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Physical laboratory 3

Task A Charge movement in electric and magnetic fields

Tasks

1. Verify formula (2) for the focal length of a short magnetic lens. Plot a graph showing $U_a = f(I_f^2)$ and calculate the focal distance f from the slope.
2. Verify the validity of formula (10) for the magnetic deflection of the electron beam. Plot graphs showing if the deflection magnitude y as a function of I_v and U_a corresponds to the formula (10).

Theory

An electron beam is used in many different electronic devices for various purposes. In such devices, it is generally needed to *focus* or *deflect* the electron beam. An old-style cathode ray tube (CRT) monitor is one of the simplest examples as it can also be used for viewing this beam on a luminescent screen.

A charged particle beam can be focused by a short magnetic lens. A short magnetic lens is a coil having a rationally symmetric magnetic field set so that it is interacting with the beam over a negligibly short part of its trajectory, focusing an originally divergent beam into a point on the screen. The focal length of the short magnetic lens f can be calculated as

$$f = 98 \frac{r}{n^2} \frac{U_a}{I_f^2}, \tag{1}$$

where r is the radius of the focusing coil, U_a is the acceleration voltage used for acceleration of the charged particle beam, n is the number of the coil threads and I_f is the current going through the focusing coil. In order to determine the focal length f and verify the validity of formula 1, we can rearrange it into

$$U_a = \frac{f n^2}{98 r} \cdot I_f^2. \tag{2}$$

If the position of the short magnetic lens does not change throughout the measurement, the graph of the dependence of U_a on the square of the focusing current I_f (i.e. $U_a = f(I_f^2)$) while keeping the point focused will be a line. Now, if we know the number of the coil threads and the dimensions of the coil, we can derive the focal length of the short magnetic lens.

The deflection of the electrons moving in a magnetic field happens through the Lorentz force (similarly to focusing) written as

$$\vec{F} = -e(\vec{v} \times \vec{B}). \tag{3}$$

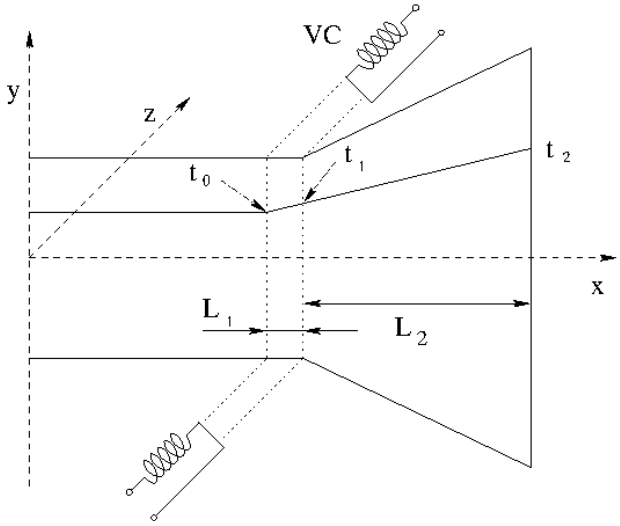

Figure 1: Force interaction of the magnetic field with an electron beam. The electrons enter the deflecting field B in time $t_0 = 0$ and stay in it for the time t_1 along the path L_1. No further deflection takes place along path L_2 during time t_2. The Lorentz force there is zero and the electron path is a line.

Let us assume the magnetic induction is perpendicular to the electron movement and that it interacts with the electrons only along the L_1 part of their path (see 1). Now we can describe the electron movement along the y axis by the following equation derived from equation (3)

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = \frac{e}{m} \cdot v_x B. \tag{4}$$

After integrating equation (4) we obtain:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{e}{m} \cdot v_x B t + C. \tag{5}$$

Assuming that the derivation $dy/dt = 0$ in time $t_0 = 0$, we obtain the integration constant $C = 0$. The speed along the y axis that the electron gains after going through the magnetic field, therefore, is

$$v_y = \frac{\mathrm{d}y}{\mathrm{d}t} \,|_{t=t_1} = \frac{e}{m} v_x B t_1, \tag{6}$$

where t_1 is the total time of flight of the electron through the deflecting field. Substituting $t_1 = L_1/v_x$, where v_x is the electron speed along the x axis, which we can obtain from the accelerating voltage as:

$$eU_a = \frac{1}{2} m v_x^2 => v_x = \left(\frac{2eU_a}{m}\right)^{\frac{1}{2}}, \tag{7}$$

we obtain

$$v_y \,|_{t=t_1} = \frac{e}{m} \cdot B \cdot L_1. \tag{8}$$

The deflection of the electron beam on the screen will be approximately

$$y = v_y t_2, \tag{9}$$

where t_2 is the time of flight from the deflecting systems to the screen, $t_2 = L_2/v_x$.

Substituting v_y from equation (8) into equation (9) , we obtain the formula for the electron beam on the screen

$$y = \sqrt{\frac{e}{2m}} L_1 L_2 \frac{B}{\sqrt{U_a}}. \tag{10}$$

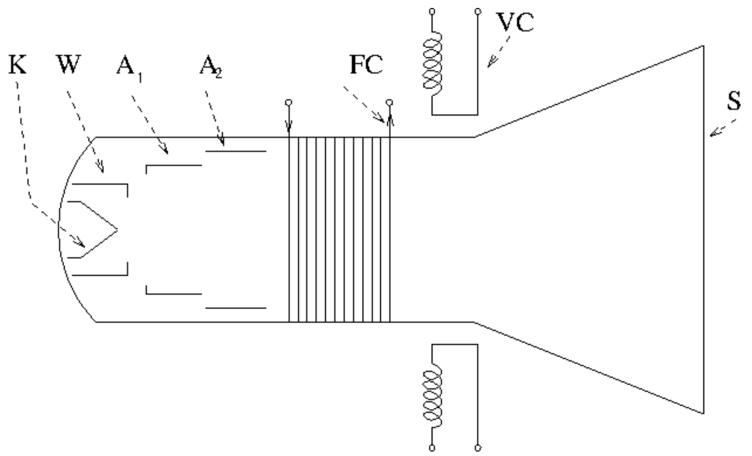

Figure 2: Schematic drawing of the used CRT with magnetic focus and magnetic deflection. K - cathode, W - Wehnelt cylinder, A_1 a A_2 - anodes, FC - focusing coil, VC - two pairs of deflection coils, S - screen.

The magnetic induction of the deflecting field B is directly proportional to the current flowing through the deflecting coil I_v. For the verification of formula (10) we will measure the dependence

$$y = f_1(I_v)$$

at constant accelerating voltage U_a and the dependence

$$y = f_2(U_a)$$

at constant deflection current I_v.

Our study will be conducted using a CRT screen, which is, in essence, a multi-electrode electron tube with an electron jet and a luminescent screen – see Fig. 2. The electron source is a hot cathode. These electrons are repulsed by a negative potential of a grid (the so-called Wehnelt cylinder) into a space in the cylinder, creating a narrow axially divergent beam. This beam then passes through the centre of two anodes A_1 and A_2, which are at a high positive potential of up to 20 kV with respect to the cathode. The anodes accelerate the electron beam to a velocity high enough to cause excitation of the luminescent layer on the screen. The anodes also take part in the (this time electrostatic) focusing of the beam.

Inside the CRT, the pressure is reduced to a value, where the mean free path of the electrons is higher than the distance between the cathode and the screen. If this wasn't the case, the movement of the electrons would be affected by their collisions with the present gas molecules. A focusing coil (a short magnetic lens) is placed outside of the tube on its circumference. It focuses the electrons to a point on the luminescent screen.

Two pairs of deflection coils are placed on the outside of the tube after the focusing coil. One pair is for deflection in the vertical direction, and the second is for the horizontal deflection of the electrons. Also, the electric field deflection can be used in some devices such as oscilloscopes.

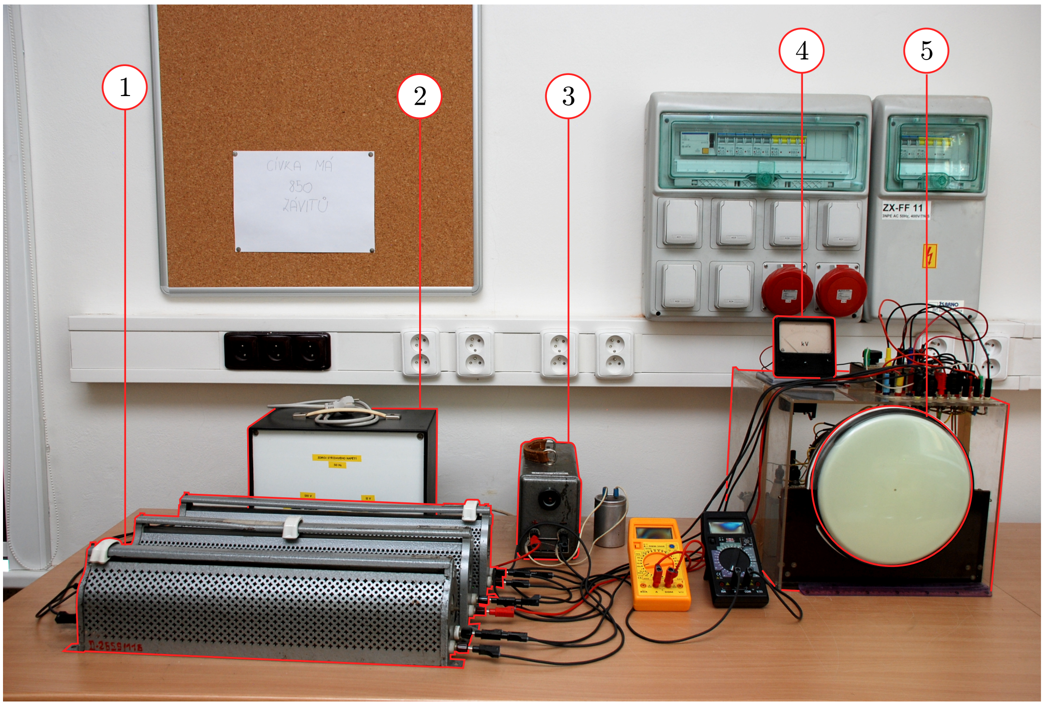

Figure 3: Measurement apparatus:: (1) Potentiometers of the control of the acceleration voltage (U_a), focus (I_f) and deflection current (I_v). (2) AC power source for the acceleration voltage and deflection current. (3) DC power source for the focus current. (4) Voltmeter for the measurement of the acceleration (anode) voltage. (5) Screen.

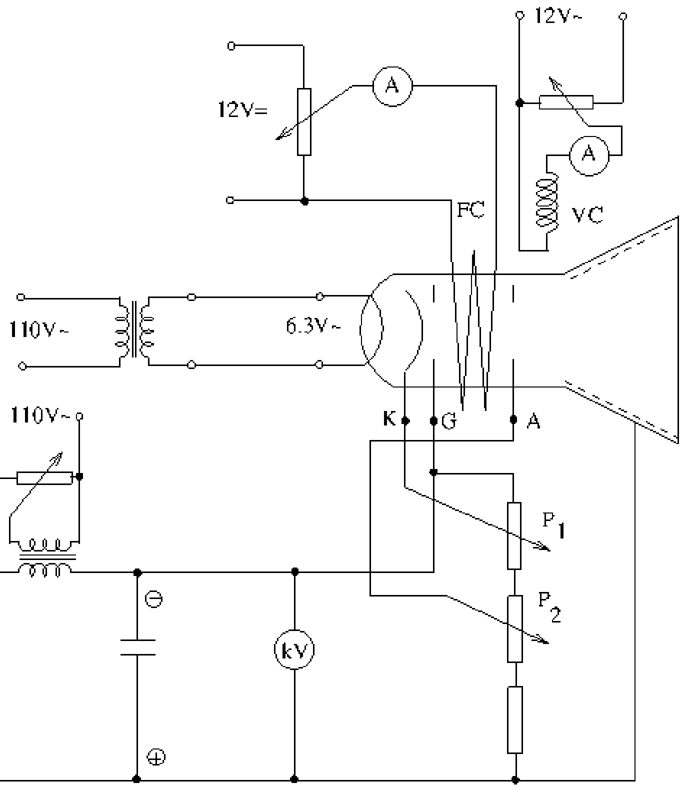

Figure 4: Electrical drawing for electrical connection of the measurement apparatus.

Experimental procedure

Connect the CRT to the power sources, voltmeters and ammeters as per Fig. 4. After connection and checks of all the circuits, set all the potentiometers so that the supplied voltages are at a minimum. Then carefully increase the voltage, one circuit at a time and check the working of the individual circuits on the screens and meters.

When verifying eq. (2), change the anode voltage and focus the picture on the screen by changing the focus current.

When studying the deflection, AC current flows through the deflection coil giving rise to an AC magnetic field. Therefore an oblong track of the beam will be visible on the screen. Its length is proportional to the amplitude of the deflection current I_v.

Owing to the luminescent screen's inertia and the human eye, we do not see a moving point, and we see a line. When measuring the $y = f(U_a)$ we will start with the lowest values of the U_a where the observed track is the widest when being still visible. We will set its length to be nearly over the whole width of the screen by setting the appropriate focus current. Now, when we increase the U_a, the track width will decrease (see (10)), and it will not go over the screen width. The track will de-focus with any change of the U_a and it needs to be re-focused by changing the focus current at every step. Measure the $y = f(I_v)$ dependence for two different constant values of U_a and then the $y = f(U_a)$ dependence for two different constant values of I_v.

Tip for the report: When graphically verifying that the measured data correspond to the theoretical formula, it can be advantageous to convert the tested formula to a linear dependence of the tested parameter.

The focus coil has a radius of $r = 2$ cm an the number of threads of $n = 1000$.

!!CAUTION!! The completed high voltage rectifier is capable of supplying a high current that can cause injury when handled carelessly!

Real-world use

Deflection and focusing of the electron beams are necessities for the practical use of electron beams. Let us take an example of a scanning electron microscope (SEM), which is in principle similar to the studied CRT. It comprises an electron source (electron gun) and different lenses that focus the electron beam onto the sample. In the CRT, this electron beam lights up the luminescent layer on the screen, while in the SEM, it interacts with the sample instead. The lens system reduces the diameter of the electron beam on the surface of the sample to maximise the resolution of the SEM. The practical beam diameter nowadays is approximately in the order of a few nanometres. The optical system also governs the beam current (number of electrons impacting the sample per unit of time) and its optical properties. A whole system of coils is used to achieve such control of the beam, whereas only one coil is used in the laboratory CRT.

Similarly to the CRT also SEM moreover contains a deflecting coil system. Their task is to change the beam's position on the sample and acquire the image in a point-by-point manner. In both the SEM and the CRT it is necessary to deflect the electron beam in two axes. The maximum deflection current (in the case of magnetic deflection) governs the maximum deflection and, therefore, the maximum magnification of the SEM. As different beam energies (different acceleration voltages) are used in electron microscopy, it is necessary to know the dependence of the deflection magnitude on the accelerating voltage. For example, when the operator images a given viewfield and changes the beam energy, the microscope system needs to change the deflection current and the acceleration voltage to keep the same viewfield.

References

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