

Homework problems #3

1. *Computer problem:* Verify the Maxwell-Boltzmann distribution of a velocity vector of magnitude v of ideal gas particles using a random number generator. Calculate the result for $N = 10^3, 10^6$ particles. Express the velocities relatively to $\sqrt{2kT/m}$. Hint: Let us assume that one has a function that generates random numbers in an interval $x \in [0, 1]$ with uniform distribution. What transformation shall be performed to get a random variable from interval $y \in [-1, 1]$? Using a computer experiment and histogram verify that a function $\text{erf}^{-1}(y)$ gives a Gaussian distribution of random variable, where erf^{-1} is an inverse function to the error function $\text{erf}(y) = 2/\sqrt{\pi} \int_0^y \exp(-t^2) dt$. Maxwell-Boltzmann distribution of a magnitude of velocity vector can be derived by combining three random variables with Gaussian distribution
2. A system with two energy levels E_0 and E_1 is populated by N indistinguishable particles in distinguishable states at temperature T . Assuming canonical distribution determine
 - (a) mean energy per particle,
 - (b) limit of mean energy for temperatures $T \rightarrow 0$ and $T \rightarrow \infty$,
 - (c) heat capacity of the system,
 - (d) limit of the heat capacity of the system for $T \rightarrow 0$ and $T \rightarrow \infty$.
3. As a result of entanglement of rotational and vibrational movement of a diatomic molecule, angular momentum depends partially on the vibrational state. In such a case, the rotational-vibrational spectrum can be approximated by

$$E_{n,l} = \hbar\omega \left(n + \frac{1}{2} \right) + \frac{\hbar^2}{2I} l(l+1) + \alpha l(l+1) \left(n + \frac{1}{2} \right), \quad (1)$$

where first two terms correspond to the vibrational and rotational movement and the last term is a small correction due to the entanglement of rotational and vibrational movement. The constant satisfy

$$\hbar\omega \gg \frac{\hbar^2}{2I} \gg \alpha. \quad (2)$$

Determine energy of an ideal gas composed of diatomic molecules for temperature

$$\hbar\omega \gg kT \gg \frac{\hbar^2}{2I}.$$

The solution should be submitted not later than on April 30th.