Statistical physics and thermodynamics: Alternative problems I.

1. Determine the density of states of free nonrelativistic particle with mass m in 1D.
2. Show that from a statistical definition of free energy $F = -kT \ln Z$ follows for entropy $S = -k \sum_n w_n \log w_n$.
3. From a partition function
$$Z = \frac{1}{N!} \left(\frac{mkT}{2\pi\hbar^3} \right)^{3/2N} V^N$$
determine c_p.
4. Using
$$\Omega = -\int_0^\infty \frac{\left(\int_0^E \rho(E')\, \mathrm{d}E' \right) \mathrm{d}E}{e^{\frac{E-\mu}{kT}} - 1}$$
determine the Landau potential of ultrarelativistic bosons in 3D.
5. Show that the fermionic function
$$F_n(y) = \frac{1}{\Gamma(n)} \int_0^\infty \frac{x^{n-1}}{e^{x-y}+1}\, \mathrm{d}x$$
can be written in terms of series as
$$F_n(y) = \sum_{j=1}^\infty (-1)^{j+1} \frac{e^{jy}}{j^n}.$$
Using this result, determine an approximate equation of state for nonrelativistic fermionic gas in 3D using first two members of series.
6. The Landau potential of bosonic gas at temperatures lower than the critical temperature T_c is
$$\Omega = -NkT \left(\frac{T}{T_\mathrm{c}} \right)^{3/2} \frac{\zeta(5/2)}{\zeta(3/2)}.$$
Determine entropy and heat capacity $C_{V,N}$.