Statistical physics and thermodynamics: Alternative problems II.

1. Consider a gas of relativistic bosons with rest mass m in 3D whose energy is given by $E=\sqrt{m^2c^4+p^2c^2}$.
   (a) Determine the density of states as a function of energy. What is the minimal energy of particles?
   (b) Calculate the integral of density of states and determine the grandcanonical potential. From the potential, calculate number of particles, entropy, and pressure.

2. The density matrix of 1D particle confined to the line of length L in the coordinate representation is
$$\rho=\frac{1}{L}\exp\left(-\frac{\pi(x-x')^2}{\lambda_T^2}\right),$$
where $\lambda_T=\sqrt{2\pi\hbar^2/mkT}$. Determine the mean value of the coordinate x of the particle.

3. Density matrices of polarized light in a plane tilded by π and $3\pi/4$ in a basis of vectors of linearly polarized light is
$$\hat{\rho}_{\pi/4}=\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}, \qquad \hat{\rho}_{3\pi/4}=\begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}. \tag{1}$$
Using $\hat{\rho}_{\pi/4}$ and $\hat{\rho}_{3\pi/4}$ determine the density matrix of unpolarized light $\hat{\rho}_{\mathrm{n}}$ and calculate $\hat{\rho}^2_{\pi/4}$, $\hat{\rho}^2_{3\pi/4}$ a $\hat{\rho}^2_{\mathrm{n}}$. Which matrices correspond to a pure state?

4. Determine entropy
$$S=\frac{k}{(2\pi\hbar)^3}\int f\ln\left(\frac{e}{f}\right)\mathrm{d}^3p\,\mathrm{d}^3q$$
of a gas described by Maxwellian distribution function
$$f=\exp\left(\frac{\mu-\varepsilon}{kT}\right),$$
where ε is the particle energy $\varepsilon=p^2/(2m)$.