Rozmery Cooprovho páru

$$\rho^2 = \frac{\int r^2 |\Psi(\vec{r})|^2 d^3\vec{r}}{\int |\Psi(\vec{r})|^2 d^3\vec{r}}$$

vlnová funkcia Cooprovho páru

$$\Psi(\vec{r}) = \frac{1}{\sqrt{\Omega}} \sum_{k} e^{i\vec{k}\cdot\vec{r}} g_{\vec{k}}$$

$\vec{r}$ - relatívna vzdialenosť elektrónov tvoriacich Cooprov pár
$g_{\vec{k}}$ - FT vlnovej funkcie relatívneho pohybu elektrónov

čitateľ

$$\int r^2 \frac{1}{\Omega} \left| \sum_{k} e^{i\vec{k}\cdot\vec{r}} g_{\vec{k}} \right|^2 d^3\vec{r} = \frac{1}{\Omega} \int r^2 \sum_{k} \sum_{k'} e^{i\vec{k}\cdot\vec{r}} e^{-i\vec{k}'\cdot\vec{r}} g_k g_{k'}^{+} d^3\vec{r} =$$

$$= \frac{1}{\Omega} \int \sum_{k} \sum_{k'} g_k g_{k'}^{+} r e^{i\vec{k}\cdot\vec{r}} r e^{-i\vec{k}'\cdot\vec{r}} d^3\vec{r} =$$

$$= \left| \begin{array}{ll} \text{využijeme toho, že platí} & \\ \nabla_k e^{i\vec{k}\cdot\vec{r}} = i r e^{i\vec{k}\cdot\vec{r}} & \nabla_{k'} e^{-i\vec{k}'\cdot\vec{r}} = -i r e^{i\vec{k}\cdot\vec{r}} \end{array} \right|$$

= využijeme prechod k integrálu a následne per partes

$$\sum_k g_k \nabla_k e^{i\vec{k}\cdot\vec{r}} \longrightarrow \frac{\Omega}{(2\pi)^3}\int_{-\infty}^{\infty} g_k \nabla_k e^{i\vec{k}\cdot\vec{r}} d^3\vec{k}$$

| per partes |

$$\int_{-\infty}^{\infty} g_k \nabla_k e^{i\vec{k}\cdot\vec{r}} d^3\vec{k} = \int_{-\infty}^{\infty} \nabla_k (g_k e^{i\vec{k}\cdot\vec{r}}) d^3\vec{k} - \int_{-\infty}^{\infty} e^{i\vec{k}\cdot\vec{r}} \nabla_k g_k d^3\vec{k}$$

využijeme Gaussovu vetu

$$\oiint_{S(V)} \vec{F}\cdot d\vec{S} = \iiint_V \nabla \vec{F} \, dV$$

$$= \frac{\Omega}{(2\pi)^3} \oiint_{S(V)} g_k e^{i\vec{k}\cdot\vec{r}} d\vec{S} - \frac{\Omega}{(2\pi)^3}\int_{-\infty}^{\infty} e^{i\vec{k}\cdot\vec{r}} \nabla_k g_k d^3\vec{k} =$$

naspäť k diskrétnomu stavu $\sum_k e^{i\vec{k}\cdot\vec{r}} \nabla_k g_k$

$$= \frac{1}{\Omega}\int \sum_k \sum_{k'} e^{i\vec{k}\cdot\vec{r}} \nabla_k g_k e^{-i\vec{k}'\vec{r}} \nabla_{k'} g_{k'}^{+} d^3\vec{r} =$$

| zameníme integrál a sumu |

$$= \frac{1}{\Omega}\sum_k \sum_{k'} \overbrace{\left(\int e^{i\vec{r}(\vec{k}-\vec{k}')} d^3\vec{r}\right)}^{\Omega\,\delta_{kk'}} \nabla_k g_k \nabla_{k'} g_{k}^{+} =$$

$$= \sum_k \nabla_k g_k \nabla_k g_k^{+} = \underline{\sum_k |\nabla_k g_k|^2}$$

menovatel:

$$\int |\Psi(\vec r)|^2 d^3 r = \frac{1}{\Omega} \int \Big| \sum_{k} e^{i\vec k \cdot \vec r} g_{\vec k} \Big| d^3 \vec r =$$

$$= \frac{1}{\Omega} \int \sum_{k} \sum_{k'} e^{i\vec k \cdot \vec r} g_k \, e^{-i\vec k' \vec r} g_{k'}^{+} \, d^3 \vec r =$$

$$= \frac{1}{\Omega} \sum_{k} \sum_{k'} \underbrace{\int e^{i\vec r(\vec k - \vec k')} d^3 \vec r}_{\Omega \delta_{kk'}} \; g_{\vec k} \, g_{k'}^{+} =$$

$$= \sum_{k} g_k \, g_k^{+} = \underline{\sum_{k} |g_k|^2}$$

$$\underline{\rho^2 = \frac{\sum_{k} |\nabla_k g_k|^2}{\sum_{k} |g_k|^2}}$$

z predn[illegible]

$$g_k = \frac{C}{2\varepsilon - 2E_f - E_b}$$

E_b = binding energy

$$\frac{\hbar \omega_D}{\exp\left(\frac{2}{V D(\varepsilon)} - 1\right)}$$

$$\rho^2 = \frac{c^2 \sum_k \left| v_k \frac{1}{2\varepsilon - 2E_F - E_b} \right|^2}{c^2 \sum_k \left| \frac{1}{2\varepsilon - 2E_F - E_b} \right|^2} =$$

využijeme výraz pre rýchlosť + integrujeme s hustotou stavov

$$\left| \begin{aligned} v_k &= \frac{\partial}{\partial k} = \frac{\partial}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial k} = \frac{\partial}{\partial \varepsilon}\frac{\partial}{\partial k}\left(\frac{\hbar^2 k^2}{2m}\right) = \\ &= \frac{\partial}{\partial \varepsilon}\frac{\hbar^2 k}{m} = \hbar \frac{\partial}{\partial \varepsilon}\frac{\hbar k}{m} = \hbar \frac{\partial}{\partial \varepsilon} v(\varepsilon) \end{aligned} \right|$$

$$= \frac{D(E_F)\,\hbar^2 v_F^2 \int_{E_F}^{E_F+\hbar\omega_D} \left[\frac{\partial}{\partial \varepsilon}\left(\frac{1}{2\varepsilon - 2E_F - E_b}\right) \right]^2 d\varepsilon}{D(E_F) \int_{E_F}^{E_F+\hbar\omega_D} \left(\frac{1}{2\varepsilon - 2E_F - E_b}\right)^2 d\varepsilon}$$

$$\left| \begin{aligned} \omega &= \varepsilon - E_F \\ d\omega &= d\varepsilon \end{aligned} \right| =$$

$$= \frac{(\hbar v_F)^2 \int_0^{\hbar\omega_D} \left(\frac{\partial}{\partial \omega}\left(\frac{1}{2\omega - E_b}\right) \right)^2 d\omega}{\int_0^{\hbar\omega_D} \frac{1}{(2\omega - E_b)^2}\, d\omega} =$$

$$= \frac{4(\hbar v_F)^2 \int_0^{\hbar\omega_D} \left(\frac{1}{2u - E_b}\right)^4 du}{\int_0^{\hbar\omega_D} \left(\frac{1}{2u - E_b}\right)^2 du} =$$

$$= \frac{4(\hbar v_F)^2 \int_{-E_b}^{2\hbar\omega_D - E_b} v^{-4}\, dv}{\int_{-E_b}^{2\hbar\omega_D - E_b} v^{-2}\, dv} =$$

$$= \frac{4(\hbar v_F)^2 \int_{-E_b}^{\infty} v^{-4}\, dv}{\int_{-E_b}^{\infty} v^{-2}\, dv} = \frac{4}{3}\frac{(\hbar v_F)^2}{E_b^2}$$

odhad velikosti

$v_F \approx 10^6\ m \cdot s^{-1}$ $\qquad \rho \approx 10^{-6}\ m = \mu m$

$E_b \sim 1\ meV$