cvičenie 10 (12)

Newtonova metóda pre systémy (nelineárnych) rovníc.

$F: \mathbb{R}^n \to \mathbb{R}^n$, $F(x) = 0$ $\quad x = (x_1, \dots, x_n)$

$$F(x) = \begin{pmatrix} f_1(x_1, \dots, x_n) \\ \vdots \\ f_n(x_1, \dots, x_n) \end{pmatrix} \qquad x_k = ((x_1)_k, \dots, (x_n)_k)$$

$$F(x) \simeq F(x_k) + J_F(x_k)(x - x_k)$$

J_F je Jacobiho matica zobrazenia F

x_{k+1} je riešenie rovnice $F(x_k) + J_F(x_k)(x - x_k) = 0$

$$F(x_k) + J_F(x_k)(x_{k+1} - x_k) = 0$$

$$J_F(x_k)(x_{k+1} - x_k) = -F(x_k)$$

$$x_{k+1} - x_k = -J_F(x_k)^{-1} \cdot F(x_k)$$

$$x_{k+1} = x_k - J_F(x_k)^{-1} \cdot F(x_k)$$

je jednoduchšie pre výpočet

označme $\delta_k = x_{k+1} - x_k \Rightarrow x_{k+1} = x_k + \delta_k$

δ_k je riešením sústavy lin. rovníc $J_F(x_k) \cdot \delta_k = -F(x_k)$

príklad. Pomocou Newtonovej metódy nájdite riešenie v prvom kvadrante nasledujúceho systému rovníc:

$y = x^2$,

$x^2 + 4y^2 = 4$.

$$F(x,y) = \begin{pmatrix} y - x^2 \\ x^2 + 4y^2 - 4 \end{pmatrix} \qquad F(x,y) = 0$$

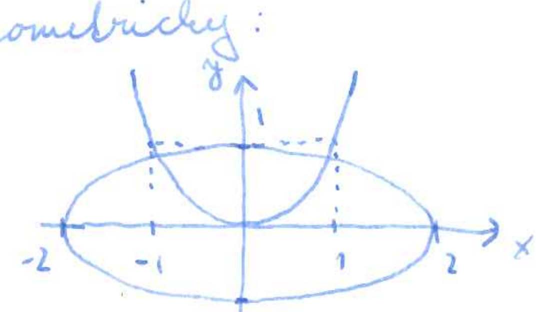
$$F(x,y) = \begin{pmatrix} f_1(x,y) \\ f_2(x,y) \end{pmatrix}$$

$$J_F(x,y) = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix} = \begin{pmatrix} -2x & 1 \\ 2x & 8y \end{pmatrix}$$

počiatočnú ~~po~~ aproximáciu určíme geometricky:

$y = x^2$ je parabola

$x^2 + 4y^2 = 4 \iff \left(\frac{x}{2}\right)^2 + y^2 = 1$ je elipsa



zvolíme $(x_0, y_0) = (1, 1)$

$$F(x_0,y_0) = F(1,1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad J_F(x_0,y_0) = J_F(1,1) = \begin{pmatrix} -2 & 1 \\ 2 & 8 \end{pmatrix}$$

$$(x_1,y_1) = (x_0,y_0) + \delta_0 \qquad J_F(x_0,y_0)\cdot\delta_0 = -F(x_0,y_0) \qquad \delta_0 = (\Delta x, \Delta y)$$

$$\left(\begin{array}{cc|c} -2 & 1 & 0 \\ 2 & 8 & -1 \end{array}\right) \sim \left(\begin{array}{cc|c} -2 & 1 & 0 \\ 0 & 9 & -1 \end{array}\right) \Rightarrow \begin{array}{l} \Delta y = -\frac{1}{9} \\ \Delta x = -\frac{1}{18} \end{array}$$

$$(x_1,y_1) = (1,1) + \left(-\frac{1}{18}, -\frac{1}{9}\right) = \left(\frac{17}{18}, \frac{8}{9}\right)$$

$$F\left(\frac{17}{18}, \frac{8}{9}\right) = \begin{pmatrix} \frac{8}{9} - \frac{289}{324} \\ \frac{289}{324} + 4\frac{64}{81} - 4 \end{pmatrix} = \begin{pmatrix} \frac{288-289}{324} \\ \frac{289+16\cdot 64 - 1296}{324} \end{pmatrix} = \begin{pmatrix} -\frac{1}{324} \\ +\frac{17}{324} \end{pmatrix}$$

$$J_F\left(\frac{17}{18}, \frac{8}{9}\right) = \begin{pmatrix} -\frac{17}{9} & 1 \\ \frac{17}{9} & \frac{64}{9} \end{pmatrix}$$

$$(x_2,y_2) = (x_1,y_1) + \delta_1$$
$$\delta_1 = (\Delta x, \Delta y)$$
$$J_F(x_1,y_1)\,\delta_1 = -F(x_1,y_1)$$

$$\left(\begin{array}{cc|c} -\frac{17}{9} & 1 & \frac{1}{324} \\ \frac{17}{9} & \frac{64}{9} & -\frac{17}{324} \end{array}\right) \sim \left(\begin{array}{cc|c} -17 & 9 & \frac{1}{36} \\ 17 & 64 & -\frac{17}{36} \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cc|c} -17 & 9 & \frac{1}{36} \\ 0 & 73 & -\frac{4}{9} \end{array}\right) \qquad \begin{array}{l} \Delta y = -\frac{4}{9\cdot 73} = -\frac{4}{657} \\ \Delta x = \dfrac{\frac{1}{36} + \frac{36}{657}}{-17} = \dfrac{\frac{73+144}{4\cdot 73\cdot 9}}{-17} = \dfrac{-217}{17\cdot 36\cdot 73} \end{array}$$

$$(x_2,y_2) = \left(\frac{17}{18}, \frac{8}{9}\right) + \delta_1 \approx \begin{pmatrix} 0{,}93958725 \\ 0{,}88280061 \end{pmatrix}$$

presné riešenie: $x \approx 0{,}9395649$

$y \approx 0{,}8827822$