① $\mathbb{R}^4$ přímka $q: [3,0,4,0] + s(2,0,1,0)$

rovina $\rho: x_1 + x_2 = 3, \quad x_3 + x_4 = 7$

Najdi přímku p se směr. vektorem

$v = (0,1,0,2)$

p protíná q i ρ.

$p \cap q$ neprázdný, přímky p a q určují rovinu

$\alpha = p \sqcup q$.

$$\alpha : [3,0,4,0] + s(2,0,1,0) + t\overset{v}{(0,1,0,2)}$$

$p \cap \rho$ neprázdný, $p \subseteq \alpha$

$\emptyset \neq p \cap \rho \subseteq \alpha \cap \rho$ $\qquad$ $\alpha \cap \rho$ neprázdný, spočítáme ho:

$$\alpha : \begin{aligned} x_1 &= 3 + 2s \\ x_2 &= t \\ x_3 &= 4 + s \\ x_4 &= 2t \end{aligned}$$

$\rho: \quad x_1 + x_2 = 3 \qquad x_3 + x_4 = 7$

$3 + 2s + t = 3 \qquad 4 + s + 2t = 7$

$2s + t = 0 \qquad s + 2t = 3$

$$\left(\begin{array}{cc|c} 2 & 1 & 0 \\ 1 & 2 & 3 \end{array}\right) \sim \left(\begin{array}{cc|c} s & t & \\ 1 & 2 & 3 \\ 0 & -3 & -6 \end{array}\right)$$

$t = 2, \quad s = -1$

$$\underline{\alpha \cap \rho} = \underline{[3,0,4,0] + (-1)(2,0,1,0)} + 2(0,1,0,2)$$

$$= \underline{[1,2,3,4]}$$

$p : [1,2,3,4] + a(0,1,0,2)$

$p \cap q = [3,0,4,0] + (-1)(2,0,1,0) = [1,0,3,0]$

② Vzdálenost přímek p, q v $\mathbb{R}^4$

$p : \overset{A}{[2,-1,6,5]} + a\overset{u}{(0,3,-1,0)}$

$q : \overset{B}{[2,3,0,3]} + b\overset{v}{(1,1,-1,0)}$

a body, kde se realizuje $P \in p$, $Q \in q$.

$=$

$\mathbb{R}^4 : \dim Z(p) + Z(q) = 2$, ortog. dopl. má dim 2

Nejdřív P, Q

$$P - Q \perp Z(p) + Z(q)$$

$$P - Q \perp u, v$$

$\langle A + au - B - bv, u \rangle = 0$ $\quad u = (0,3,-1,0)$

$, v \rangle = 0$ $\quad v = (1,1,-1,0)$

$B - A = (0,4,-6,-2)$

$$a\langle u,u\rangle - b\langle v,u\rangle = \langle B-A,u\rangle$$
$$a\langle u,v\rangle - b\langle v,v\rangle = \langle B-A,v\rangle$$

$$a\,10 - b\,4 = 18$$
$$a\,4 - b\,3 = 10$$

$$\left(\begin{array}{cc|c} 10 & -4 & 18 \\ 4 & -3 & 10 \end{array}\right) \sim \left(\begin{array}{cc|c} 5 & -2 & 9 \\ 4 & -3 & 10 \end{array}\right)$$

$10-36$

$$\sim \left(\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & -7 & 14 \end{array}\right) \qquad b=-2 \quad a=1$$

$$P = A + 1\cdot u = [2,2,5,5]$$
$$Q = B - 2v = [0,1,2,3]$$

$$\mathrm{dist}(p,q) = \|P-Q\| = \|(2,1,3,2)\| = \sqrt{4+1+9+4} = \underline{\underline{\sqrt{18}}}$$

p přímka dim 1

q rovina dim 2 $\quad$ di$(Z(p)+Z(q)) = 3$

3 rovnice a 3 neznámých

$$\left(\begin{array}{ccc|c} * & * & * & * \\ 0 & * & * & * \\ 0 & 0 & * & * \end{array}\right)$$

Lepší $(Z(p)+Z(q))^{\perp}$

Projekce $P-Q$ do $(\quad)^{\perp}$

$P-Q$ = projekce$(P-Q)$ do $(\quad)^{\perp}$

③ Kvadr. forma $q : U \to \mathbb{R}$

báze $\alpha = (u_1, u_2)$ souřadnice x_1, x_2

$$q(u) = 8x_1^2 - 6x_1x_2 + 5x_2^2$$

$$f(u,v) = 8x_1z_1 - 3x_1z_2 - 3x_2z_1 + 5x_2z_2$$

$(id)_{\alpha,\beta} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$

$$\beta = (2u_1 + 3u_2, u_1 + 2u_2)$$

y_1, y_2 souřadnice v β

Najít vyjádření kvadr. formy v bázi β.

$$q(u) = \cdot\cdot\, y_1 \cdot\cdot\, y_2 \cdot\cdot$$

① Pomocí matice kvadr. formy v bázi α $\quad A = \begin{pmatrix} 8 & -3 \\ -3 & 5 \end{pmatrix}$

Chceme matici v bázi β

$$B$$

$$B = (id)_{\alpha,\beta}^T \cdot A \cdot (id)_{\alpha,\beta}$$

$$B = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 8 & -3 \\ -3 & 5 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 7 & 9 \\ 2 & 7 \end{pmatrix}} \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 41 & 25 \\ 25 & 16 \end{pmatrix} !$$

$$q(u) = 41y_1^2 + 50y_1y_2 + 16y_2^2$$

2. řešení $\alpha = (u_1, u_2)$

$$\beta = (2u_1+3u_2, u_1+2u_2)$$

$$\beta = \alpha \cdot (id)_{\alpha,\beta}$$

$$(2u_1+3u_2, u_1+2u_2) = (u_1, u_2)\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \quad !$$

$$L(u)_\alpha = (id)_{\alpha,\beta}(u)_\beta \quad !$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\underline{x_1} = 2y_1 + y_2$$
$$\underline{x_2} = 3y_1 + 2y_2$$

$$q(u) = 8x_1^2 - 6x_1x_2 + 5x_2^2$$
$$= 8(2y_1+y_2)^2 - 6(2y_1+y_2)(3y_1+2y_2)$$
$$+5(3y_1+2y_2)^2 =$$
$$= \underline{8(4}y_1^2 + 4y_1y_2 + y_2^2) - 6(6y_1^2 + 7y_1y_2$$
$$+ 2y_2^2) + 5(9y_1^2 + 12y_1y_2 + 4y_2^2$$

$$= \underbrace{(32-36+45)}_{41} y_1^2 + \underbrace{(32-42+60)}_{50} y_1 y_2$$

$$+ \underbrace{(8-12+20)}_{16} y_2^2$$

3. řešení – kreativita

$$\underset{\alpha}{\left(\begin{array}{cc|c} 8 & -3 & u_1 \\ -3 & 5 & u_2 \\ \hline u_1 & u_2 & \end{array}\right)} \overset{2.}{\rightsquigarrow} \underset{\beta}{\left(\begin{array}{cc|c} \multicolumn{2}{c|}{B} & 2u_1+3u_2 \\ & & u_1+2u_2 \\ \hline 2u_1+3u_2 & u_1+2u_2 & \end{array}\right)}$$

$$\sim \left(\begin{array}{cc|c} \overset{7}{16-9} & \overset{9}{-6+15} & 2u_1+3u_2 \\ \overset{2}{8-6} & \overset{7}{-3+10} & u_1+2u_2 \\ \hline u_1 & u_2 & \end{array}\right)$$

stejné sloupc. úpravy

$$\sim \left(\begin{array}{cc|c} \overset{41}{14+27} & \overset{25}{7+18} & 2u_1+3u_2 \\ \overset{25}{4+21} & \overset{16}{2+14} & u_1+2u_2 \\ \hline 2u_1+3u_2 & u_1+2u_2 & \end{array}\right)$$

$$q(u) = 41y_1^2 + 50y_1y_2 + 16y_2^2$$

④ $\varphi : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

$\alpha = (v_1, v_2, v_3)$ a vl. vektorů

k vl. číslům $1, -\frac{1}{2}, -3$

$$\varphi(v_1) = v_1$$
$$\varphi(v_2) = -\frac{1}{2} v_2$$
$$\varphi(v_3) = -3v_3$$

(1) $u = 20v_1 + 21v_2$

(2) $u = 5v_1 + 16v_2 + 4v_3$

| $\varphi^n(u) = ?$

Konverguje $\varphi^n(u)$ k nějakému vektoru.

$\mathbb{R}^3$ je norm. prostor.

$$u = a v_1 + b v_2 + c v_3$$
$$\varphi(u) = a\,\varphi(v_1) + b\,\varphi(v_2) + c\,\varphi(v_3)$$
$$= a\,1.v_1 + b\left(-\tfrac{1}{2}\right)v_2 + c\,(-3)v_3$$
$$\varphi^2(u) = a\cdot 1\,\varphi(v_1) + b\cdot\left(-\tfrac{1}{2}\right)\varphi(v_2) + c\,(-3)\,\varphi(v_3)$$
$$= a\cdot 1\cdot 1\,v_1 + b\left(-\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)v_2 + c(-3)(-3)\,v_3$$

$\boxed{1.b}$ $$\varphi^n(u) = a\cdot 1^n v_1 + b\left(-\frac{1}{2}\right)^n v_2 + c\left(-3\right)^n v_3$$

φ^n je lineární

$$\varphi^n(u) = a\,\varphi^n(v_1) + b\,\varphi^n(v_2) + c\,\varphi^n(v_3)$$

$$= a\cdot 1^n v_1 + b\left(-\frac{1}{2}\right)^n v_2 + c\left(-3\right)^n v_3$$

$$\begin{array}{ccc} \swarrow {\scriptstyle n\to\infty} & \swarrow & \downarrow \qquad \nearrow \infty \\ \underline{a v_1} & b\cdot 0\cdot v_2 = \vec{0} & \|(-3)^n v_3\| \end{array}$$

$c = 0$ $\varphi^n(u)$ konverguje k $a v_1 + \vec{0}$

$= a v_1$

$c \neq 0$ $c(-3)^n v_3$ nekonv., ostatní konvergují

tedy $\varphi^n(u)$ nekonverguje.