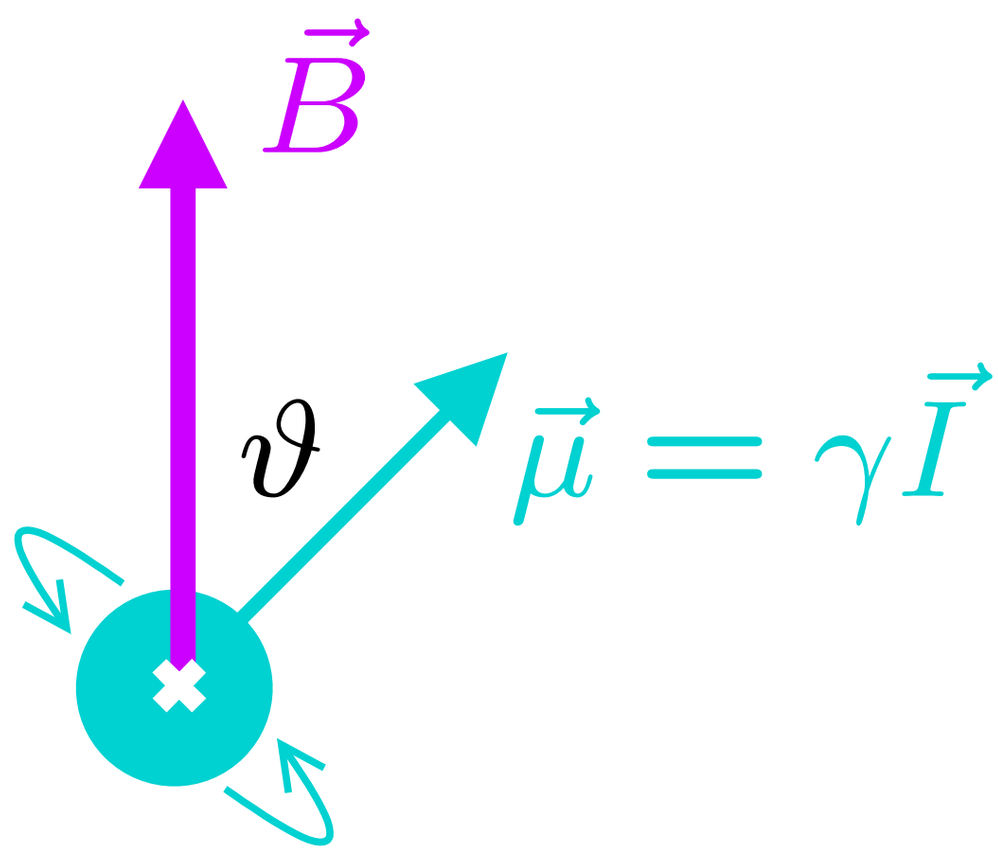
Lecture 1: Nuclear magnetic resonance

Magnetic moment

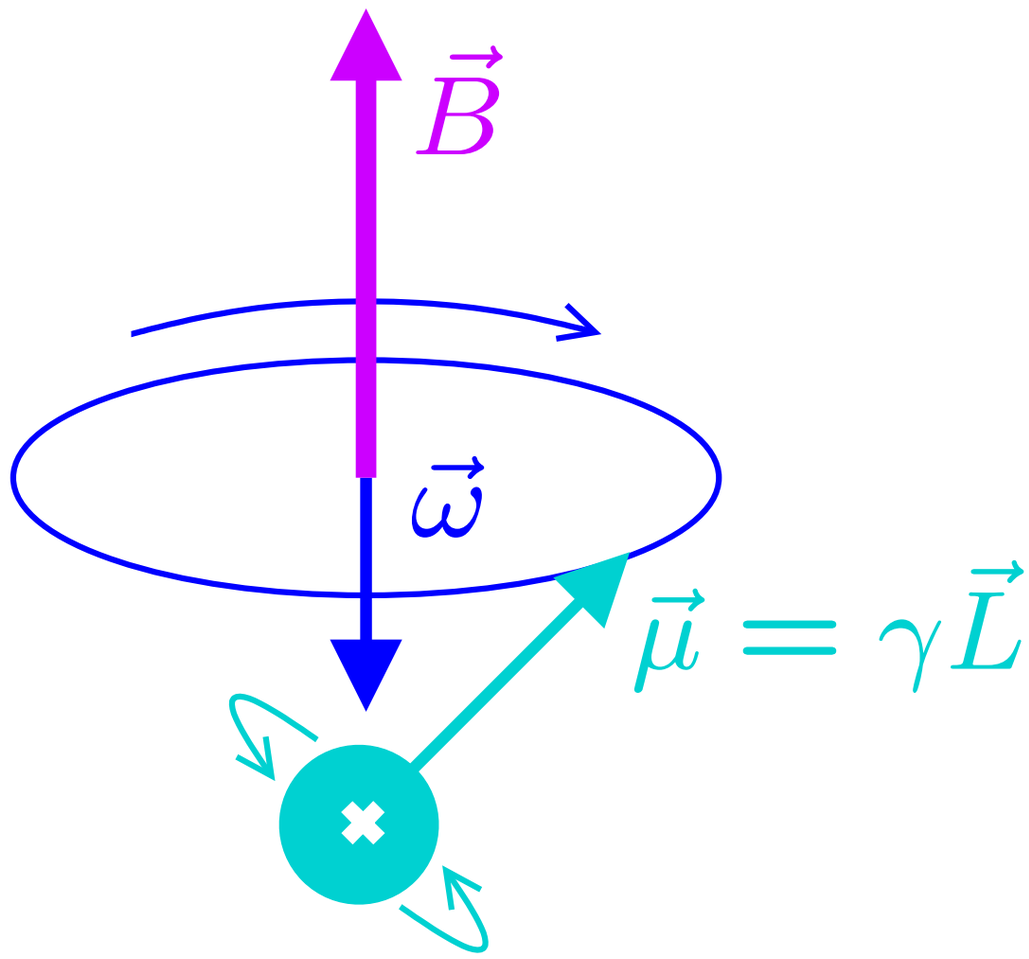
$$\vec{\mu} = \gamma \vec{I}$$

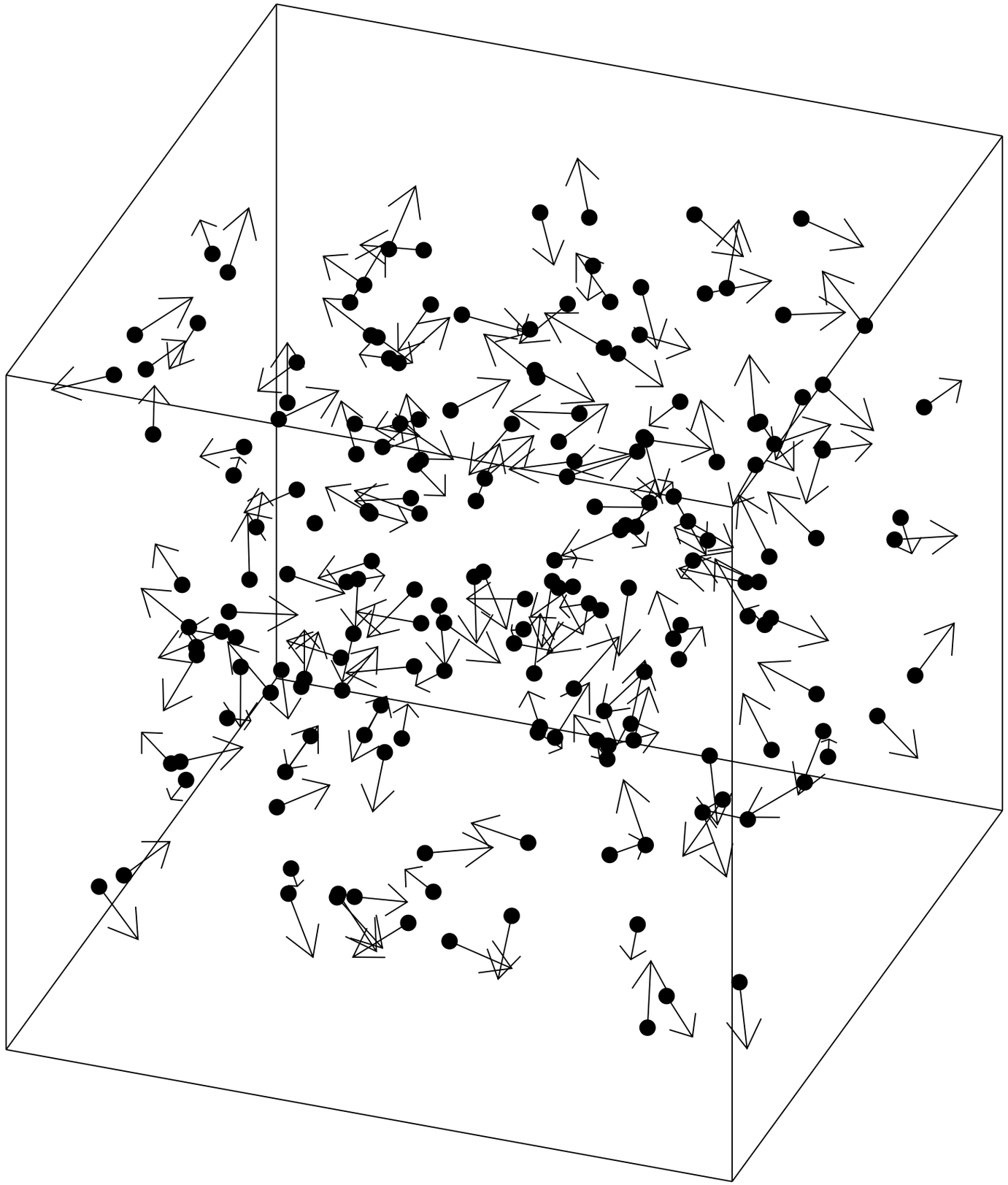
Energy

$$E = -\vec{\mu} \cdot \vec{B} = -|\mu||B| \cos\theta$$

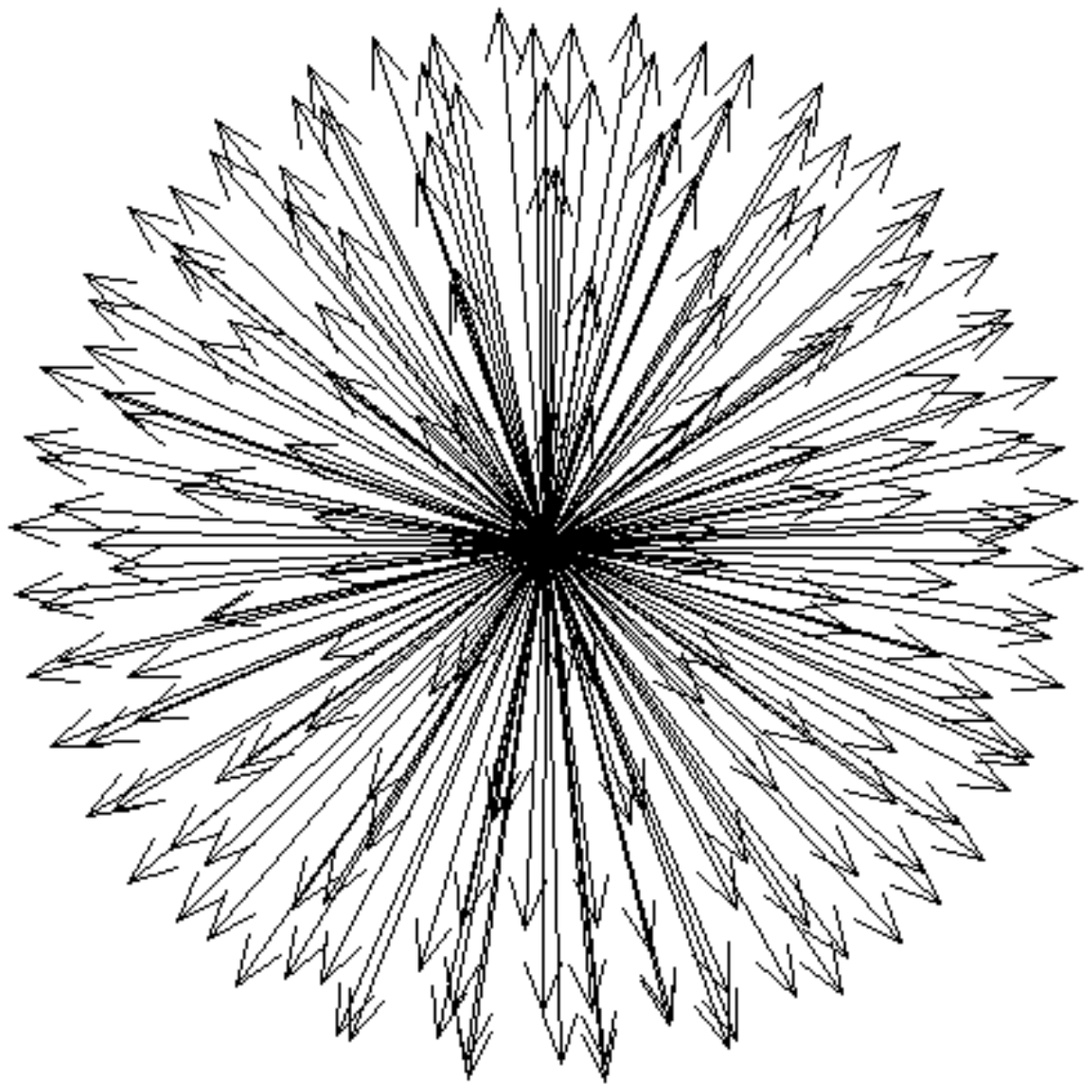


Angular precession frequency $\vec{\omega} = -\gamma \vec{B}$

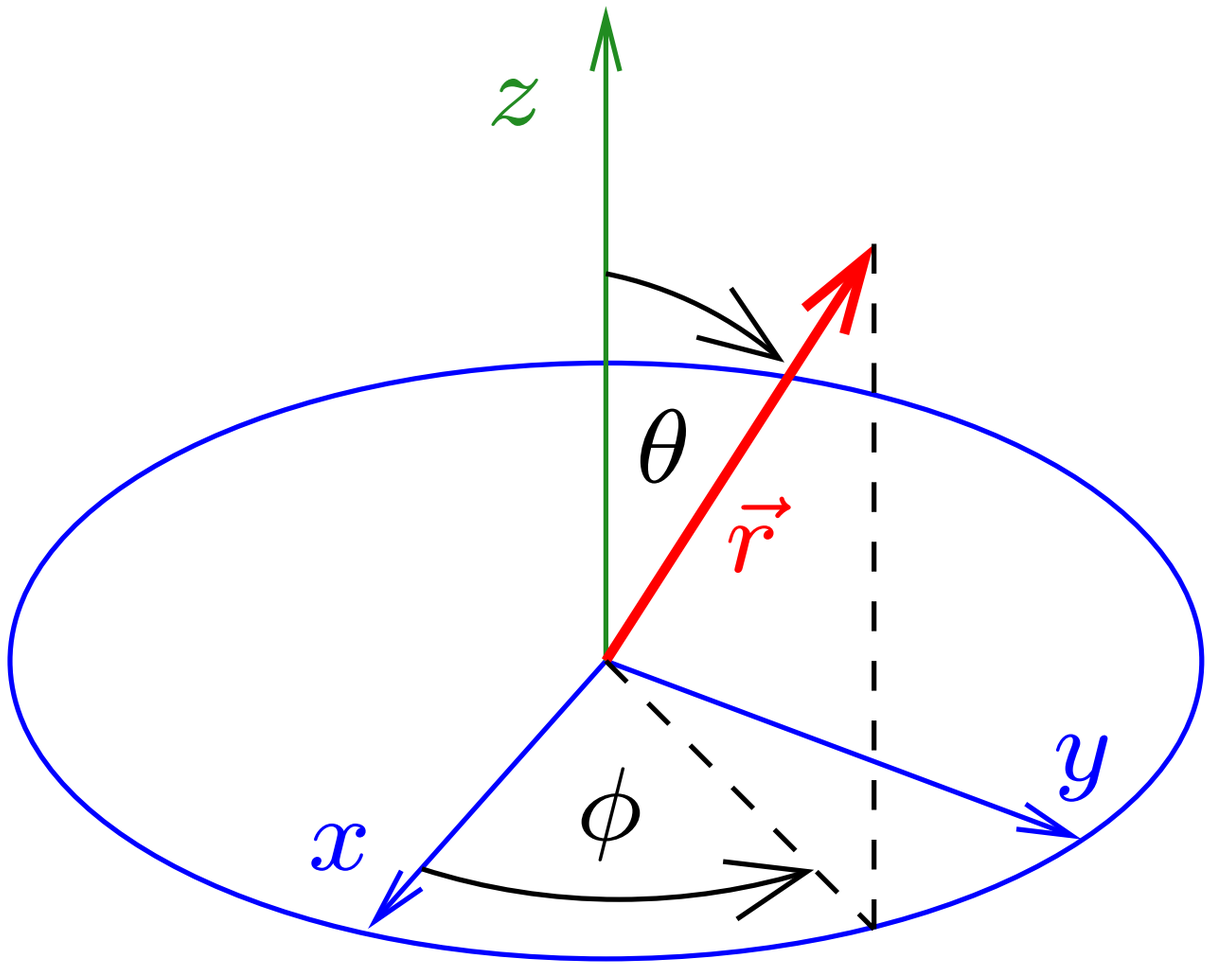


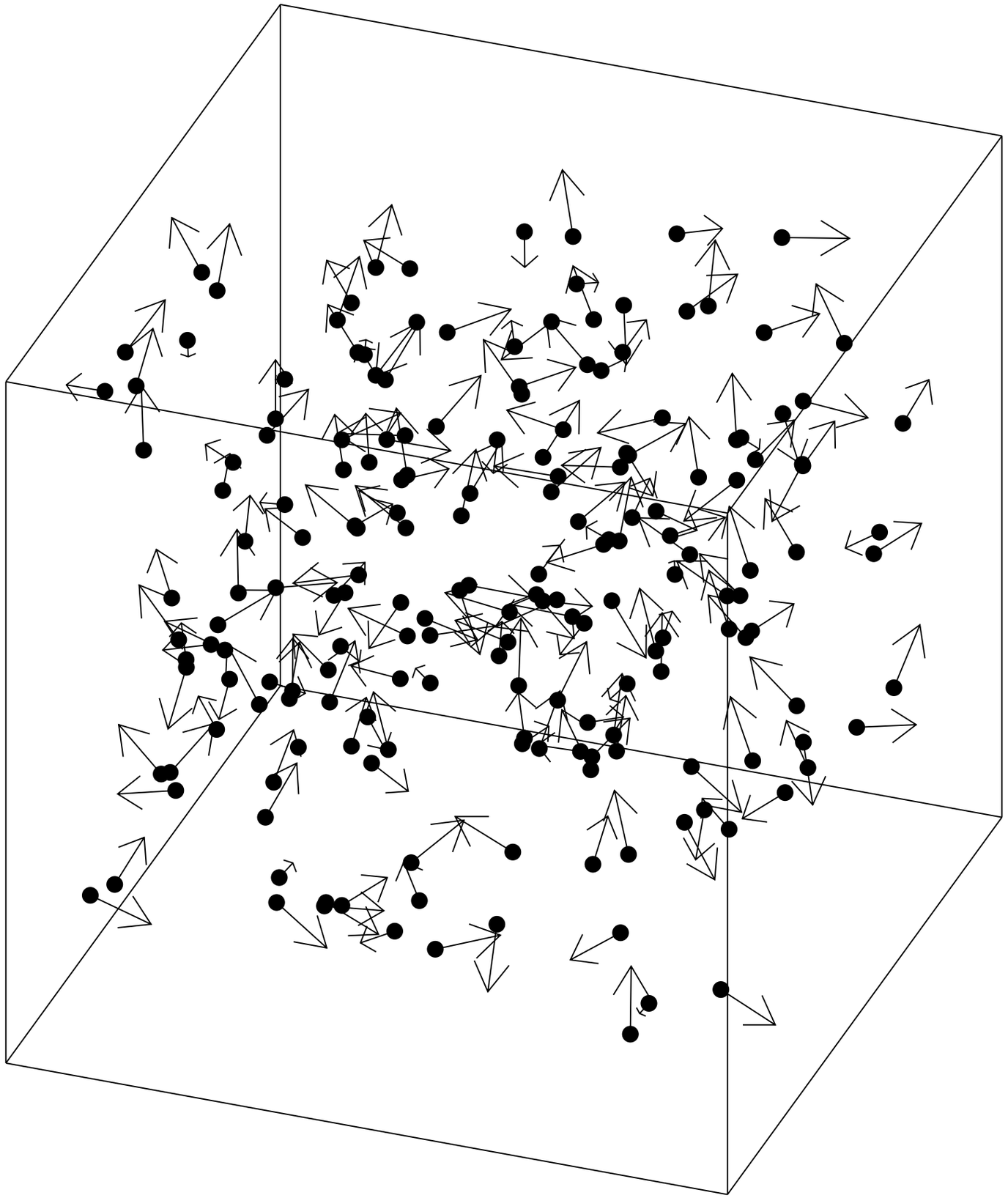


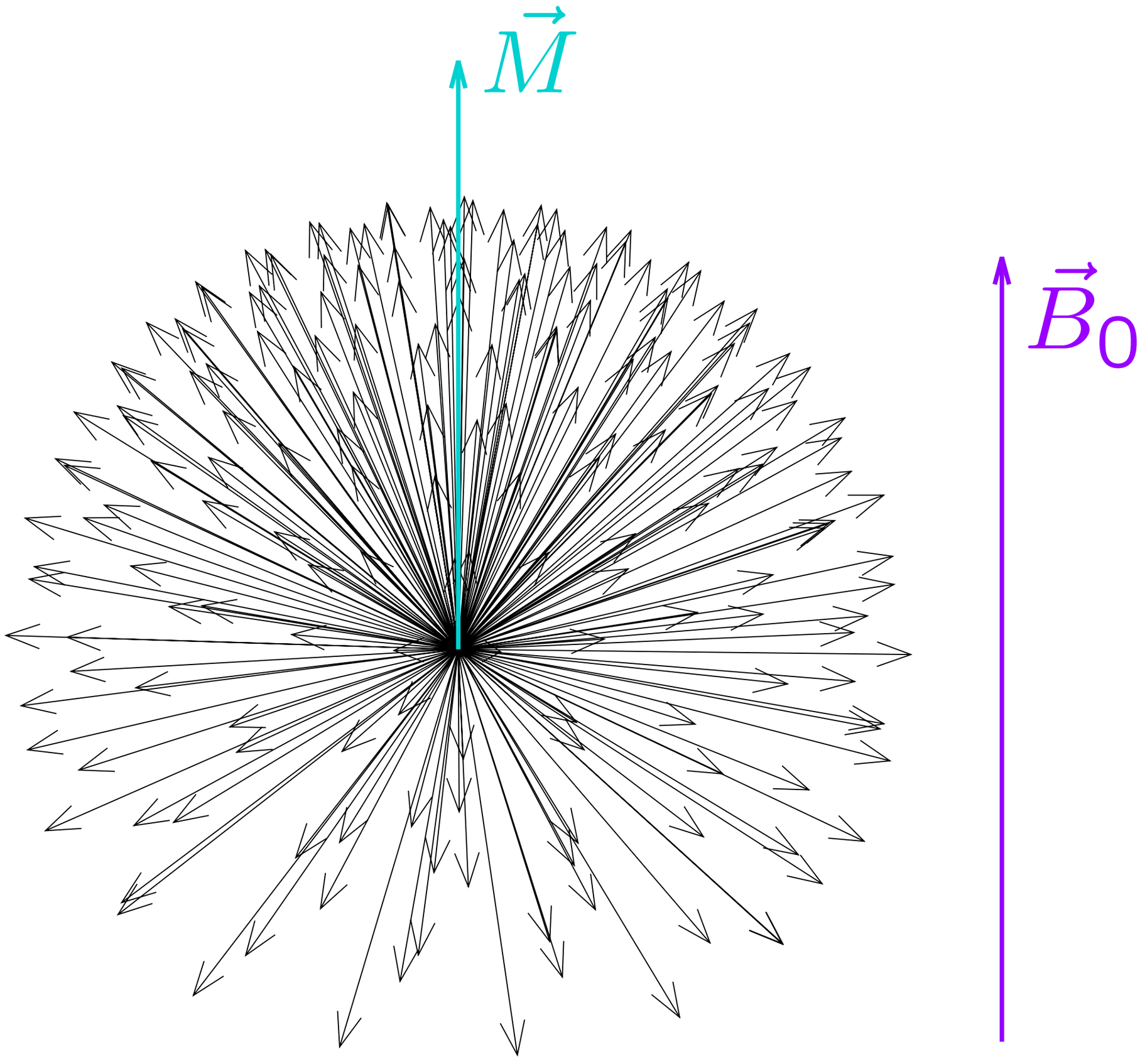
$\vec{M} = (\vec{\mu}_1 + \vec{\mu}_2 + \vec{\mu}_3 + \vec{\mu}_4 + \vec{\mu}_5 + \vec{\mu}_6 + \cdots)/V$ Magnetization



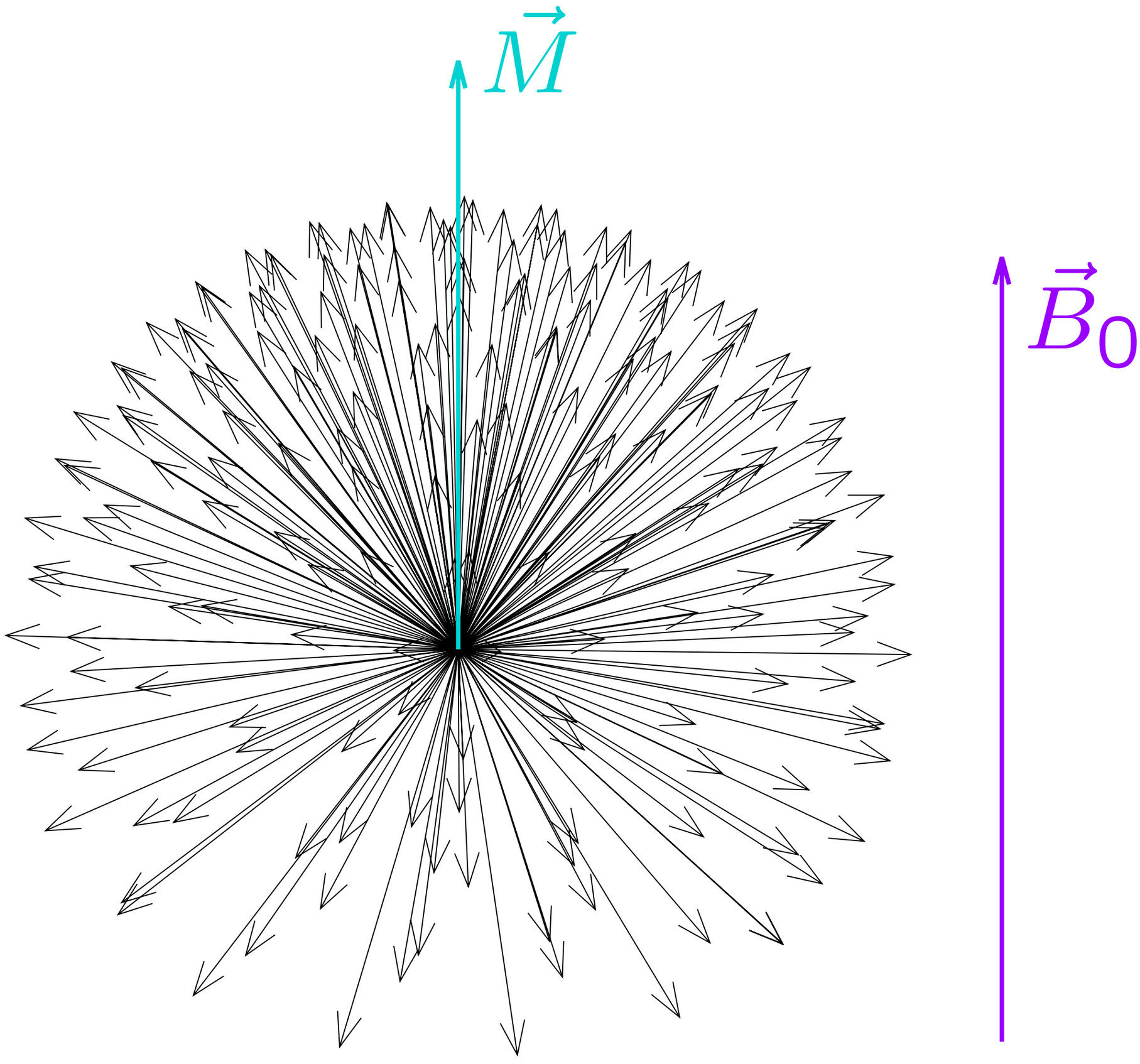
Spherical coordinates



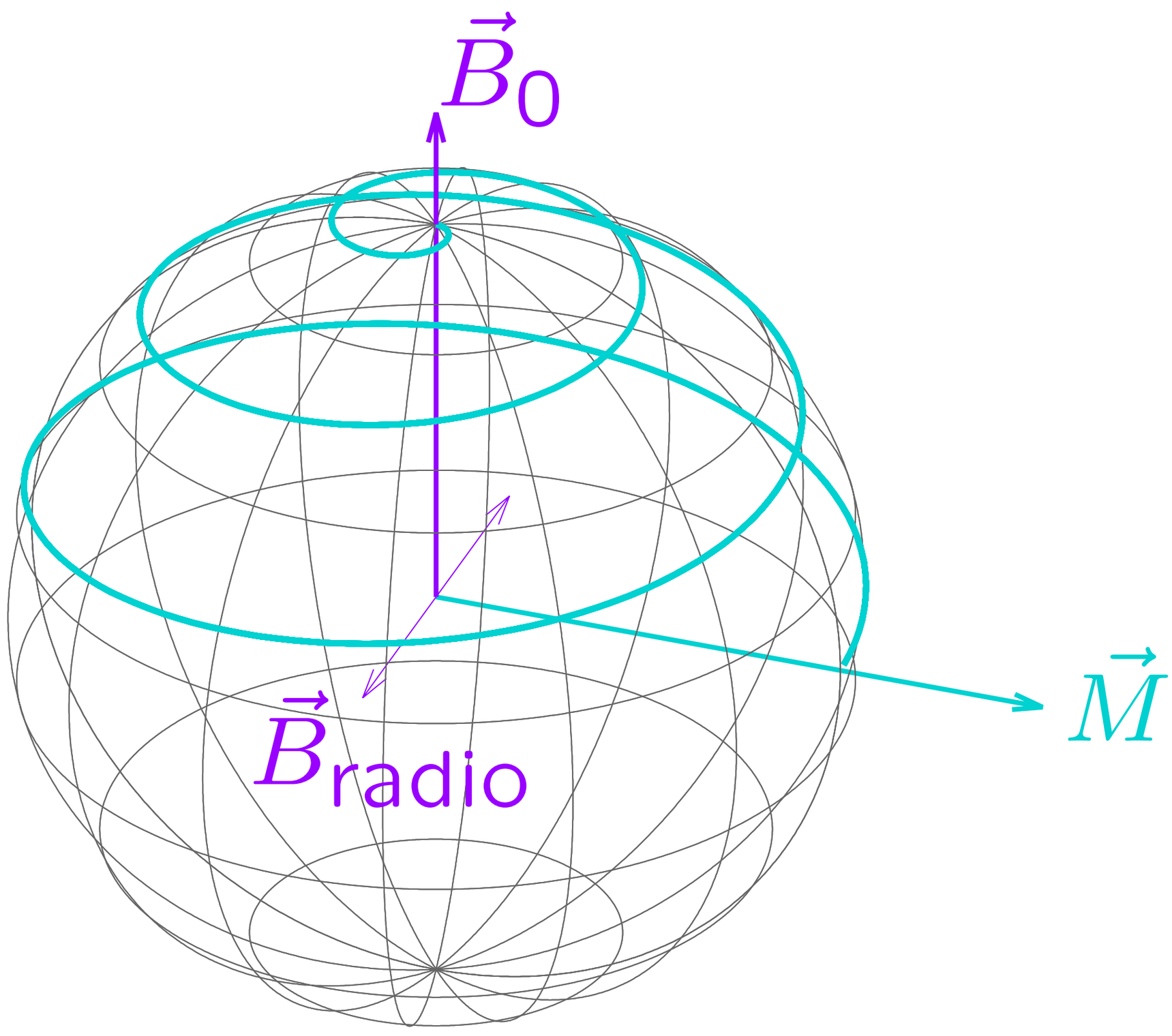


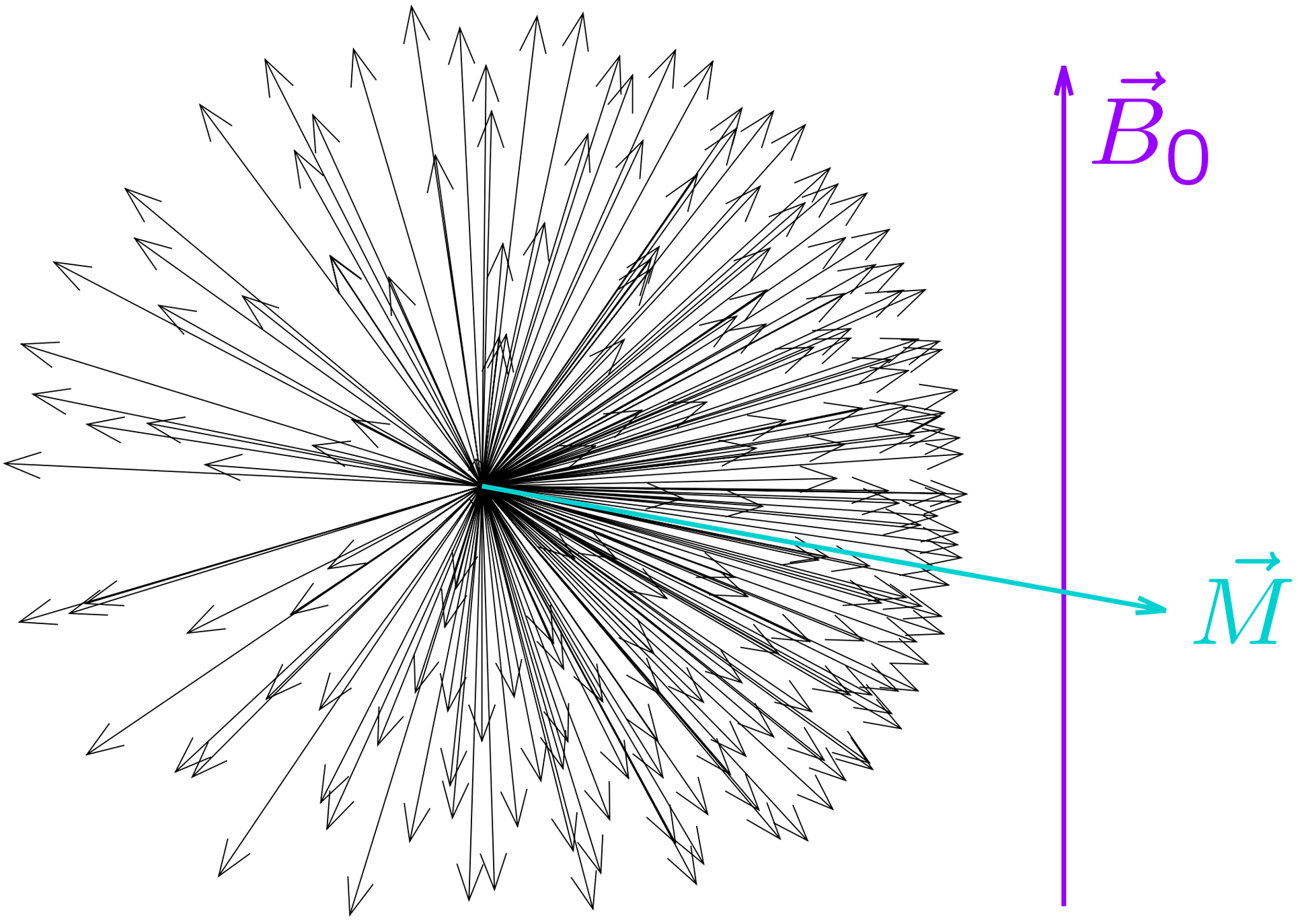

$\vec{M}$
$\vec{B}_0$

Longitudinal polarization

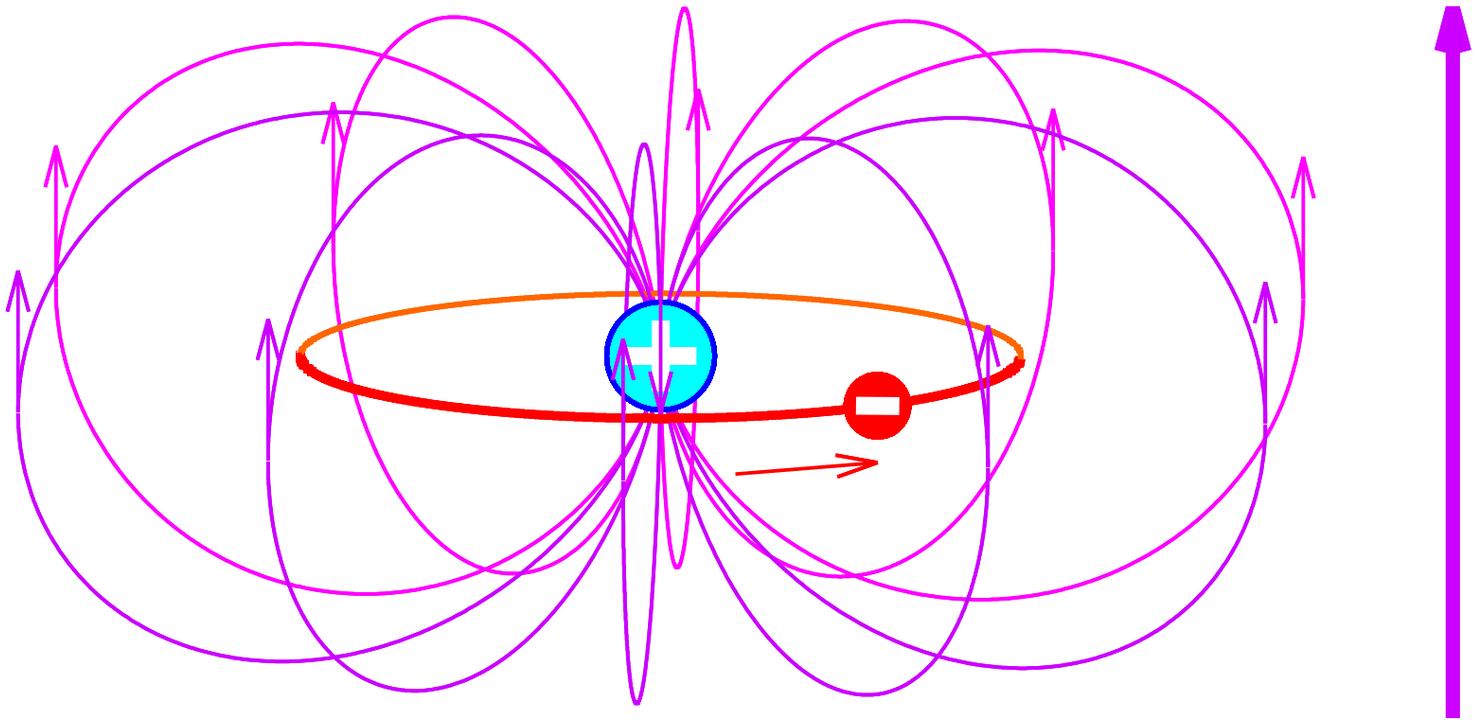

$\vec{M}$
$\vec{B}_0$

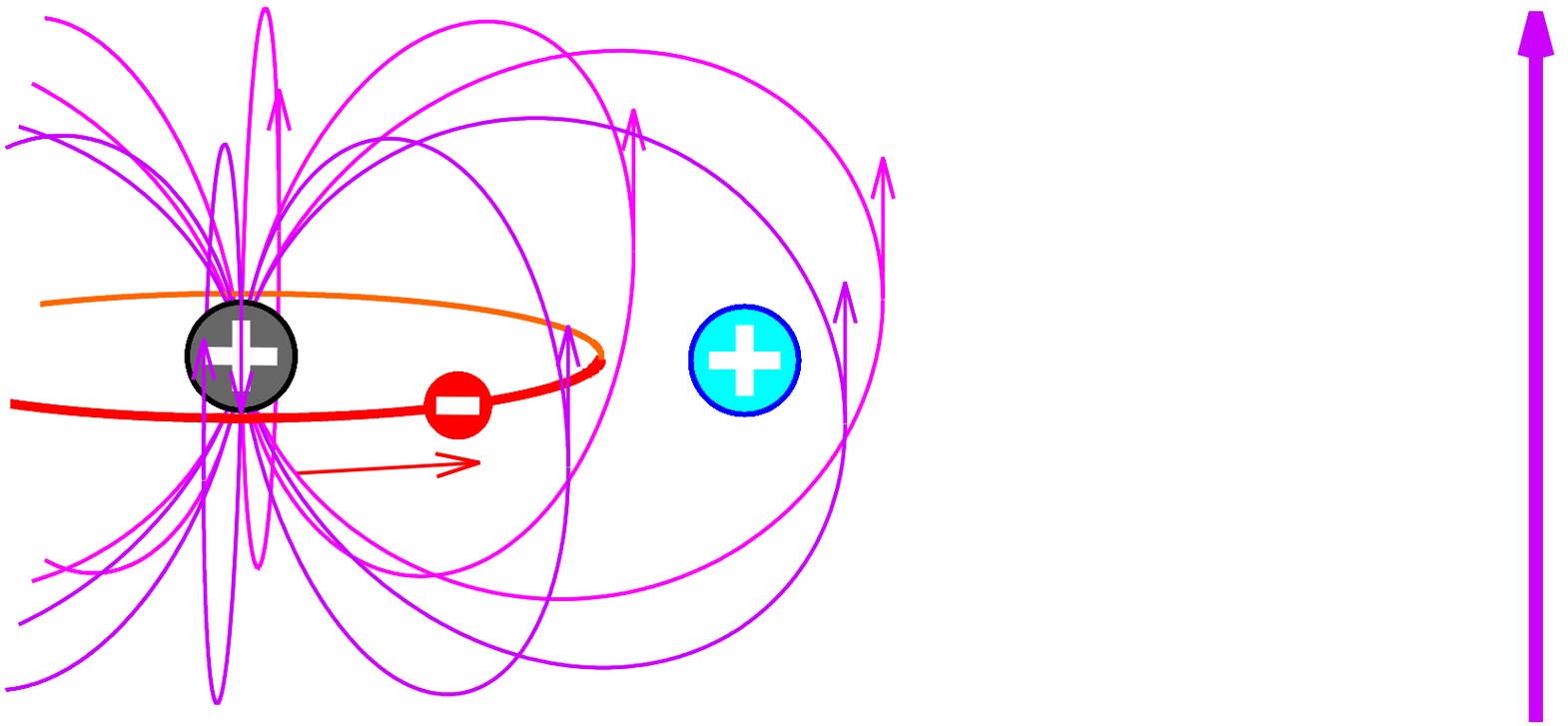
Flipping magnetization

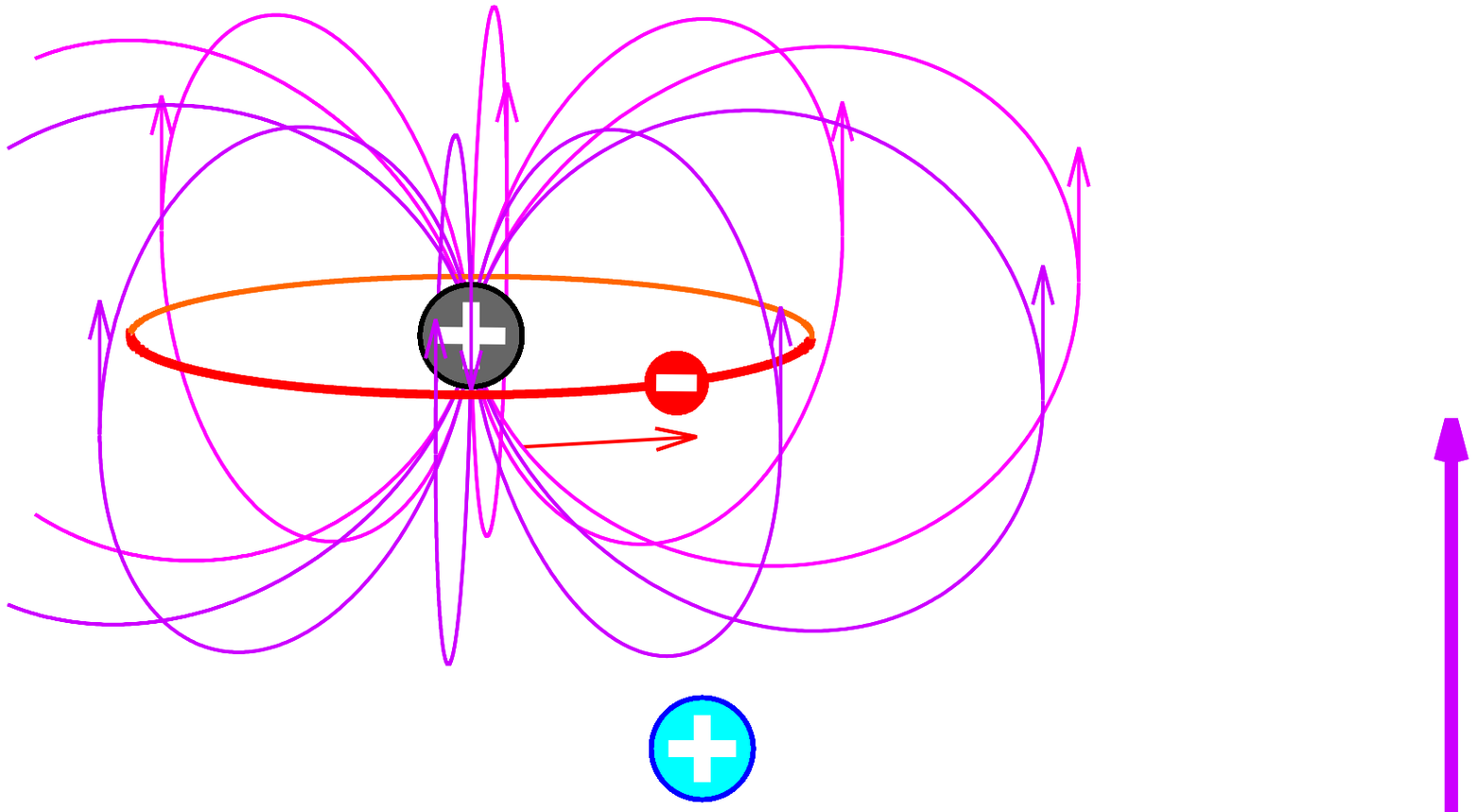

$\vec{B}_0$
$\vec{B}_{\text{radio}}$
$\vec{M}$


$\vec{B}_0$
$\vec{M}$

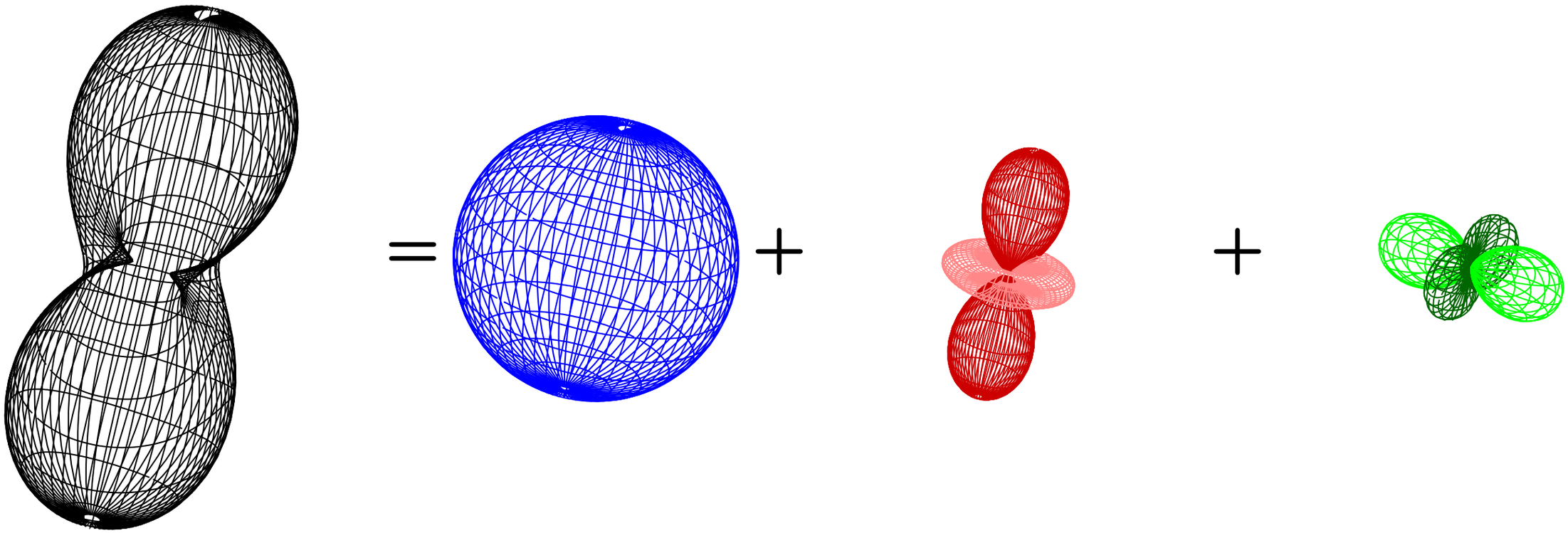
Transverse polarization







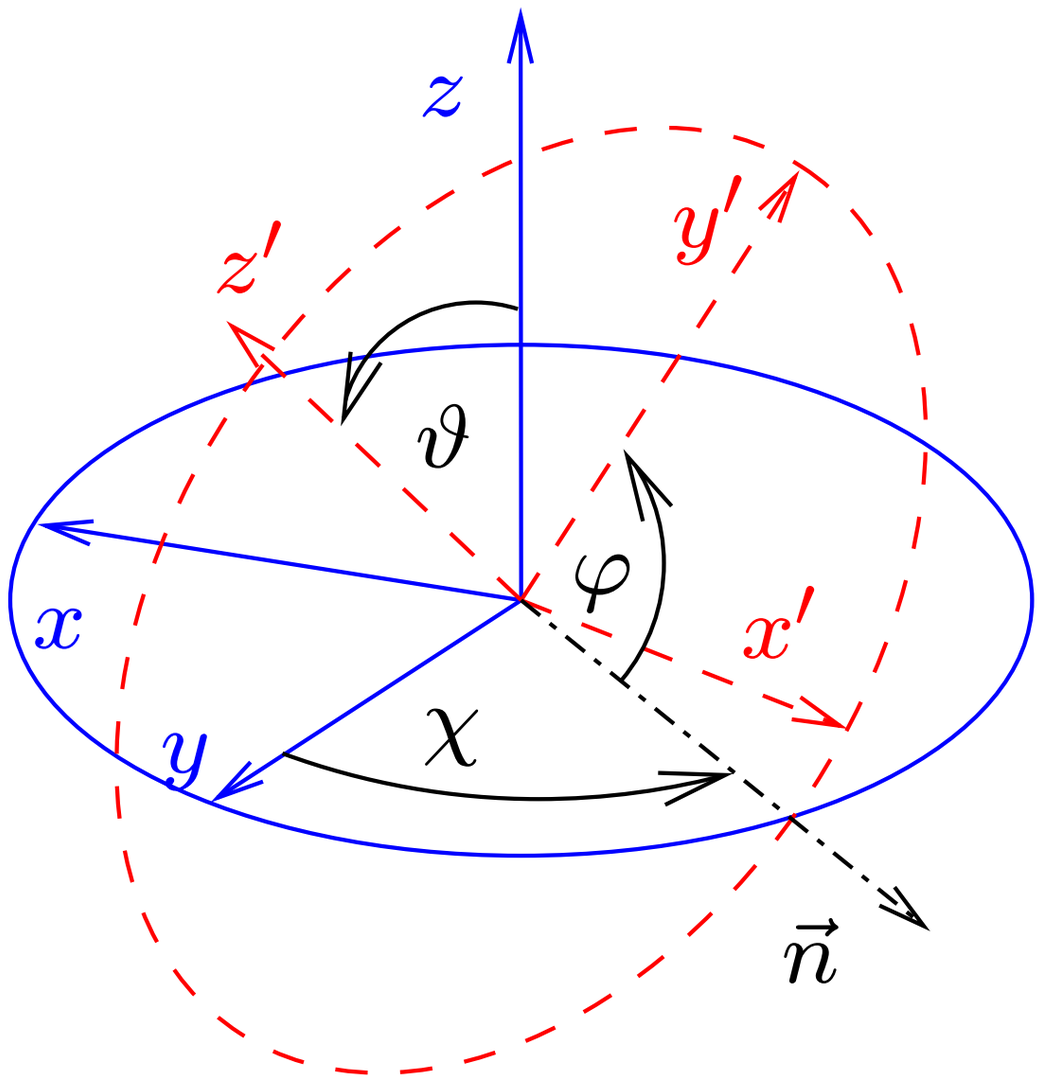
$$\begin{pmatrix} \delta_{XX} & 0 & 0 \\ 0 & \delta_{YY} & 0 \\ 0 & 0 & \delta_{ZZ} \end{pmatrix} = \delta_{\mathsf{i}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \delta_{\mathsf{a}} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \delta_{\mathsf{r}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$$\delta_{\mathsf{i}} = \tfrac{1}{3}\mathsf{Tr}\{\underline{\delta}\} = \tfrac{1}{3}(\delta_{XX} + \delta_{YY} + \delta_{ZZ})$$

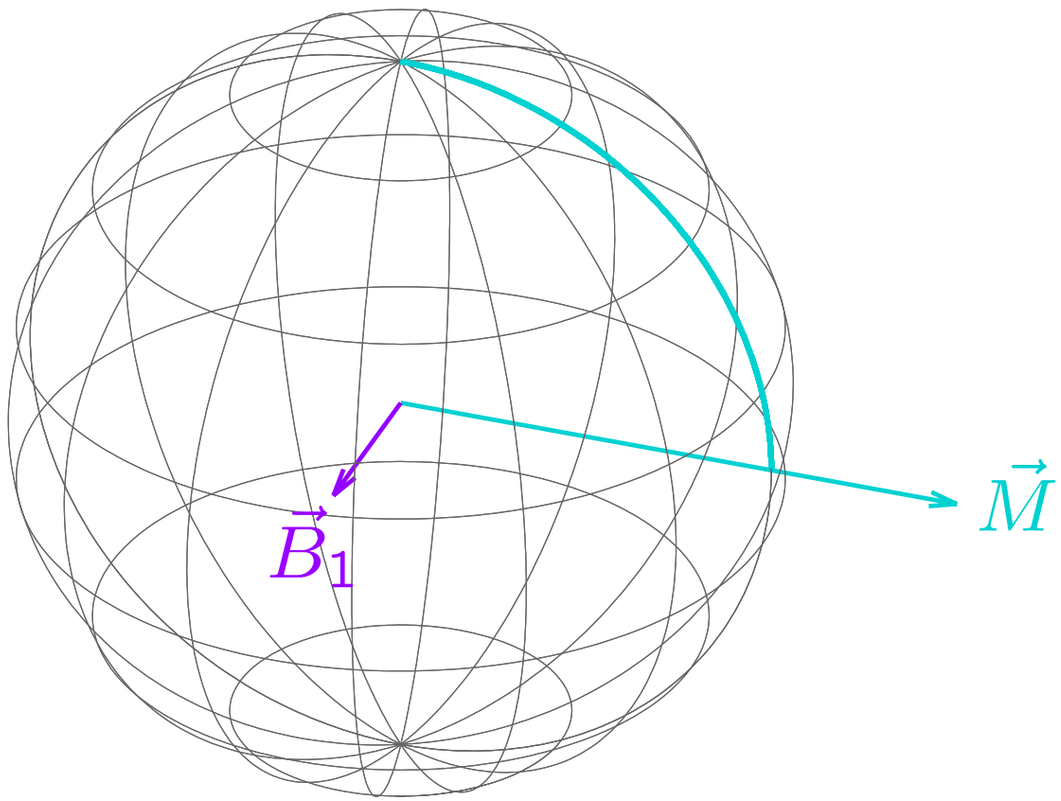
$$\delta_{\mathsf{a}} = \tfrac{1}{3}\Delta_{\delta} = \tfrac{1}{6}\left(2\delta_{ZZ} - (\delta_{XX} + \delta_{YY})\right)$$

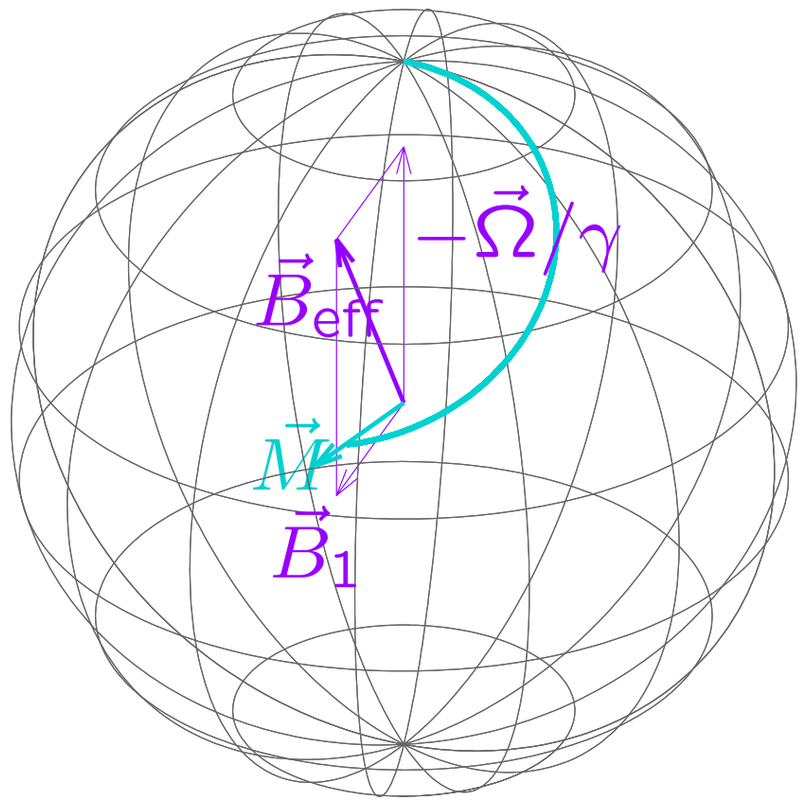
$$\delta_{\mathsf{r}} = \tfrac{1}{3}\eta_{\delta}\Delta_{\delta} = \tfrac{1}{2}(\delta_{XX} - \delta_{YY})$$

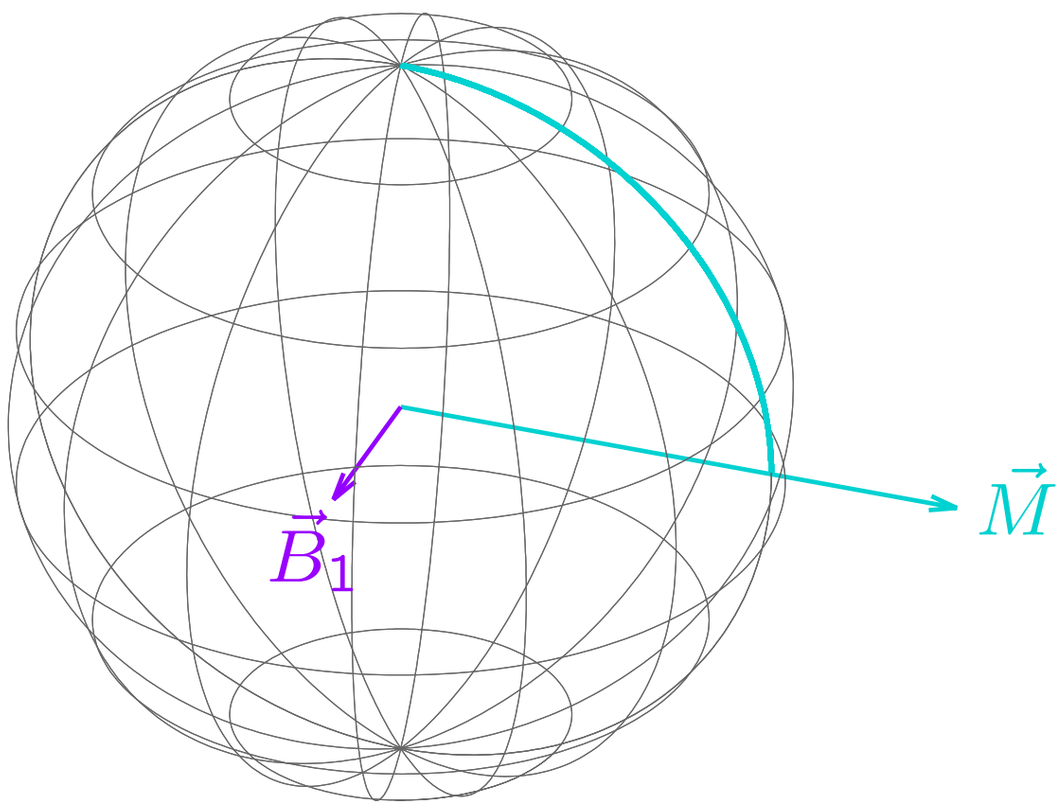


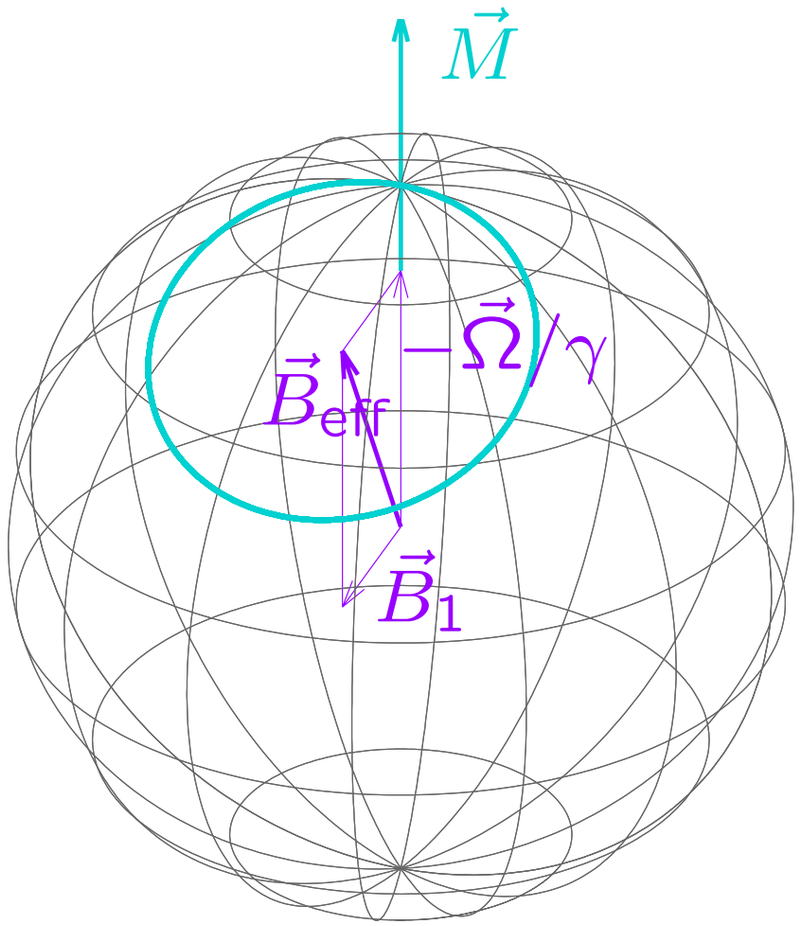
$$\vec{B}_{\mathrm{e}} = \delta_{\mathrm{i}} B_0 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \delta_{\mathrm{a}} B_0 \begin{pmatrix} 3\sin\vartheta\cos\vartheta\cos\varphi \\ 3\sin\vartheta\cos\vartheta\sin\varphi \\ 3\cos^2\vartheta - 1 \end{pmatrix}$$

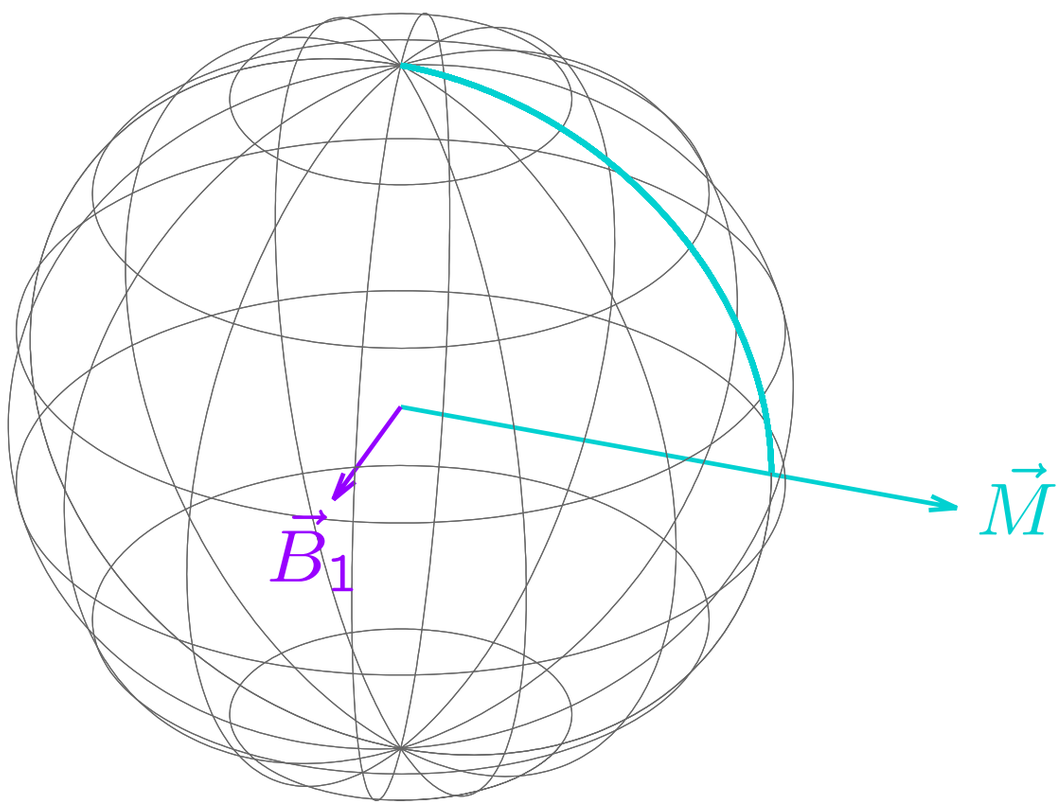
$$+ \delta_{\mathrm{r}} B_0 \begin{pmatrix} -(2\cos^2\chi - 1)\sin\vartheta\cos\vartheta\cos\varphi + 2\sin\chi\cos\chi\sin\vartheta\sin\varphi \\ -(2\cos^2\chi - 1)\sin\vartheta\cos\vartheta\sin\varphi - 2\sin\chi\cos\chi\sin\vartheta\cos\varphi \\ +(2\cos^2\chi - 1)\sin^2\vartheta \end{pmatrix}$$

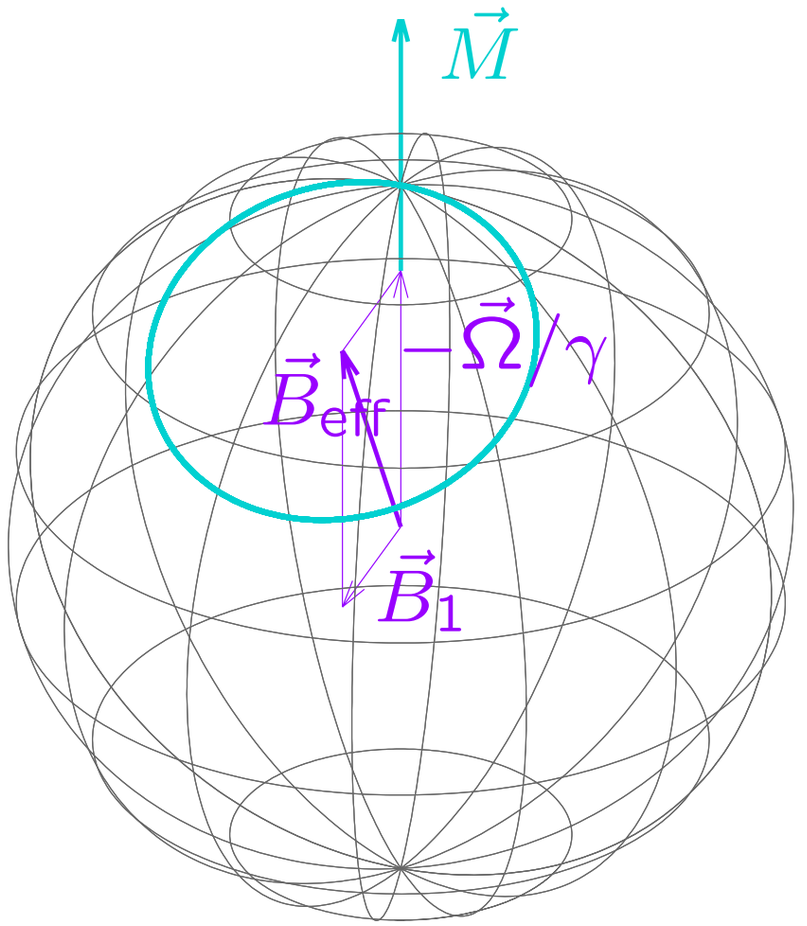

$\vec{B}_1$
$\vec{M}$


$\vec{B}_{\text{eff}}$
$-\vec{\Omega}/\gamma$
$\vec{M}$
$\vec{B}_1$


$\vec{B}_1$
$\vec{M}$


$\vec{M}$
$\vec{B}_{\text{eff}}$
$-\vec{\Omega}/\gamma$
$\vec{B}_1$


$\vec{B}_1$
$\vec{M}$


$\vec{M}$
$\vec{B}_{\text{eff}}$
$-\vec{\Omega}/\gamma$
$\vec{B}_1$