Department of Physical Electronics
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Physical laboratory 3

Task C Thermoelectron emission

Tasks

1. Measure the dependence of the anode current on the anode voltage $I_a = f(U_a)$, where U_a is in the range from $-5\mathrm{V}$ to $500\mathrm{V}$, for two different values of the heat current I_f and plot those measuerements in a graph.

   - Plot the start-up area of the anode current I_a in $\ln I_a = f(U_a)$ coordinates and determine the electron temperature.
   - Use the $\ln I_{nas} = \sqrt{U_a}$ coordinates to work with the area of the saturated anode current and determine the current increase due to the Schottky effect. Compare the experimentally obtained value and the value calculated from (11). You can estimate the electric field intensity near the surface of the cathode according to (13).

2. Determine the anode voltage U_a, for which the anode current is already saturated, $I_a = I_{nas}$.

3. By measuring the dependence of the saturated anode current on the neat current $I_{nas} = f(I_f)$ determine the work function of tungsten w using the Richardson – Dushman line.

Thermoelectron emission study

Releasing free electrons from the surface of metals is called electron emission. Emission occurs when electrons gain enough energy for overcoming the attractive forces stopping them from their emission from the metal's surface. The electrons can gain this energy by different means. One of the most common and most effective ways is the heat emission known as *thermionic emission*, *thermoelectron emission*, *thermoelectronic emission* or *thermoemission* being the emission of electrons at high temperatures of the metal and *photoemission* being the emission of electron due to short-wavelength irradiation of the metal's surface. The electrons can also be released due to a high electric field in the order of $10^8 - 10^{10}\,\mathrm{Vm^{-1}}$ without heating the metal. This case is called an *electron autoemission* or *cold emission*. Emission due to the bombardment of charged particles is called a *secondary emission*. This assignment is focused on the study of thermoelectron emission. The history of heat emission dates back to the 1870s, and nowadays, thermoemission is used for the generation of electron rays in many electronic devices (electron microscopes, electron tubes, cathode ray tubes, etc.). It is also used for the analysis of forces binding the electrons in the matter.

Work function

Metals at a sufficiently high temperature emit electrons. Only electrons with high enough energy for overcoming the attractive forces between the electron and the metal can be emitted. The

energy of the attractive forces is called the work function (w). The number of all the electrons released from the cathode at a given temperature is given by the saturated emission current. Its magnitude depends on the temperature of the metal constituting the cathode (T) and the work function. This dependence is shown in the Richardson – Dushman equation

$$I_{\text{nas}} = BT^2 \exp(-w/kT), \tag{1}$$

where B is a constant describing the different parameters such as the cathode surface or the thermionic emission constant and k is the Boltzmann constant. The derivation of this equation and the formula for the thermionic emission constant can be found for example in[3]. We can use equation (1) for the measurement of the work function by rewriting it to the form of the Richardson line. We rearrange the equation and use logarithms

$$\ln(I_{\text{nas}}/T^2) = \ln B - w/kT. \tag{2}$$

We denote $y = \ln(I_{\text{nas}}/T^2)$ and $x = 1/T$. Therefore we get a line in the new coordinates as

$$y = (-w/k) \cdot x + \ln B. \tag{3}$$

From the slope of this line, we can calculate the work function w of the given metal. Work function examples for different metals are shown in the following table.

element	w [eV]		element	w [eV]
Cs	1.9		W	4.5
Ba	2.5		Fe	4.7
Th	3.4		Ni	5.1
Ta	4.1		Pt	5.4

Cathode temperature determination

It is necessary to know the temperature of the cathode for plotting the dependence (2). We can determine this temperature based on the known dependence of the conductor resistance on the temperature. The tungsten filament resistance (R_t) can be described as

$$R_t = \frac{\rho d}{S} (1 + \alpha t), \tag{4}$$

where $\rho = 4.89 \times 10^{-8}$ Ωm at $0\,^o$C, d is the filament length, S is the filament cross-section, $d/S = 7{,}76 \cdot 10^6$ m^{-1}, $\alpha = 4.83 \times 10^{-3}$ K^{-1} is the temperature coefficient of resistivity and t is the temperature in degrees Celsius. The resistance of the filament can be determined by Ohm's law from the value of the measured heat current I_{f} and the voltage drop at the cathode U_{f}.

Distribution of electrons according to their energies

It can be shown that the kinetic energy of electrons emitted from a surface of the metal has a Maxwellian distribution. The electron energy distribution can be experimentally measured by the retarding field method. The principle of this method is in introducing an anode voltage U_{a} with the variable polarity between a cathode and an anode with a suitable geometric configuration. By measuring the dependence of the anode current I_{a} on the anode voltage U_{a} (for $U_{\text{a}} < 0$ as well), one can obtain the electron energy distribution function in its integral shape. If we decrease the value of the anode voltage from positive to negative values, we decelerate the electrons by the anode's electric field. In other words, more and more electrons do not hold true to the condition

$$\frac{1}{2}mv^2 \geq -eU_{\text{a}} \qquad (U_{\text{a}} < 0) \tag{5}$$

and they, therefore, can not reach the cathode, and they do not take part in the current flowing between the cathode and the anode. Different cathode temperatures will yield a different number of electrons of different energy distributions.

One can, therefore, verify the validity of the Maxwellian electron energy distribution by measuring the VA characteristics in the region of negative anode voltage. One can also determine the
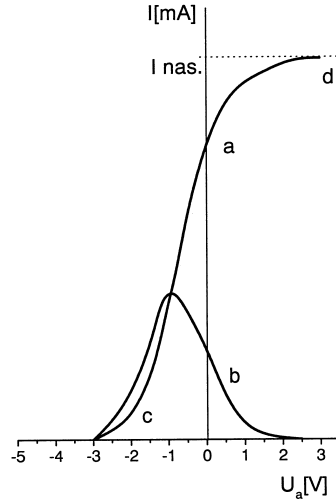

Figure 1: a - integral, b - differential shape of the electron energy distribution. Region c is the region of the start-up current, region d is the region of the saturated current.

emitted electron temperature from the VA characteristic. The start-up region can be described as

$$I_a = I_0 \exp\left(\frac{eU_{\mathrm{a}}}{kT}\right). \tag{6}$$

The electron temperature can be determined after performing the logarithm of the previous formula from the slope $a = \frac{e}{kT}$ in the linear region of the dependence $\ln I_a = f(U_{\mathrm{a}})$ in the start-up current region. This temperature can be compared to the one determined in the task focused on the work function calculation.

Schottky effect

The presence of a strong electric field near the metal surface causes a decrease in the cathode work function. Figure 2 shows the potential near the surface of the metal. Without the presence of the magnetic field, the potential has an approximately rectangular step shape. For the electron to be released from the metal, it has to have energy greater than the work function w. The value of the work function is given by the depth of the Fermi level, that is by the difference of the potential of the Fermi level of the vacuum potential.

If an electric field of the intensity E is introduced, its potential is superimposed with the step potential. The step potential is transformed into a potential barrier of finite thickness. Also, the height of the barrier is lower. Therefore, the electron emission is facilitated by two processes. On the one hand, the work function decreases to w', and on the other hand, the finite thickness of the barrier gives rise to a non-zero probability of the electron to tunnel through the barrier. The shape of the superimposed potential also shows that the electrons are accelerated away from the cathode from a certain distance.

It can be shown that the work function of the electron in a metal w is decreased by w_{p} at the presence of an electric field as per

$$w_{\mathrm{p}} = \sqrt{\frac{e^3 E}{4\pi\epsilon_0}} \tag{7}$$
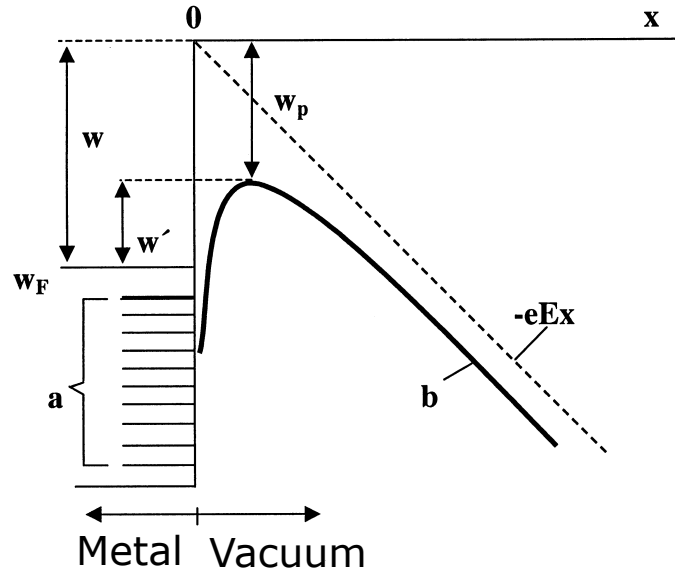
Figure 2: Shottky effect.

and therefore the new value of the work function w' is

$$w' = w - w_{\mathrm{p}} = w - \sqrt{\frac{e^3 E}{4\pi\epsilon_0}}, \tag{8}$$

where ϵ_0 is the permittivity of vacuum. Richardson – Dushman equation of the saturated emission current becomes

$$I'_{\mathrm{nas}} = BT^2 \exp(-w'/kT) = BT^2 \exp(-w/kT) \exp(w_{\mathrm{p}}/kT) = I_{\mathrm{nas}} \exp(w_{\mathrm{p}}/kT), \tag{9}$$

where I_{nas} is the saturated emission current without the presence of the electric field. From this we obtain

$$\ln I'_{\mathrm{nas}} = \ln I_{\mathrm{nas}} + w_{\mathrm{p}}/kT \tag{10}$$

and we can see that $I'_{\mathrm{nas}} > I_{\mathrm{nas}}$. Therefore, decrease in the work function by w_{p} leads to an increase a of the saturated emission current. After substitution of w_{p}, we can write

$$\ln I'_{\mathrm{nas}} = \ln I_{\mathrm{nas}} + \sqrt{\frac{e^3}{4\pi\epsilon_0 k^2 T^2}} \times \sqrt{E}. \tag{11}$$

Formula 11 contains the electric field intensity near the surface of the cathode. Numerical methods are necessary to determine the exact value of the electric field in the present configuration. However, the electric field can be approximately estimated by assuming that in the closest vicinity of the cathode, it does not significantly differ from the electric field in a cylindrical capacitor. In such a case, the electric field intensity E near the surface of a cylindrical electrode with radius R can be described by

$$E = U_{\mathrm{a}} \frac{1}{r \ln(R/r)}, \tag{12}$$

where U_{a} is the anode voltage. In our simplified case, the anode has a virtual radius D corresponding to the distance of the hot cathode and the anode. It is obvious that the real geometry significantly differs from a cylindrical capacitor. Numerical calculations have shown that formula
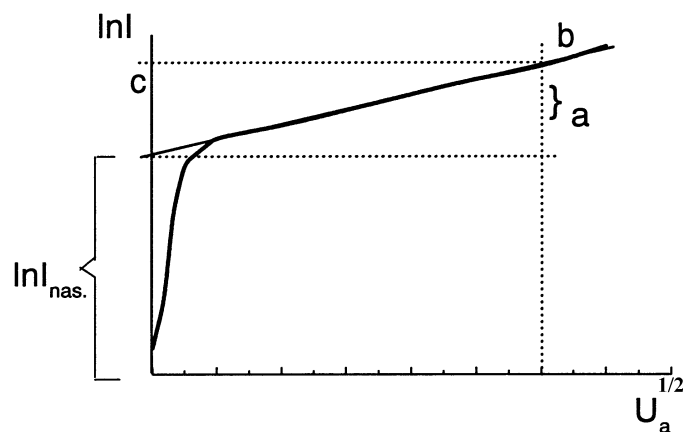
Figure 3: The VA characteristic in the $\ln I$ and $U_a^{1/2}$ coordinates.

(12) needs to be multiplied by a factor of $(L-D)/D$, where D is the distance of the hot cathode, and the anode and L is the distance between the anode and the planar cold part of the cathode placed behind the hot cathode that is used to homogenise the electric field between the electrodes. The electric field intensity near the present hot cathode can be, therefore, calculated according to

$$E = U_a \frac{L-D}{D} \frac{1}{r\ln(R/r)}. \tag{13}$$

As the electric field intensity is proportional to the anode voltage U_a, the logarithm of the anode current $\ln I'_{nas}$ must be directly proportional to the anode voltage square root $\sqrt{U_a}$, see figure 3.

Measurement setup description

The measurement is carried out with the apparatus connected as per figure 5. The measurement is semi-automatic due to a computer-controlled power source Agilent E3631A that also serves as a measurement device. Agilent E3631A has several outputs. The 0–6 V, 0–5 A output is used as the current source in the cathode circuit serving for the heating of the tungsten filament. A hot enough filament emits electrons due to the thermionic emission. These electrons can be collected at an anode depending on the potential difference between the cathode and the anode (anode voltage). The anode voltage can be controlled by the $0-20$V output of the power source. The voltage range is not high enough to carry out the whole experiment. Therefore, a voltage converter multiplying the output voltage by the factor of 25 is connected to this output. A switch can disconnect this converter for more precise measurement in the region of low anode voltages. Another switch is used to change the voltage polarity on the electrodes. The anode voltage is monitored by a Protek 506 voltmeter connected to the computer through a serial port. The anode current is monitored by an Agilent 34410A digital ammeter connected to the computer by USB. During the measurement, the measured data are plotted as well as saved to a text file for later analysis.

Experimental procedure

- You will measure with voltages up to 500 V, mind your own safety first!
- Do not exceed the maximum allowed values of the heating current and anode voltage.

1. Connect the circuits as per figure 4 and wait for a check by the teacher before turning anything on.

2. Run „Termoemise" (MTE) from the desktop.

3. Connect all the measurement devices (Agilent E3631A, Agilent 34410A a Protek 506) in the menu Zařízení (Devices) and sub-menu Připojit (Connect). The taskbar at the bottom of the window (see fig. 6) will indicate a successful connection of the devices.

4. Click Zapnout výstupy (Turn on outputs). This will activate the power source outputs and the pull bars and switches for the anode current and anode voltage in the software.

5. Choose the correct experiment in menu Měření (Measurement). Výstupní práce is for the measurement of the work function by measuring the dependence of the saturated ion current on the heating current or Schottkyho efekt for the measurement of the Schottky effect for the measurement of the dependence of the anode current on the anode voltage.

6. While measuring the Schottky effect measure the VA characteristic of the diode ($I_a = f(U_a)$) for two different values of the heating current I_f. The heating current must not exceed 2 A! Consider the measurement range of the anode voltage U_a before the experiment.

7. Set the desired values of the heating current and wait for several minutes (at least 10) for its stabilisation.

8. Use the manual switch to change the polarity of the applied voltage. You need to use the voltage converter to use a voltage higher than 20 V. Make sure that $U_a = 0$ V before engaging the converter. Then use the switch „zesilovač" in the software.

9. Save the measured data by clicking Uložit měření (Save measurement) in the Soubor (File) menu after each task.

10. Determine the anode voltage U_a corresponding to the saturated anode current region from the VA characteristic.
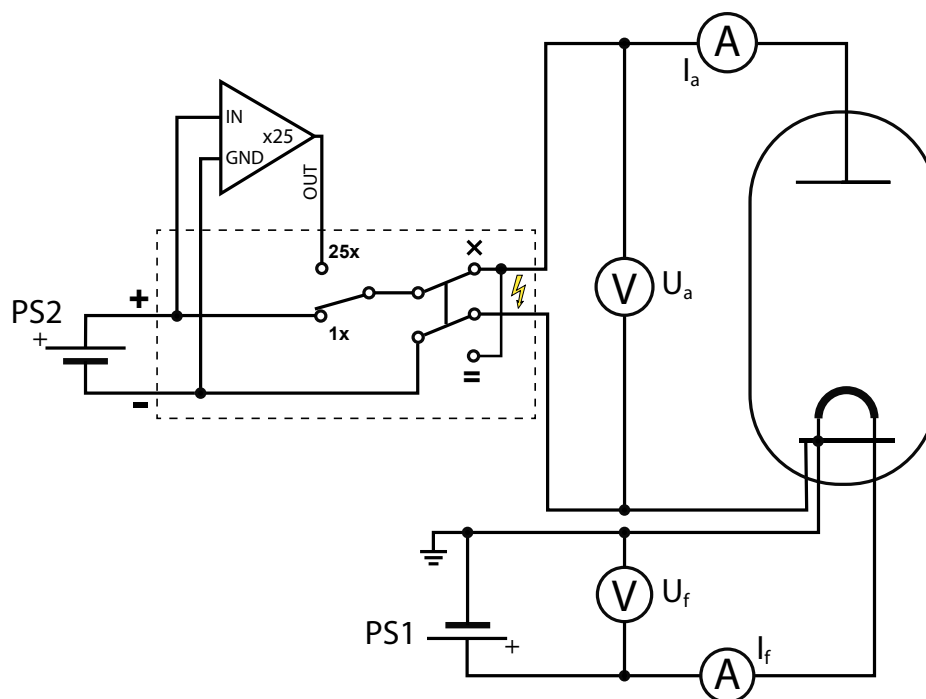

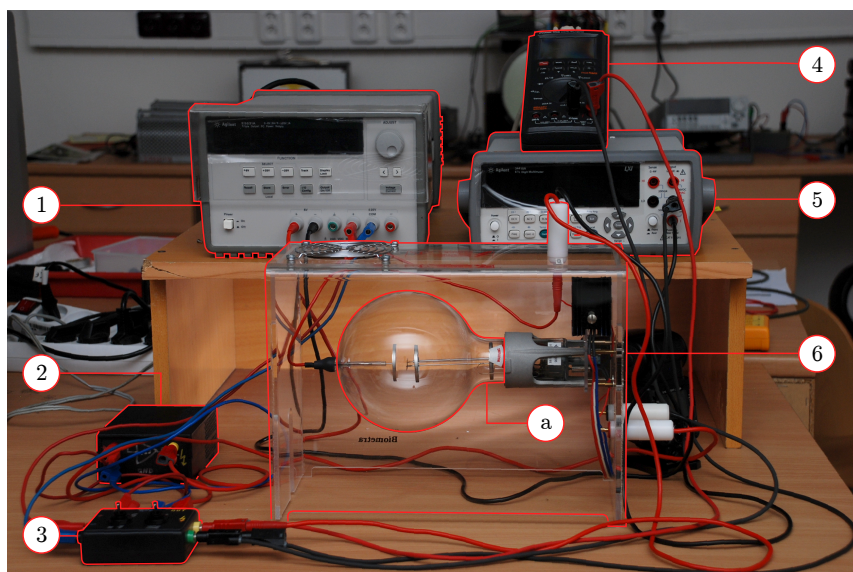Figure 4: Electrical diagram of the diode connection used for the study of thermoemission.

Figure 5: Diagram of the diode connection used for the study of thermoemission. 1 – Agilent E3631A power source, 2 – voltage converter, 3 – voltage polarity and voltage converter switch, 4 – Protek 506 multimeter, 5 – Agilent 34410A multimeter, 6 – transparent diode cover: a) vacuum tube containing the hot cathode and planar anode.

11. While measuring the work function in the Výstupní práce regime, measure the $I_{\mathrm{nas}} = f(I_{\mathrm{f}})$ dependence at a chosen anode voltage U_{a}. Always wait several minutes after every change of the heating current I_{f} for its stabilisation.

12. After you finish the measurement, click Vypnout výstup (turn off the output) and within the Zařízení (Devices) menu and the Odpojit (Disconnect), disconnect all the measurement devices.

Notes

- Maximum heating current: $I_{\mathrm{f}} = 2\,\mathrm{A}$
- Maximum anode voltage: $U_{\mathrm{a}} = 500\,\mathrm{V}$ (while using the voltage converter)
- Cathode: tungsten (W), tabulated value of the work function: $w = 4.5\,\mathrm{eV}$
- Cathode diameter: $2r \approx 0.09\,\mathrm{mm}$, cathode length: $d = 50\,\mathrm{mm}$, cathode geometry factor $d/S = 7{,}76 \cdot 10^6\,\mathrm{m}^{-1}$
- Anode radius: $R = 17\,\mathrm{mm}$
- Distance between the anode and the hot cathode: $D = 15\,\mathrm{mm}$
- Distance between the anode and the cold cathode: $L = 25\,\mathrm{mm}$

References

[1] T. Chudoba a kol.: Fyzikální praktikum III., skripta Přír. fak. UJEP v Brně, SPN Praha 1986.

[2] L.N. Dobrecov: Elektronová a iontová emise, Nakladatelství ČSAV, Praha 1956.

[3] O. Vybíhal: Automatizace měření ve fyzikálním praktiku, bakalářská práce, PřF MU 2011. https://is.muni.cz/auth/th/175317/prif_b/
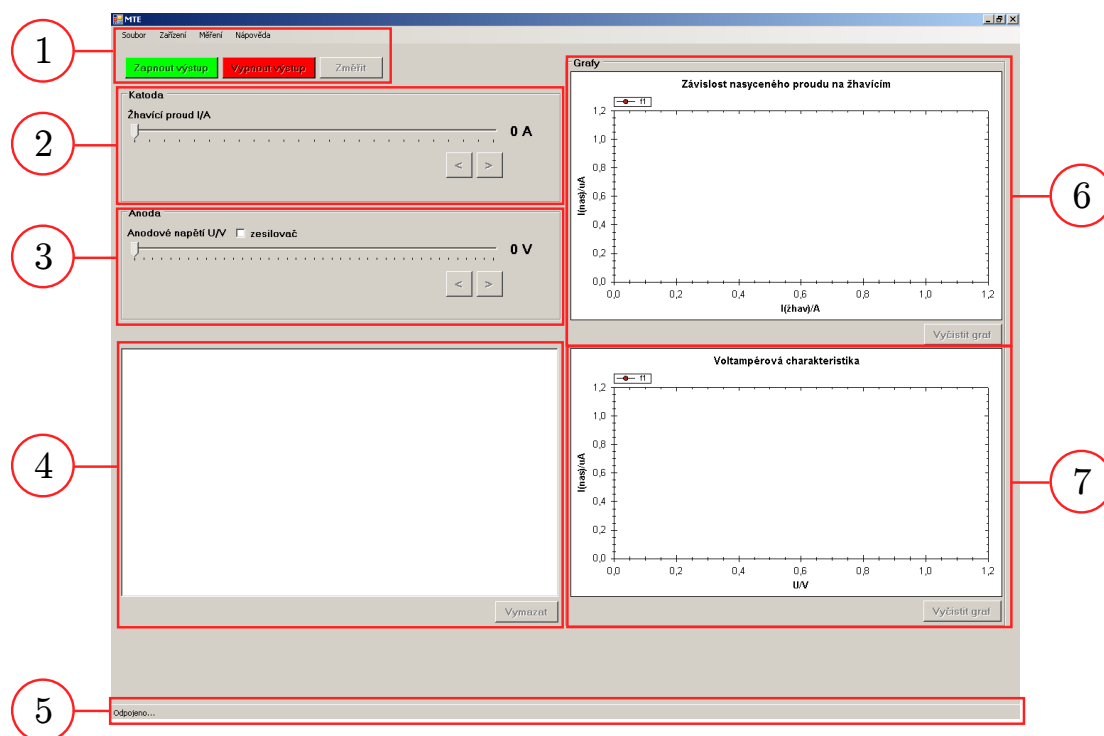
Figure 6: The user interface of the MTE software. 1 – main menu, activation of the connected devices and measurement start, 2 – heating current control, 3 – anode voltage control and (zesilovač = turn on the 25 times voltage converter), 4 – data registered by the individual measurement devices, 5 – taskbar indicating possible connection errors, 6 – graph of the dependence of the anode current on the heating current used for the measurement of the work function, 7 – graph of the dependence of the anode current on the anode voltage used for the measurement of the Schottky effect.