Homework problems #4

1. *Computer problem*: Approximate the value of Riemann function $\zeta(3/2)$ a) using a numerical integration of
$$\zeta(n) = \frac{1}{\Gamma(n)} \int_0^\infty \frac{x^{n-1}}{e^x - 1}\,\mathrm{d}x,$$
b) by calculating sum of
$$\zeta(n) = \sum_{k=1}^{\infty} \frac{1}{k^n}.$$

2. From the Landau potential of extremely relativistic bosonic gas
$$\Omega = -\frac{8\pi g V}{(2\pi\hbar)^3} \frac{(k_\mathrm{B}T)^4}{c^3} B_4\left(\frac{\mu}{k_\mathrm{B}T}\right) \tag{1}$$
determine the number of particles, the entropy, and energy of the gas. In the limit of very high temperatures, determine the specific heat $c_{V,N}$ and the state equation $p = p(N,V,T)$.

3. Let us consider ideal Fermi-Dirack gas with particle energy proportional to the momentum via $\varepsilon \propto p^s$. The gas is closed in a box with energy V in n dimensional space. Show that the pressure P is
$$PV = \frac{s}{n}E, \tag{2}$$
and that the adiabatic equation (S and N is constant) is
$$PV^{1+\frac{s}{n}} = \text{const.} \tag{3}$$
Show that for $T \to \infty$ the heat capacity becomes
$$c_{V,N} = \frac{n}{s}N. \tag{4}$$

4. Let us assume that our Universe is a spherical cavity with radius 10^{28} cm in thermal equilibrium and opaque walls.
   (a) If the cavity temperature is 3 K, estimate the total number of photons and their energy in the cavity.
   (b) If the temperature of the cacity is 0 K and the Universe contains 10^{80} electrons, estimate the Fermi momentum of these electrons.

The solution should be submitted not later than on May 14th.