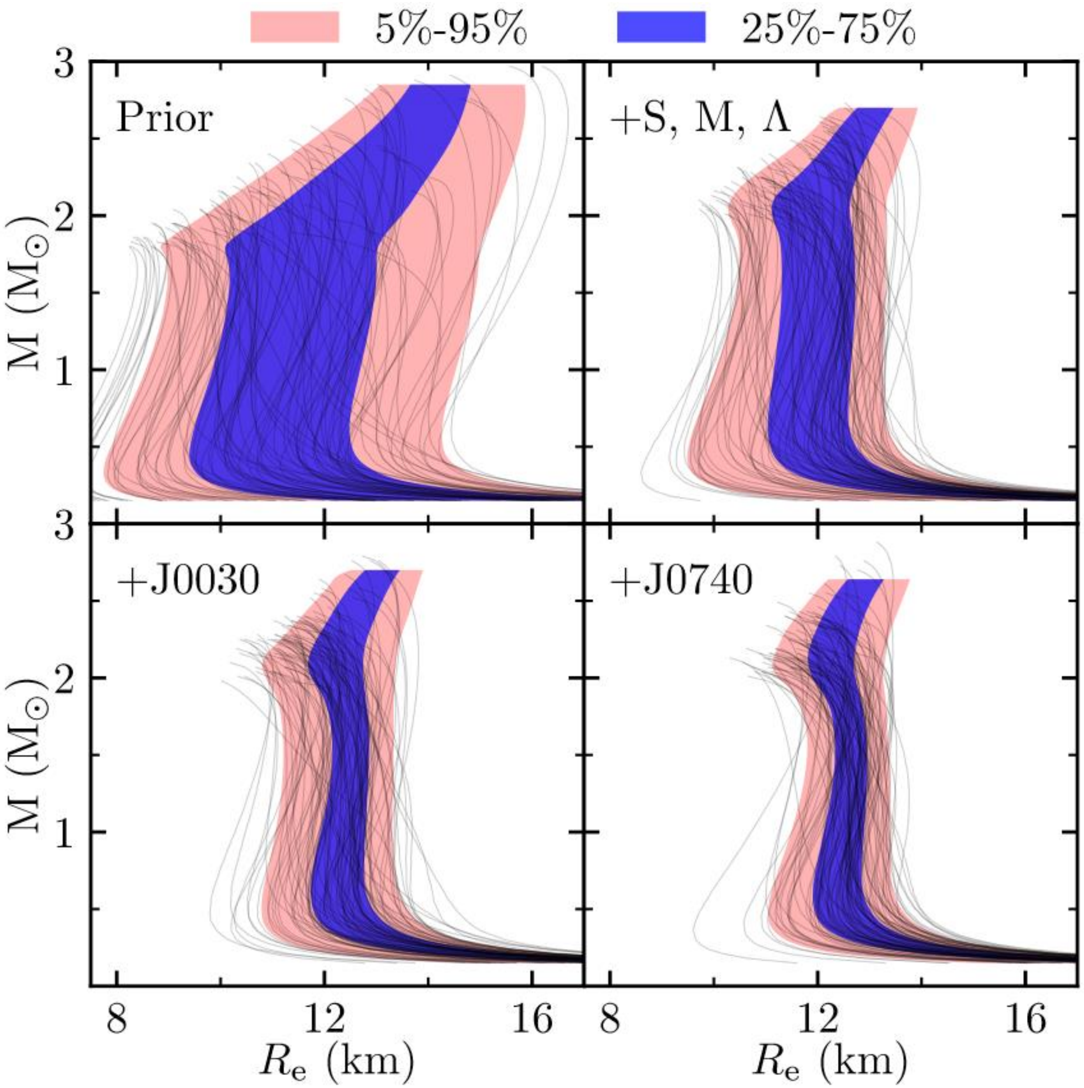



Model: two circular spots,
pure hydrogen model atmospheres
that allow for the possibility
of partial ionization.

2105.06979

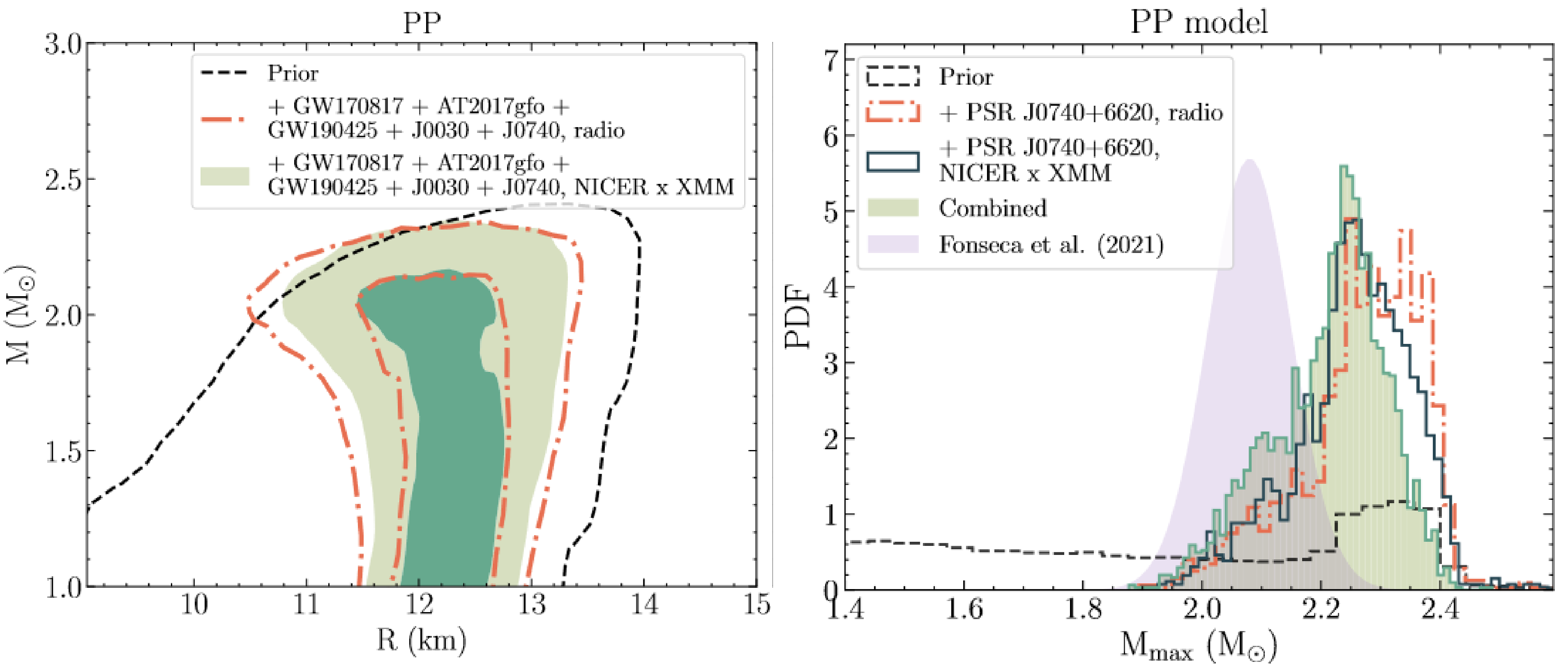
# PSR J0740+6620 and EoS




2105.06979

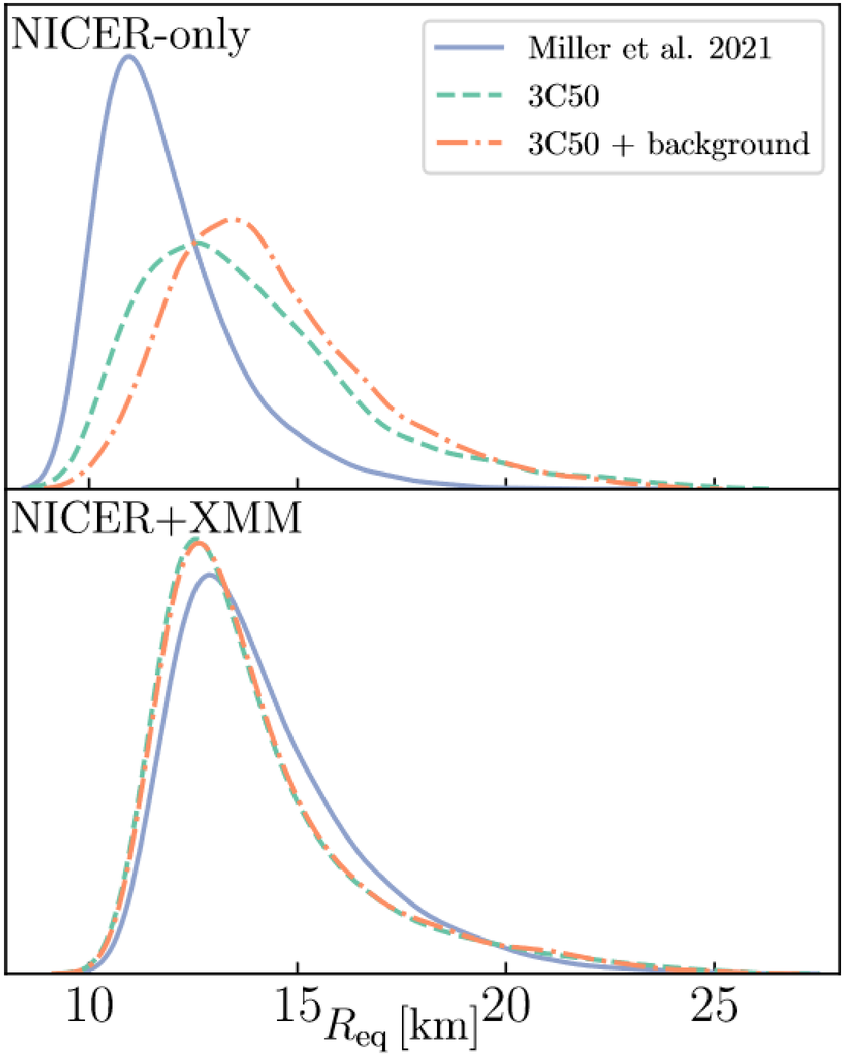
# Altogether now

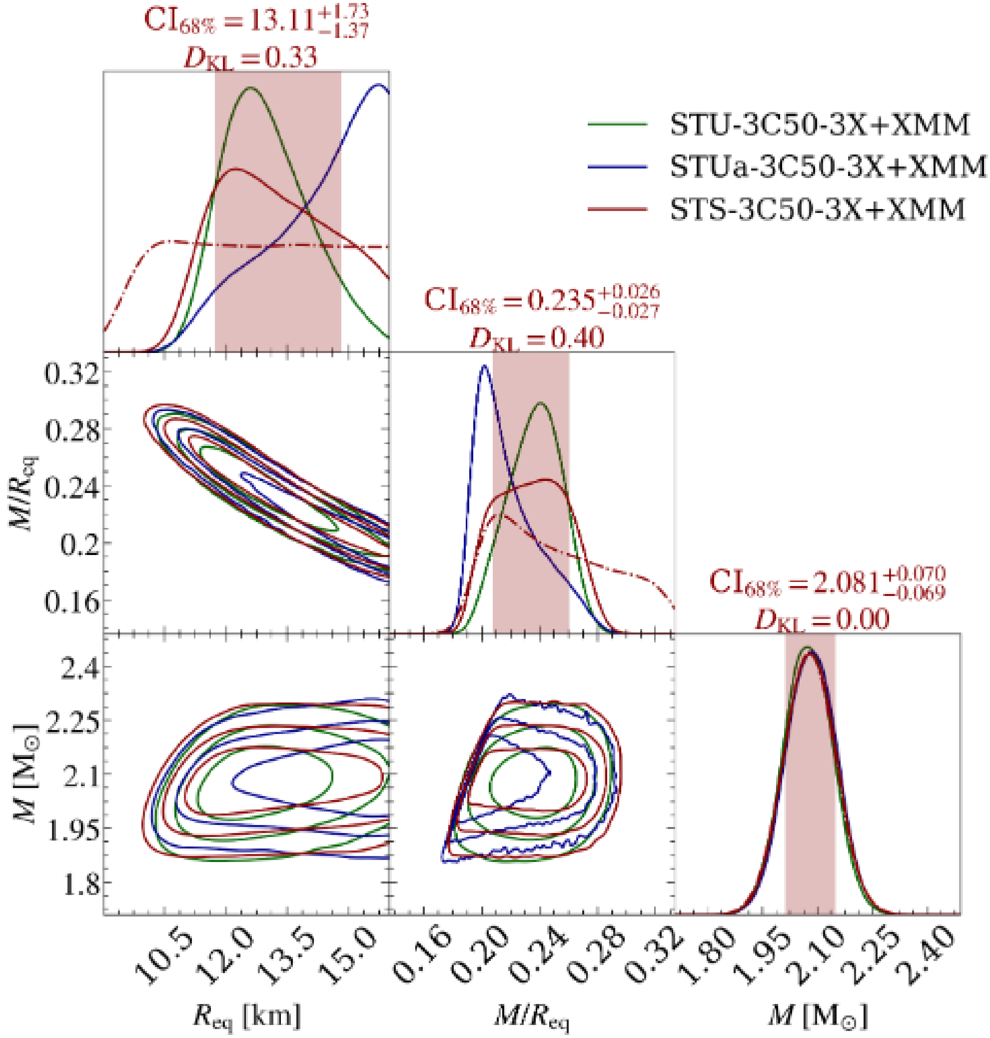



Joint constraints based on NICER, GW, etc.

2105.06981

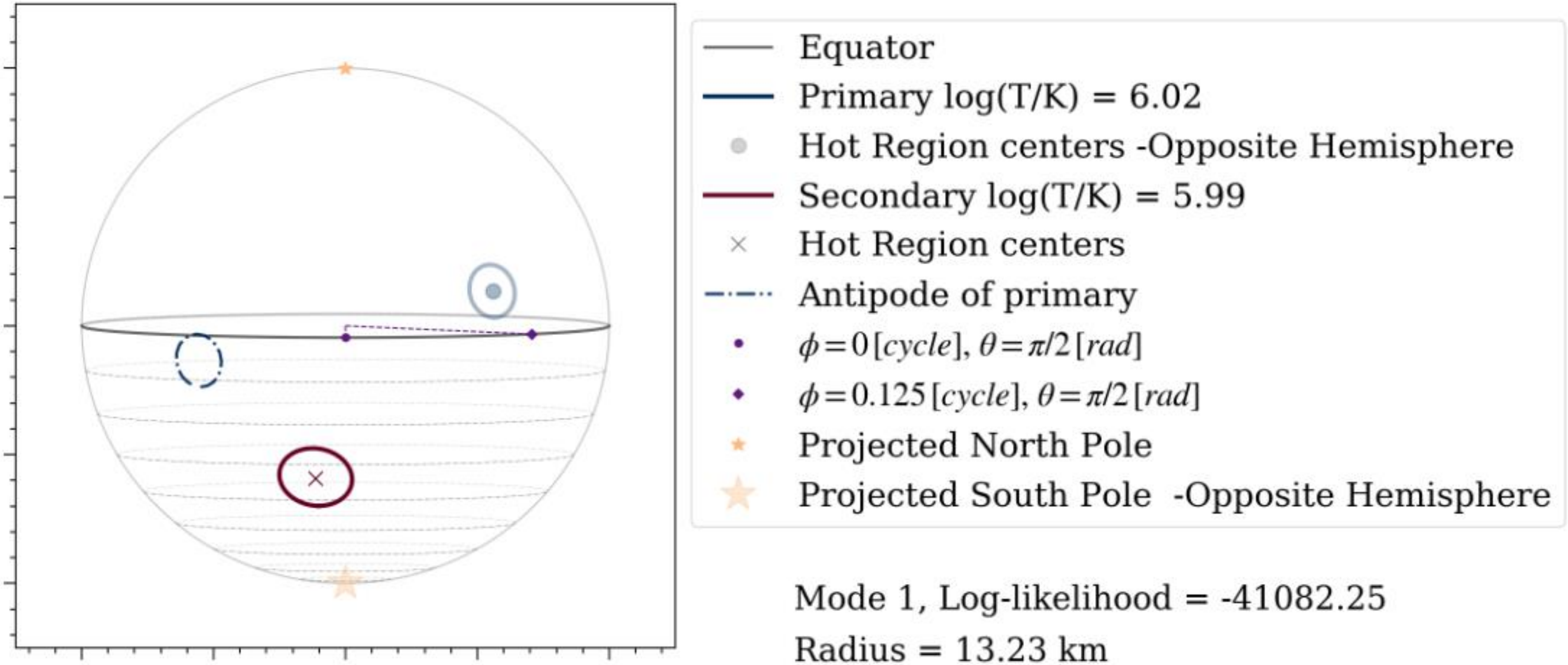
# New analysis for PSR J0740+6620



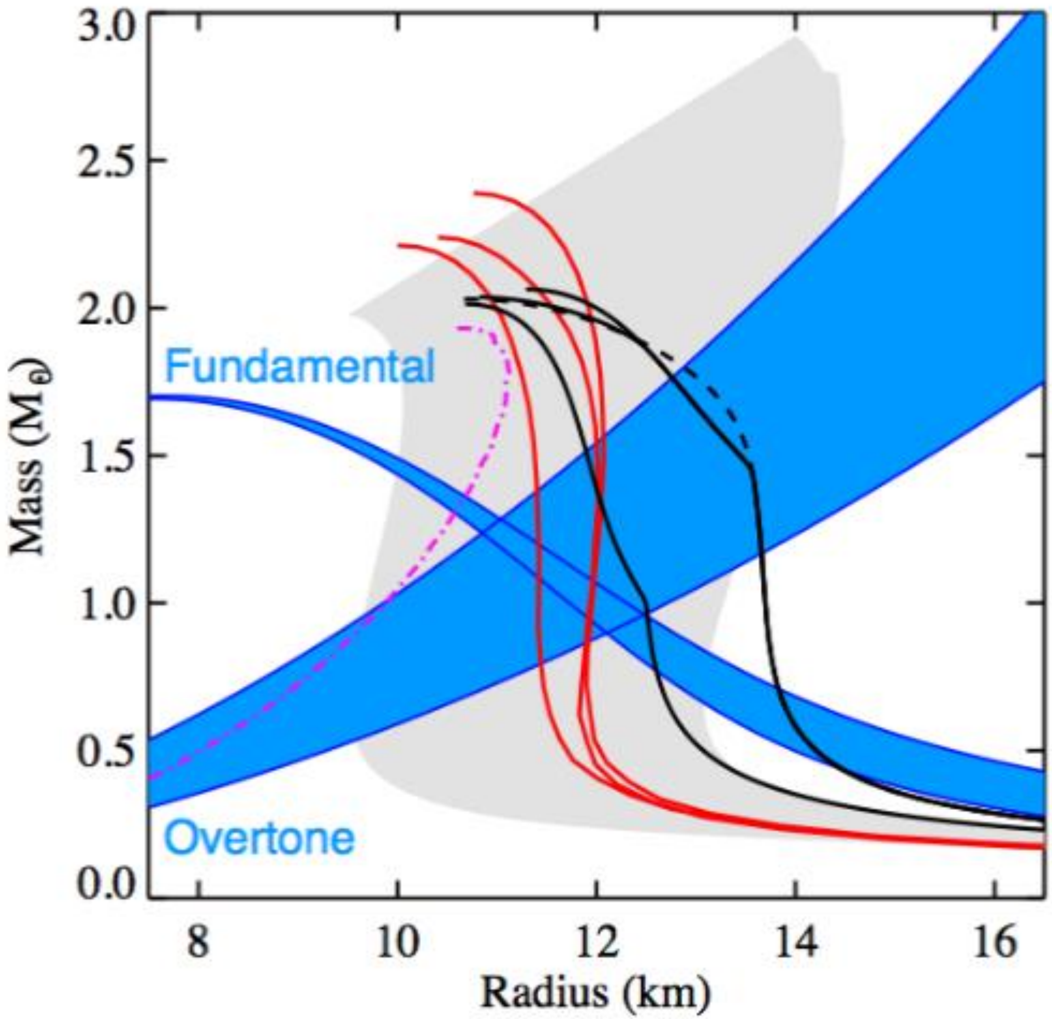

2209.12840

# Geometry of hot regions: non-centered and (non-)dipole




2209.12840

# Astroseismology



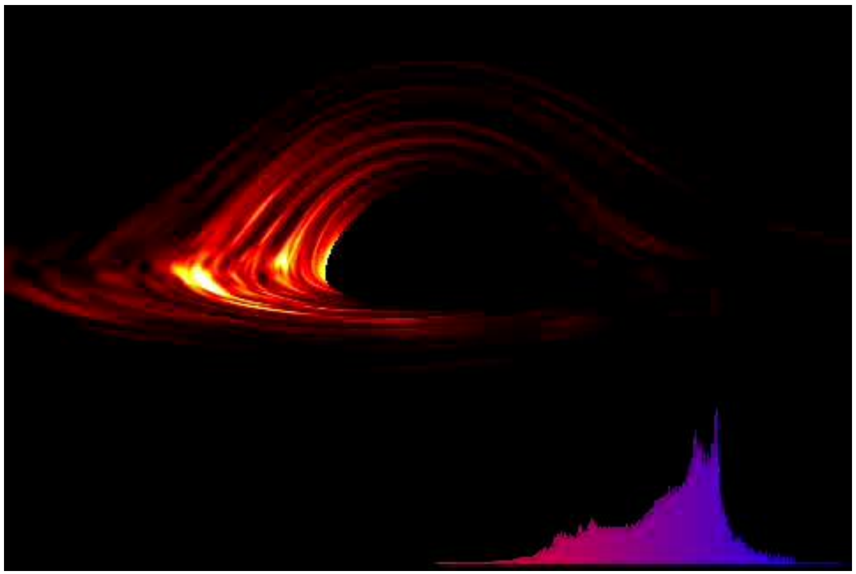

M – R diagram showing the seismological constraints for the soft gamma-ray repeater SGR 1806–20 using the relativistic torsional crust oscillation model of Samuelsson and Andersson (2007), in which the 29 Hz QPO is identified as the fundamental and the 625 Hz QPO as the first radial overtone. The neutron star lies in the box where the constraints from the two frequency bands overlap.

This is a simplified model.

1602.01081

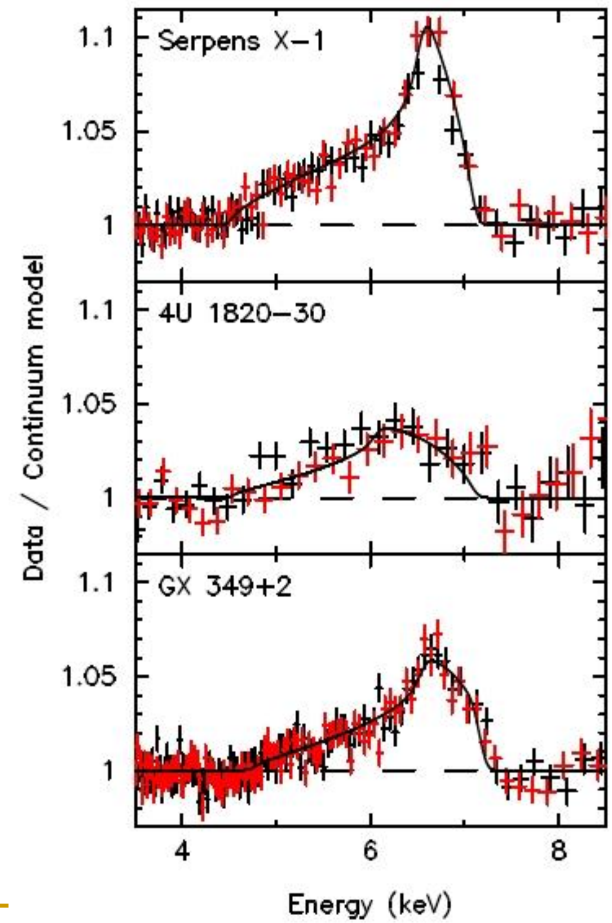
# Fe K lines from accretion discs

Measurements of the inner disc radius provide upper limits on the NS radius.



Ser X-1 <15.9+/-1
4U 1820-30 <13.8+2.9-1.4
GX 349+2 <16.5+/-0.8
(all estimates for 1.4 solar mass NS)

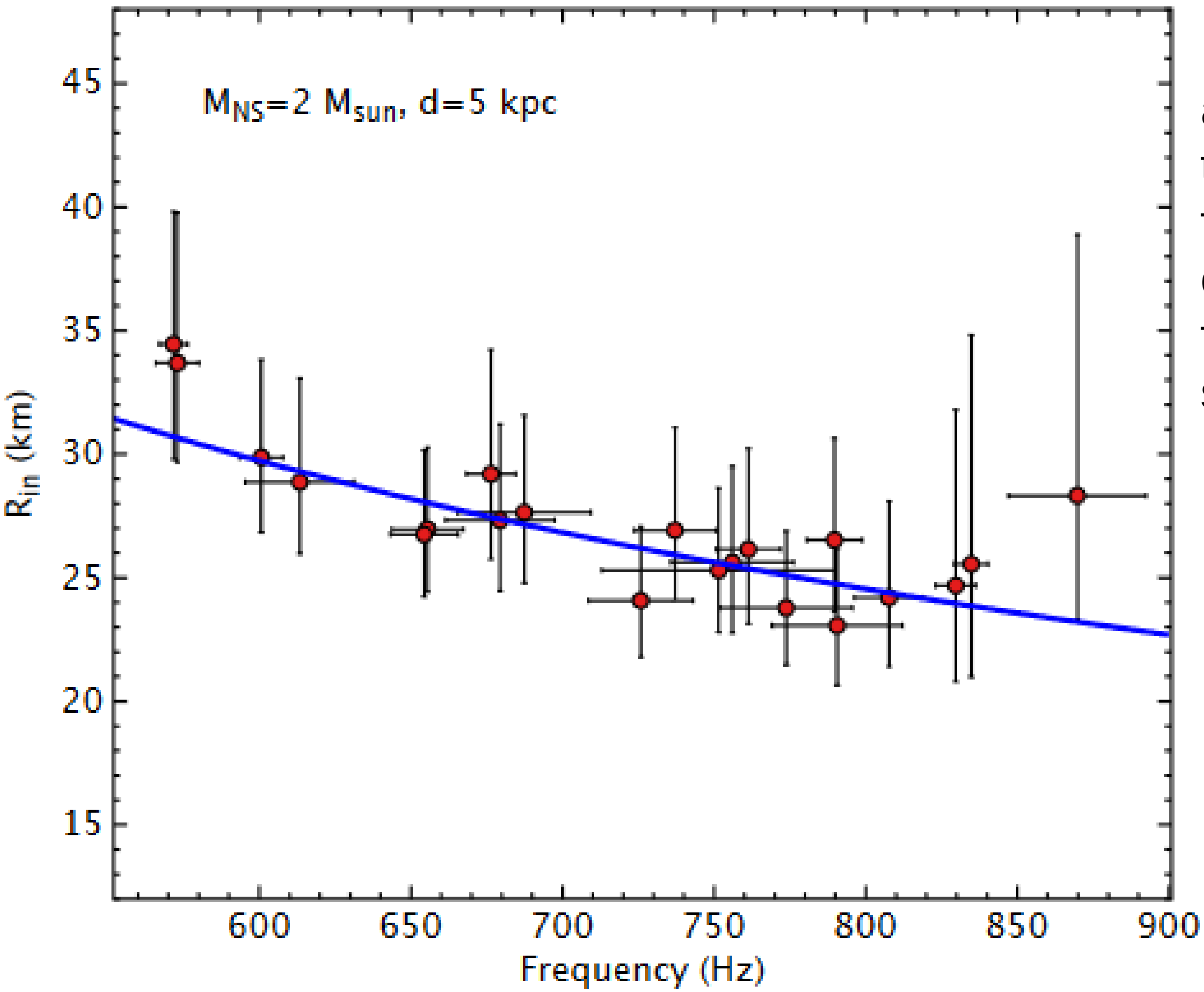
[Cackett et al. arXiv: 0708.3615]




Suzaku observations

**See also Papito et al. arXiv: 0812.1149,**
**a review in Cackett et al. 0908.1098, and theory in 1109.2068.**

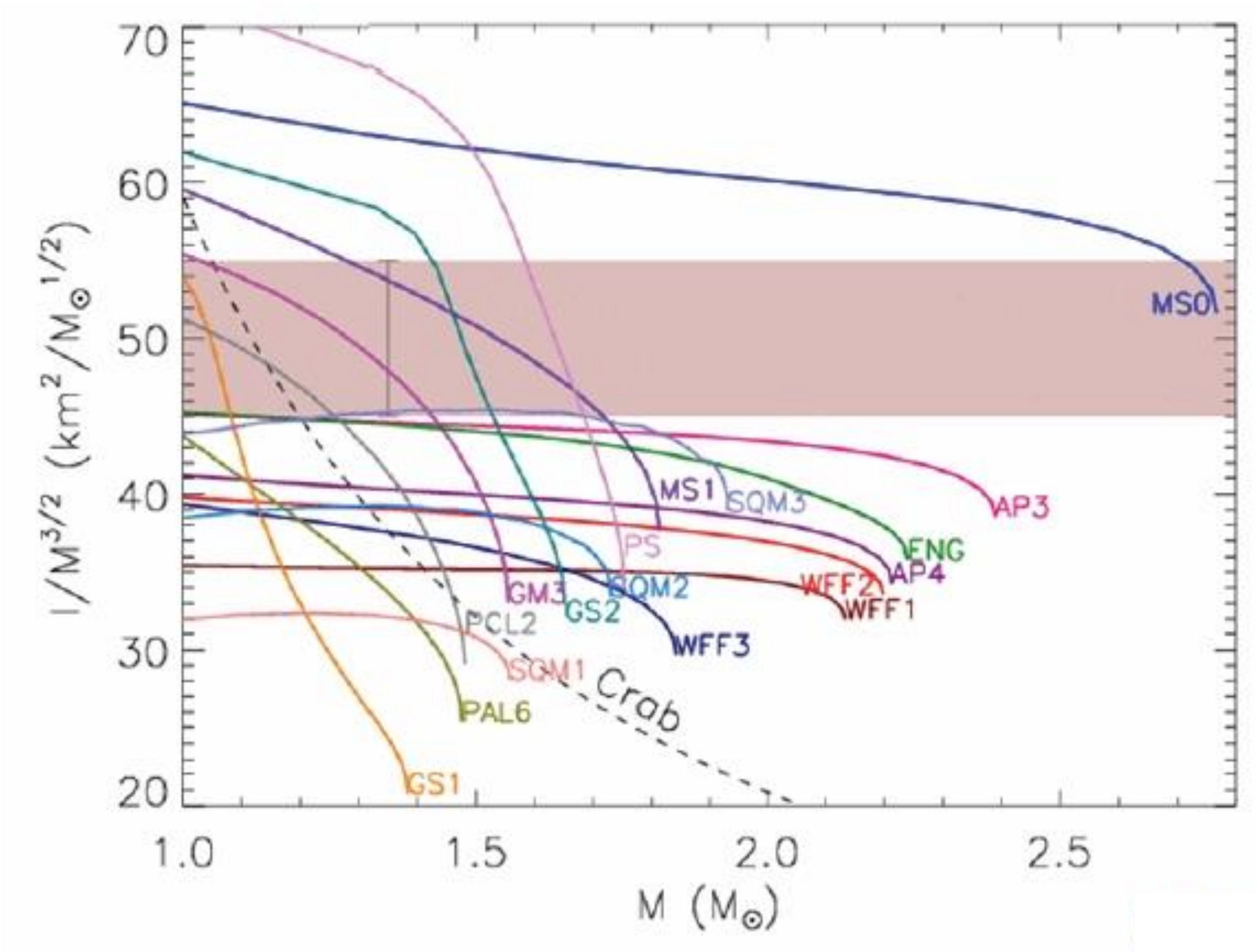
# Fits from QPOs




Inner radius of the accretion disc, from fits to the energy spectra, as a function of the frequency of the lower kHz QPO, from fits to the power spectra, in 4U 1608–52

2010.08291

# Limits on the moment of inertia




Spin-orbital interaction

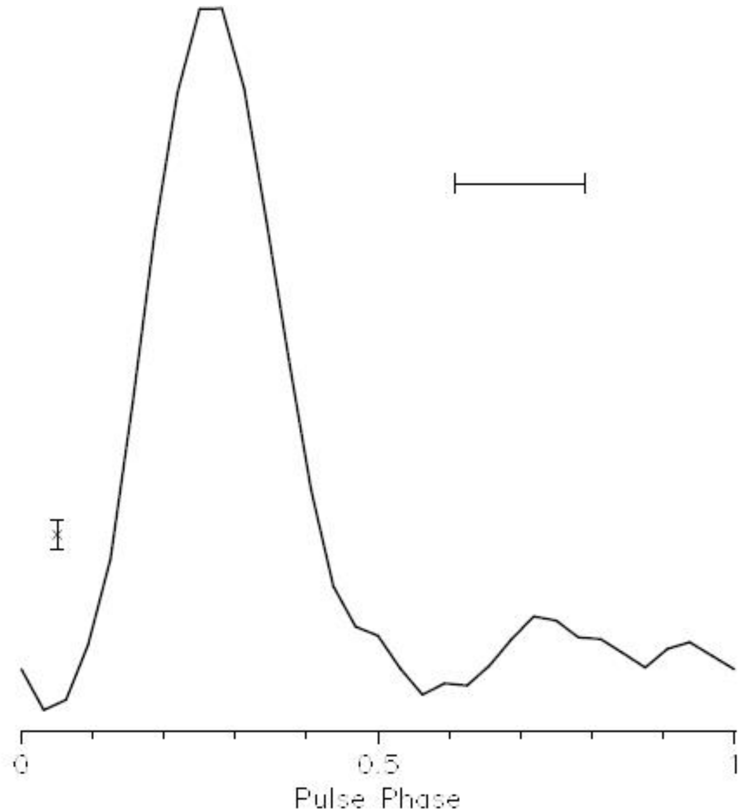
PSR J0737-3039
(see Lattimer, Schutz
astro-ph/0411470)

The band refers to a *hypothetical* 10% error. This limit, hopefully, can be reached in several years of observ.

See a more detailed discussion in 1006.3758

# Most rapidly rotating PSR

716-Hz eclipsing binary radio pulsar in the globular cluster Terzan 5

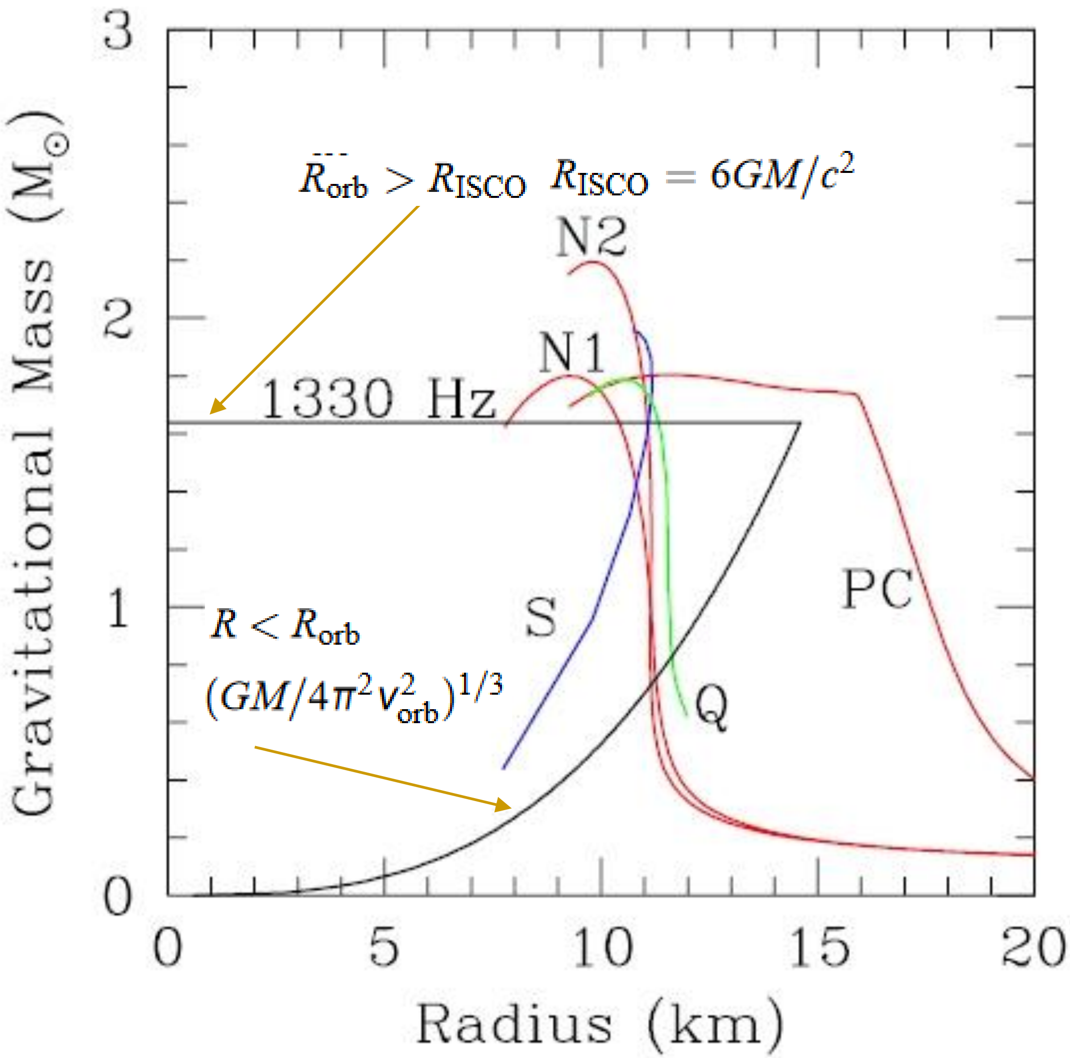



Previous record
(642-Hz pulsar B1937+21)
survived for more than 20 years.

Rotation starts to be important
from periods ~3 msec.

Jason W.T. Hessels et al. astro-ph/0601337

# QPO and rapid rotation



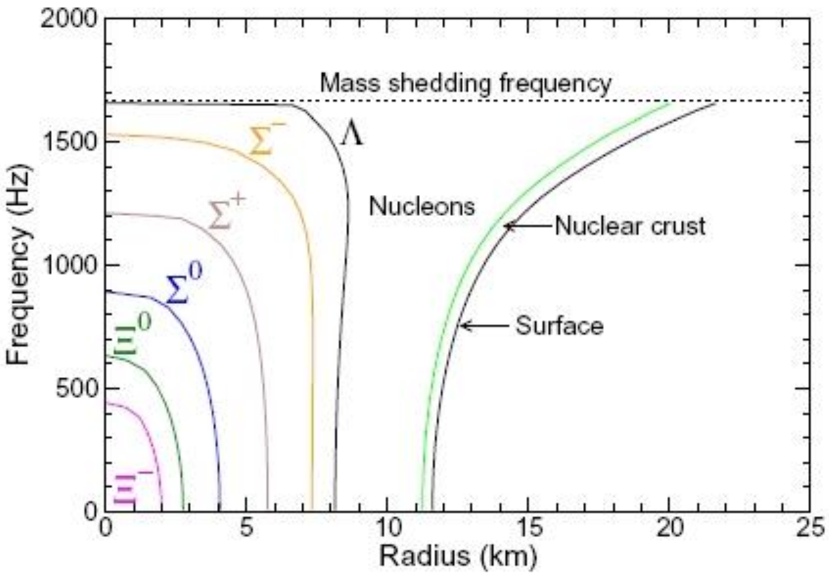

XTE J1739-285
1122 Hz
P. Kaaret et al.
astro-ph/0611716

1330 Hz – one of the highest QPO frequency

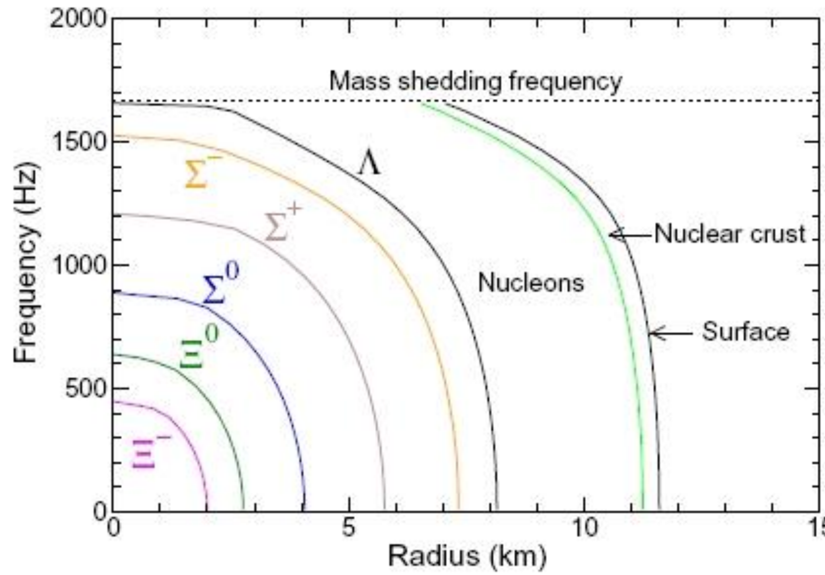
The line corresponds to the interpretation, that the frequency is that of the last stable orbit, $6GM/c^2$

Miller astro-ph/0312449

# Rotation and composition




(equatorial)



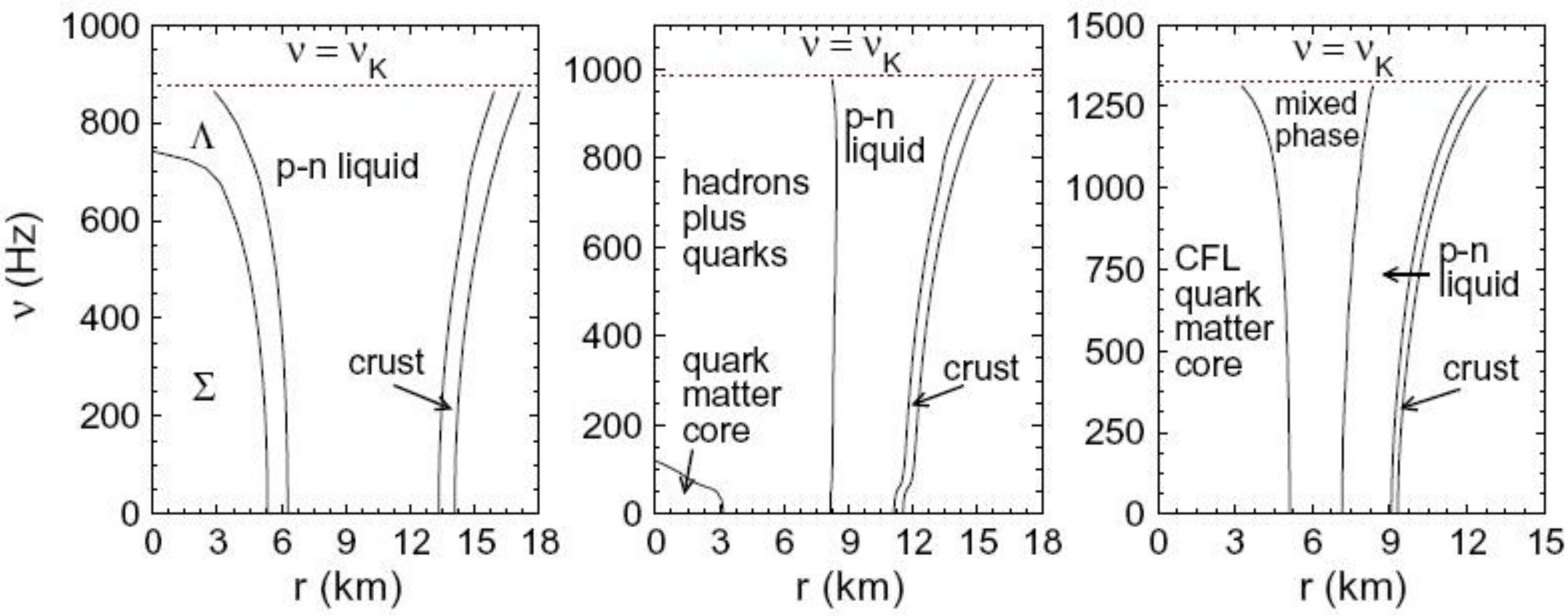

(polar)

Computed for a particular model:
density dependent relativistic Brueckner-Hartree-Fock (DD-RBHF)

(Weber et al. arXiv: 0705.2708)

Detailed study of the influence of rotation onto structure and composition is given in 1307.1103

# Rotation and composition




**hyperon** **quark-hybrid** **quark-hybrid (quarks in CFL)**

(Weber et al. arXiv: 0705.2708) 1.4 solar mass NS (when non-rotating)

# GW170817: deformability Λ

Many papers are published based on detection of GW signal from GW170817: 1803.00549, 1804.08583, 1805.09371, 1805.11579, 1805.11581, 1901.04138.

$$\tilde{\Lambda} = \frac{16}{13}\frac{(12q+1)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5},$$

$$q = m_2/m_1 \leq 1$$

$$\Lambda_{1,2} = \frac{2}{3}k_2\left(\frac{R_{1,2}c^2}{Gm_{1,2}}\right)^5$$

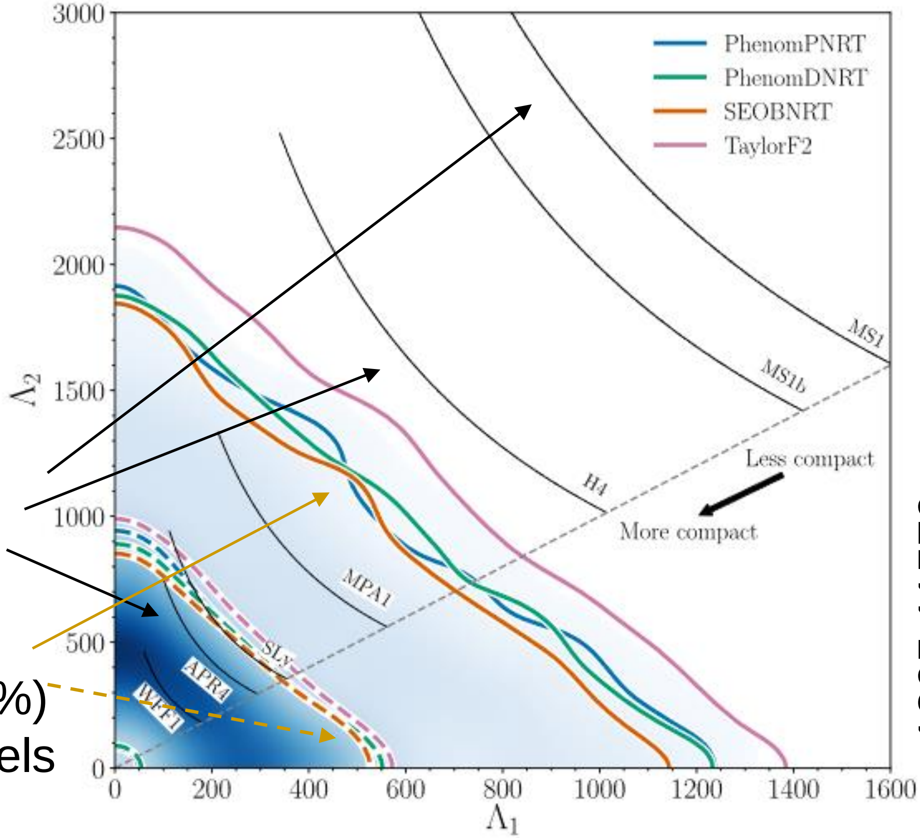
$$\beta = Gm/(Rc^2)$$

$k_2 \sim \beta^{-1}$
$\Lambda \sim \beta^{-6}$

Low spin priors

Solid – theoretical EoS

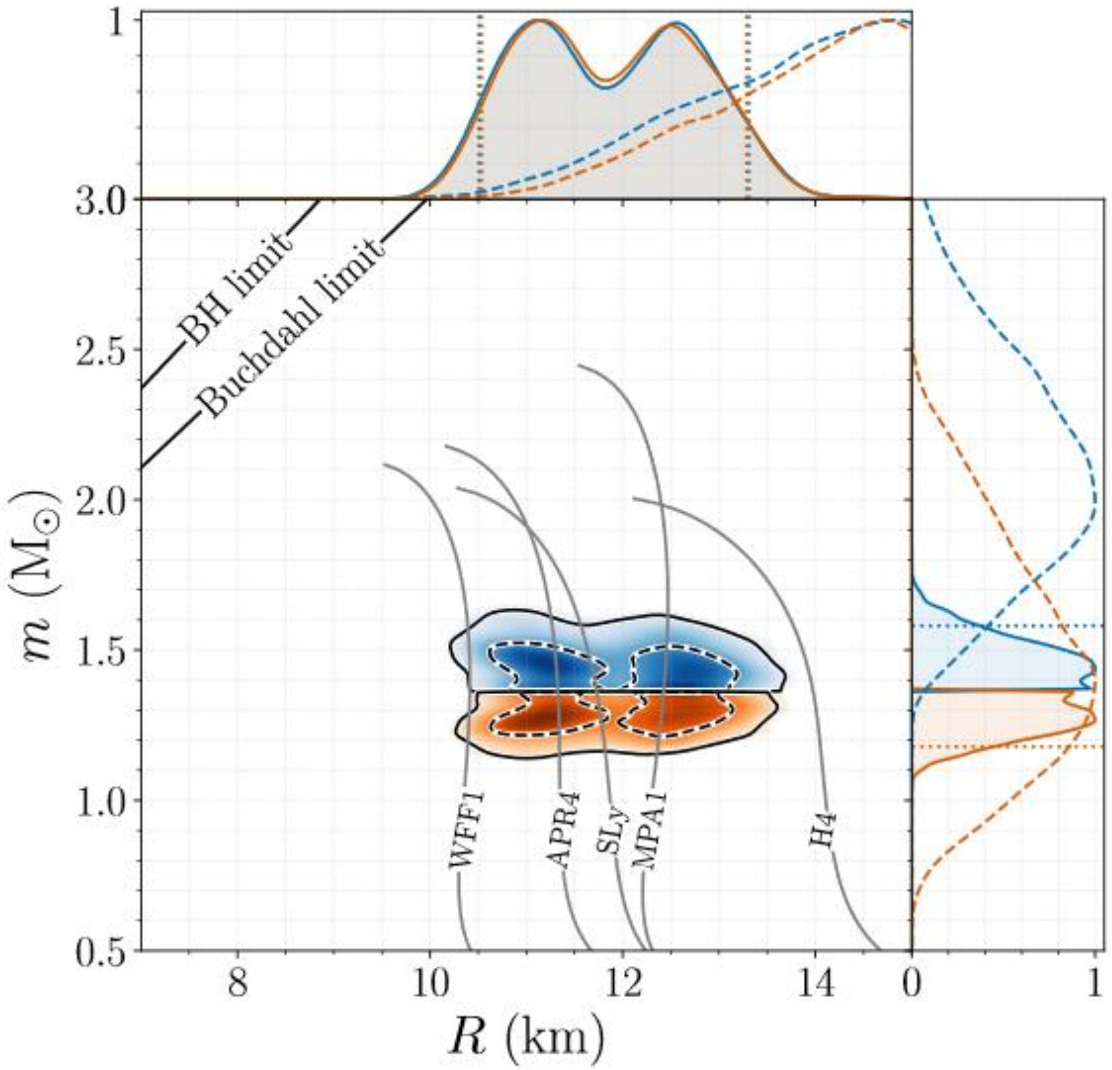
Colored – limits
(dashed 50%, solid 90%)
for four waveform models




1805.11579

Collapse to a BH after ~1 sec? (1901.04138)

# GW170817: M-R




$R_1 = 11.9^{+1.4}_{-1.4}\,\mathrm{km}$

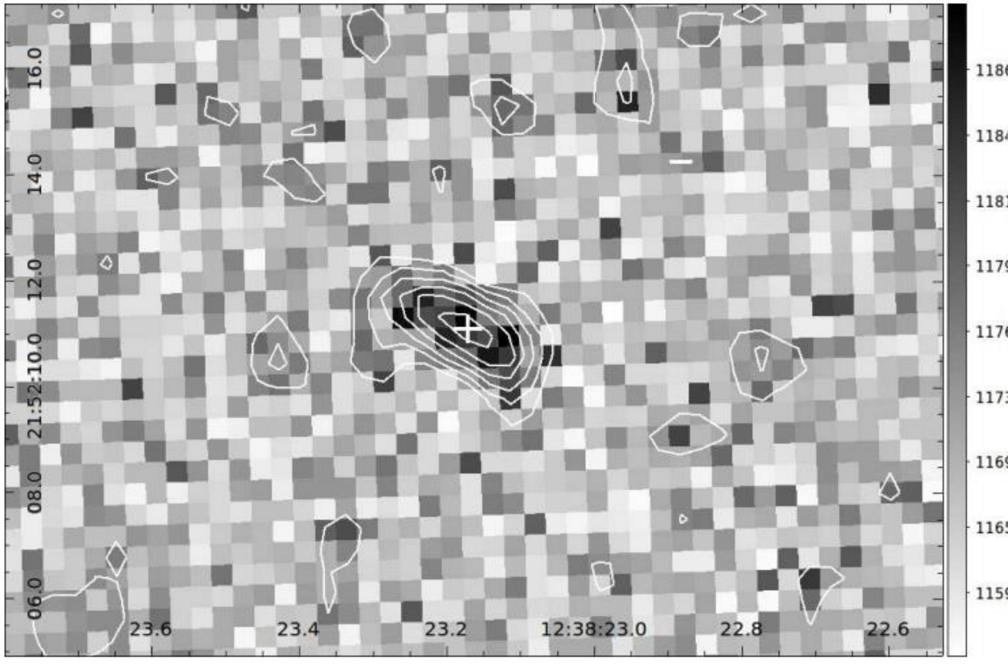
$R_2 = 11.9^{+1.4}_{-1.4}\,\mathrm{km}$

1805.11581

# Microlensing and weak lensing

In the future (maybe already with Gaia)
it can possible to determine NS mass with lensing.
Different techniques can be discussed:
photometric (normal) microlensing (1009.0005),
astrometric microlensing, weak lensing (1209.2249).

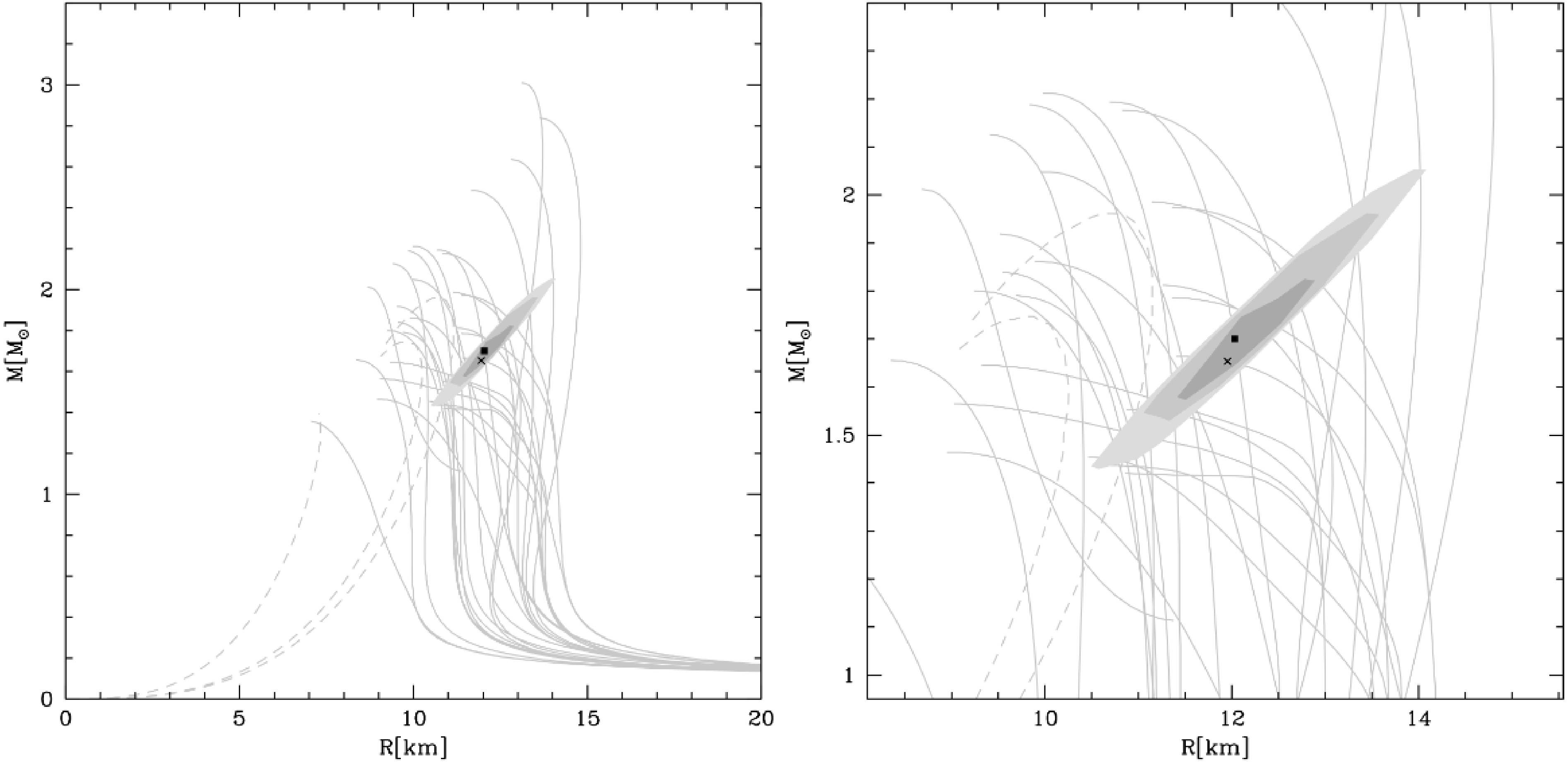
See recent studies
in 2107.13697, 2107.13701




1209.2249

# ATHENA

Using only spectra M and R can be determined within 3-10% and 2-8%, respectively.




1912.01608

# References

- Observational Constraints on Neutron Star Masses and Radii 1604.03894
  The review is about X-ray systems
- Mass, radii and equation of state of neutron stars 1603.02698
  The review about different kinds of measurements, including radio pulsars. Recent lists of mass measurements for different NSs.
- Measuring the neutron star equation of state using X-ray timing 1602.01081
  The review about EoS and X-ray measurements
- The masses and spins of neutron stars and stellar-mass black holes 1408.4145
  The review covers several topics. Good brief description of radio pulsar mass measurements.
- Properties of DNS systems. 1706.09438
  The review covers all aspects of observations, formation and evolution.
- Testing the equation of state of neutron stars with electromagnetic observations. 1806.02833 The BIG review describes observational tests of the EoS.
- Birth events, masses and the maximum mass of Compact Stars. 2011.08157 The review covers mass measurements and birth rates.