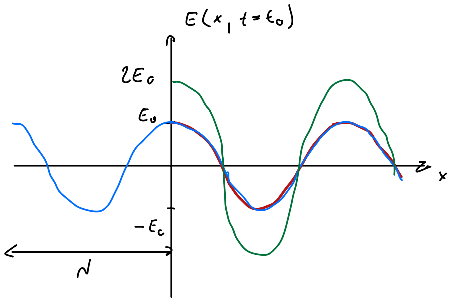
7. seminár (5.4.2024)

VLNOVÁ OPTIKA

Skladanie vĺn (interferencia):

$$E(x,t) = E_0 \cos(kx - \omega t + \varphi); \quad k = \frac{2\pi}{\lambda} \quad \omega = \frac{2\pi}{T} \quad \varphi \text{... počiatočná fáza}$$

Konštruktívna interferencia:

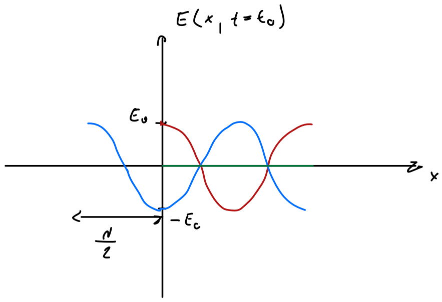


$$\left.\begin{aligned} E_1(x, t=t_0) &= E_0 \cos(kx - \omega t_0) \\ E_2(x, t=t_0) &= E_0 \cos(kx - \omega t_0 + 2\pi) \end{aligned}\right\} E_1 + E_2$$

$\Delta\varphi = 0, 2\pi, 4\pi, \dots \rightarrow$ posun $\Delta r = 0, \lambda, 2\lambda, \dots$

↳ zmena počiatočnej fázy E_2 voči E_1

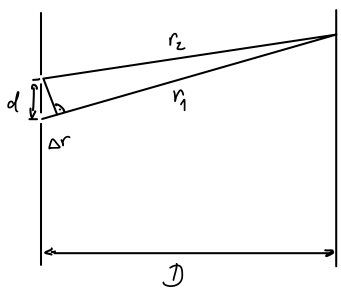
Deštruktívna interferencia:



$$\left.\begin{aligned} E_1(x, t=t_0) &= E_0 \cos(kx - \omega t_0) \\ E_2(x, t=t_0) &= E_0 \cos(kx - \omega t_0 + \pi) \end{aligned}\right\} E_1 + E_2$$

$\Delta\varphi = \pi, 3\pi, 5\pi \dots \rightarrow$ posun $\Delta r = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \dots$

Difrakcia na dvojštrbine

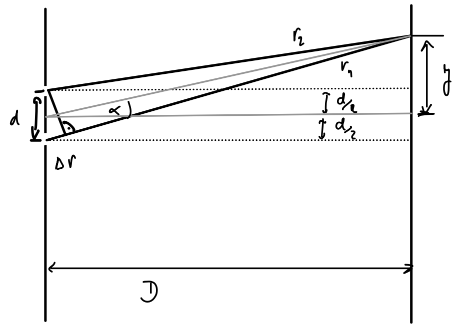


Δr ... dráhový rozdiel $\Delta r = r_1 - r_2$

podmienka maxima: $\boxed{\Delta r = m\lambda}$ $m = 0, 1, 2, \dots$

podmienka minima: $\boxed{\Delta r = (m - \frac{1}{2})\lambda}$ $m = 1, 2, 3, \dots$

24



$d = 0{,}1\,mm = 10^{-4}\,m$

$\lambda = 546\,nm = 5{,}46 \cdot 10^{-7}\,m$

$D = 2\,m$

Franhofferove aproximácia

$d \ll \sqrt{\lambda D} \Rightarrow 10^{-4} \ll \sqrt{2 \cdot 5{,}46 \cdot 10^{-7}}$

$10^{-4} \ll \sqrt{10^{-6}}$

$10^{-4} \ll 10^{-3}$ Ok!

$r_1 = \sqrt{D^2 + \left(y + \frac{d}{2}\right)^2} \qquad r_2 = \sqrt{D^2 + \left(y - \frac{d}{2}\right)^2}$

$$\Delta r = r_1 - r_2 = \sqrt{D^2 + \left(y + \frac{d}{2}\right)^2} - \sqrt{D^2 + \left(y - \frac{d}{2}\right)^2} = D\left[\sqrt{1 + \underbrace{\left(\frac{y + d/2}{D}\right)^2}_{\ll 1}} - \sqrt{1 + \underbrace{\left(\frac{y - d/2}{D}\right)^2}_{\ll 1}}\right]$$

- $y \ll D$ a $d \ll D \Rightarrow$ Taylorov rozvoj $\quad \sqrt{1+\varepsilon} \approx 1 + \frac{1}{2}\varepsilon$ ak $\varepsilon \ll 1$

$$\Delta r \approx D\left[1 + \frac{1}{2}\left(\frac{y}{D} + \frac{d}{2D}\right)^2 - 1 - \frac{1}{2}\left(\frac{y}{D} - \frac{d}{2D}\right)^2\right] = \frac{D}{2}\left[\frac{y^2}{D^2} + \frac{yd}{D^2} + \frac{d^2}{4D^2} - \frac{y^2}{D^2} + \frac{yd}{D^2} - \frac{d^2}{4D^2}\right]$$

$$= \frac{D}{2}\,\frac{2yd}{D^2} = \frac{yd}{D}$$

$$\boxed{\Delta r \approx \frac{yd}{D}}$$

a) $\alpha_{m1} = ?$

$y_{m1} = ?$

$\Delta r = (m + \frac{1}{2})\lambda \quad \rightarrow \quad (m - \frac{1}{2})\lambda = \frac{yd}{D} \Rightarrow y_m = \frac{D\lambda}{d}(m - \frac{1}{2})$

$\Delta r = \frac{yd}{D}$

$y_{m1} = \frac{D\lambda}{d}\left(1 - \frac{1}{2}\right)$

$y_{m1} = \frac{2 \cdot 5{,}46 \cdot 10^{-7}}{10^{-4}} \cdot \frac{1}{2} = 5{,}46 \cdot 10^{-3}\,m = 5{,}46\,mm$

$\operatorname{tg} \alpha_{m1} = \frac{y_{m1}}{D} = \frac{\lambda(m_1 - \frac{1}{2})}{d} = \frac{5{,}46 \cdot 10^{-7} \cdot \frac{1}{2}}{10^{-4}} = 2{,}73 \cdot 10^{-3} = 0{,}00273$

$\Rightarrow \alpha_{m1} = 0{,}156°$

b) $y_{M10} = ?$

$\alpha_{M10} = ?$

$\Delta r = M\lambda$ $\rightarrow$ $M\lambda = \frac{yd}{D}$ $\rightarrow$ $y_M = \frac{M\lambda D}{d} = \frac{10\cdot 5{,}46\cdot 10^{-7}\cdot 2}{10^{-4}} = 10{,}92\cdot 10^{-2}\,m = 10{,}92\,cm$

$\Delta r = \frac{yd}{D}$

$tg\,\alpha_{M10} = \frac{y_{M10}}{D} = \frac{M\lambda}{d} = \frac{10\cdot 5{,}46\cdot 10^{-7}}{10^{-4}} = 5{,}46\cdot 10^{-2}$

$\rightarrow \alpha_{M10} = 3{,}13^\circ$

c) $I = I(y) = ?$ $\quad I = |E_c|^2 = E_c\cdot E_c^*$ $\quad E_c = E_1 + E_2$

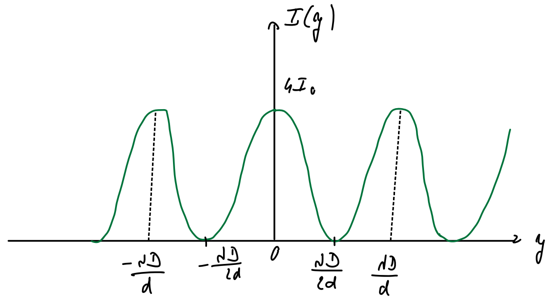
predp. rovinné vlny + $|\bar{E}_1| = |\bar{E}_2| \equiv E_0$

$$I = |E_c|^2 = E_0^2 + E_0^2 + 2E_0^2\cos\Delta\varphi = 2E_0^2(1+\cos\Delta\varphi) = 4E_0\cos^2\left(\frac{\Delta\varphi}{2}\right)$$

$2\cos^2\frac{x}{2} = 1+\cos x$

$E_0^2 \equiv I_0 \rightarrow I = 4I_0\cos^2\frac{\Delta\varphi}{2}$ $\quad \boxed{\Delta\varphi = \frac{2\pi}{\lambda}\Delta r}$ $\quad \Delta\varphi = \frac{2\pi}{\lambda}\cdot\frac{yd}{D}$

$$I(y) = 4I_0\cos^2\left(\frac{\pi d}{\lambda D}y\right)$$



$\max(I(y)) \Leftrightarrow \cos\left(\frac{\pi d}{\lambda D}y\right) = 1 \Leftrightarrow \frac{\pi d}{\lambda D}\cdot y = n\cdot\pi$; $n = \mathbb{Z}$

$y_{max} = \frac{n\lambda D}{d}$