

Homework problems #5

1. *Computer problem:* a) Make a function that calculates numerically an integral

$$F_{3/2}(y) = \frac{1}{\Gamma(3/2)} \int_0^\infty \frac{x^{1/2}}{e^{x-y} + 1} dx.$$

Calculate $F_{3/2}(0)$. b) Using an appropriate numerical method find $\tilde{\mu}_x$ that fulfills

$$F_{3/2}(\tilde{\mu}_x) = x \equiv \frac{N\lambda_T^3}{gV}$$

for a given x . Calculate $\tilde{\mu}_{80}$, $\tilde{\mu}_{10}$ and $\tilde{\mu}_{0.1}$. c) Plot a graph of function

$$N(\varepsilon, \tilde{\mu}_x) = \frac{1}{e^{\varepsilon - \tilde{\mu}_x} + 1}$$

for determined values $\tilde{\mu}_{80}$, $\tilde{\mu}_{10}$, and $\tilde{\mu}_{0.1}$.

2. Evaluate the mean value of free particle hamiltonian from quantum mechanical partition function

$$Z = \frac{V}{\lambda_T^3}. \quad (1)$$

Calculate in momentum representation.

3. Density matrix of left-handed and right-handed polarized light in a basis of vectors of linearly polarized light is

$$\hat{\rho}_L = \begin{pmatrix} 1/2 & i/2 \\ -i/2 & 1/2 \end{pmatrix}, \quad \hat{\rho}_R = \begin{pmatrix} 1/2 & -i/2 \\ i/2 & 1/2 \end{pmatrix}. \quad (2)$$

Using $\hat{\rho}_L$ and $\hat{\rho}_R$ determine the density matrix of unpolarized light $\hat{\rho}_n$ and calculate $\hat{\rho}_L^2$, $\hat{\rho}_R^2$ and $\hat{\rho}_n^2$. Which matrices correspond to a pure state?

4. Density matrix of a harmonic oscillator with a hamiltonian

$$\hat{H} = \frac{1}{2m} \hat{p}^2 + \frac{1}{2} m \omega^2 \hat{x}^2$$

has in a position space form of

$$\rho(x, x', T) = \sqrt{\frac{m\omega}{\pi\hbar} \tanh\left(\frac{\hbar\omega}{2kT}\right)} \exp\left\{-\frac{m\omega}{2\hbar \sinh\left(\frac{\hbar\omega}{kT}\right)} \left[(x^2 + x'^2) \cosh\left(\frac{\hbar\omega}{kT}\right) - 2xx' \right]\right\}.$$

(a) Calculate the mean value of energy $E = \langle \hat{H} \rangle$.

(b) Show that for $T \rightarrow \infty$ the equipartition theorem holds for the mean value of energy.

5. Approximately calculate the heat capacity at the constant volume for a gas with interatomic potential $U(r)$ (the unknown integral can be denoted appropriately). Particles can be considered as point masses.

6. The approximate solution of the Boltzmann equation in a presence of temperature gradient $T = T_0 + \alpha y$ is

$$f = f_0 + \alpha \tau v_y \frac{T_0}{2(T_0 - \alpha y)^{7/2}} \left(\frac{p^2}{mk(T_0 - \alpha y)} - 5 \right) n_0 \left(\frac{2\pi\hbar^2}{mk} \right)^{3/2} e^{-\frac{p^2}{2mk(T_0 - \alpha y)}}, \quad (3)$$

where f_0 is equilibrium velocity distribution. Calculate a mean value of momentum flux $\langle mv_y |v_y| \rangle$; in a velocity distribution you can approximate $T_0 - \alpha y \approx T_0$. The nonzero momentum flux causes, for example, the motion of a light mill.

The solution should be submitted not later than on May 27th.