Homework problems #5

1. *Computer problem*: a) Make a function that calculates numerically an integral

$$F_{3/2}(y) = \frac{1}{\Gamma(3/2)} \int_0^\infty \frac{x^{1/2}}{e^{x-y}+1}\,\mathrm{d}x.$$

   Calculate $F_{3/2}(0)$. b) Using an appropriate numerical metod find $\tilde{\mu}_x$ that fulfills

$$F_{3/2}(\tilde{\mu}_x) = x \equiv \frac{N\lambda_T^3}{gV}$$

   for a given x. Calcullate $\tilde{\mu}_{80}$, $\tilde{\mu}_{10}$ a $\tilde{\mu}_{0.1}$. c) Plot a graph of function

$$N(\varepsilon, \tilde{\mu}_x) = \frac{1}{e^{\varepsilon - \tilde{\mu}_x}+1}$$

   for determined values $\tilde{\mu}_{100}$, $\tilde{\mu}_1$, and $\tilde{\mu}_{0.01}$.

2. Evaluate the mean value of free particle hamiltonian from quantum mechanical partition function

$$Z = \frac{V}{\lambda_\mathrm{T}^3}. \tag{1}$$

   Calculate in momentum representation.

3. Density matrix of left-handed and right-handed polarized light in a basis of vectors of linearly polarized light is

$$\hat{\rho}_\mathrm{L} = \begin{pmatrix} 1/2 & i/2 \\ -i/2 & 1/2 \end{pmatrix}, \qquad \hat{\rho}_\mathrm{R} = \begin{pmatrix} 1/2 & -i/2 \\ i/2 & 1/2 \end{pmatrix}. \tag{2}$$

   Using $\hat{\rho}_\mathrm{L}$ and $\hat{\rho}_\mathrm{R}$ determine the density matrix of unpolarized light $\hat{\rho}_\mathrm{n}$ and calculate $\hat{\rho}_\mathrm{L}^2$, $\hat{\rho}_\mathrm{R}^2$ a $\hat{\rho}_\mathrm{n}^2$. Which matrices correspond to a pure state?

4. Density matrix of a harmonic oscillator with a hamiltonian

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2$$

   has in a position space form of

$$\rho(x,x',T) = \sqrt{\frac{m\omega}{\pi\hbar}\tanh\left(\frac{\hbar\omega}{2kT}\right)}\exp\left\{-\frac{m\omega}{2\hbar\sinh\left(\frac{\hbar\omega}{kT}\right)}\left[\left(x^2+x'^2\right)\cosh\left(\frac{\hbar\omega}{kT}\right)-2xx'\right]\right\}.$$

   (a) Calculate the mean value of energy $E = \langle\hat{H}\rangle$.

   (b) Show that for $T\to\infty$ the equipartition theorem holds for the mean value of energy.

5. Approximately calculate the heat capacity at the constant volume for a gas with interatomic potencial $U(r)$ (the unknownd integral can be denoted appropriatelly). Particles can be considered as point masses.

6. The approximate solution of the Boltzmann equation in a presence of temperature gradient $T = T_0 + \alpha y$ is

$$f = f_0 + \alpha\tau v_y \frac{T_0}{2(T_0-\alpha y)^{7/2}}\left(\frac{p^2}{mk(T_0-\alpha y)} - 5\right) n_0 \left(\frac{2\pi\hbar^2}{mk}\right)^{3/2} e^{-\frac{p^2}{2mk(T_0-\alpha y)}}, \tag{3}$$

   where f_0 is equilibrium velocity distribution. Calculate a mean value of momentum flux $\langle m v_y |v_y|\rangle$; in a velocity distribution you can approximate $T_0 - \alpha y \approx T_0$. The nonzero momentum flux causes, for example, the motion of a light mill.

The solution should be submitted not later than on May 26th.