I

$$A(\omega) = \frac{1}{Z} \sum_{m,n} e^{-\beta E_n} \langle n|\hat{c}|m\rangle\langle m|\hat{c}^+|n\rangle \, \delta(\omega - E_m + E_n)$$

$$B(\omega) = \frac{1}{Z} \sum_{m,n} e^{-\beta E_n} \langle n|\hat{c}^+|m\rangle\langle m|\hat{c}|n\rangle \, \delta(\omega + E_m - E_n)$$

$$C(\omega) = A(\omega) + B(\omega)$$

---

1. $\int d\omega \, C(\omega) = 1$

$$\int d\omega \, A(\omega) = \frac{1}{Z} \sum_{m,n} e^{-\beta E_n} \langle n|\hat{c}|m\rangle \langle m|\hat{c}^+|n\rangle =$$

$$= \frac{1}{Z} \sum_{n} e^{-\beta E_n} \langle n|\hat{c}\hat{c}^+|n\rangle$$

$$\int d\omega \, B(\omega) = \frac{1}{Z} \sum_{n} e^{-\beta E_n} \langle n|\hat{c}^+\hat{c}|n\rangle$$

$$\int d\omega \, C(\omega) = \frac{1}{Z} \sum_{n} e^{-\beta E_n} = 1$$

Fermi function

2. $\langle \hat{n} \rangle = \int d\omega \, C(\omega) \, f(\omega)$

side remark

$\int d\omega \, B(\omega) = \langle \hat{n} \rangle$, but usually we know $C(\omega)$ not $A(\omega)$ and $B(\omega)$ separately

$$\int d\omega\, A(\omega) f(\omega) = \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} \langle n|\hat{c}|m\rangle\langle m|\hat{c}^+|n\rangle \frac{1}{1+e^{\beta(E_m-E_n)}}$$

$$\delta(\omega \pm E_m + E_n) \Rightarrow \quad \omega = E_m - E_n$$

$$= \frac{1}{Z} \sum_{n,m} e^{-\beta E_m} \langle n|\hat{c}^+|m\rangle\langle m|\hat{c}|n\rangle \frac{1}{1+e^{\beta(E_n-E_m)}}$$

$$\int d\omega\, B(\omega) f(\omega) = \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} \langle n|\hat{c}^+|m\rangle\langle m|\hat{c}|n\rangle \frac{1}{1+e^{\beta(E_n-E_m)}}$$

$$\int d\omega\, (A(\omega)+B(\omega)) f(\omega) = \frac{1}{Z} \sum_{n,m} \langle n|\hat{c}^+|m\rangle\langle m|\hat{c}|n\rangle \frac{e^{-\beta E_n} + e^{-\beta E_m}}{1+e^{\beta(E_n-E_m)}} =$$

$$= \frac{1}{Z} \sum_{n,m} \langle n|\hat{c}^+|m\rangle\langle m|\hat{c}|n\rangle e^{-\beta E_n} \frac{1+e^{\beta(E_n-E_m)}}{1+e^{\beta(E_n-E_m)}}$$

$$= \frac{1}{Z} \sum_{n} \langle n|\hat{c}^+\hat{c}|n\rangle e^{-\beta E_n}$$

$$= \langle \hat{n} \rangle$$

(3) $C(\omega)$ in a non-interating sytem

III

$$H = \sum_\alpha \epsilon_\alpha \hat{a}_\alpha^+ \hat{a}_\alpha \qquad \hat{c}_i = \xi_{i\alpha} \hat{a}_\alpha$$

$$A_{ii}(\omega) = \frac{1}{Z} \sum_{n,m} e^{-\beta E_n} \langle n | \hat{c}_i | m \rangle \langle m | \hat{c}_i^+ | n \rangle \delta(\omega - E_m + E_n)$$

$$= \frac{1}{Z} \sum_{n,m} \sum_{\alpha,\beta} e^{-\beta E_n} \xi_{i\alpha} \overline{\xi_{i\beta}} \langle n | \hat{a}_\alpha | m \rangle \langle m | \hat{a}_\beta^+ | n \rangle \delta(\omega - E_m + E_n)$$

$$= \frac{1}{Z} \sum_{n,m,\alpha} e^{-\beta E_n} |\xi_{i\alpha}|^2 \langle n | \hat{a}_\alpha | m \rangle \langle m | \hat{a}_\alpha^+ | n \rangle \delta(\omega - E_m + E_n)$$

$$= \frac{1}{Z} \sum_{n,\alpha} e^{-\beta E_n} |\xi_{i\alpha}|^2 \delta(\omega - \epsilon_\alpha)$$

orbital α is empty

$$B_{ii}(\omega) = \frac{1}{Z} \sum_{n,m,\alpha} e^{-\beta E_n} |\xi_{i\alpha}|^2 \langle n | \hat{a}_\alpha^+ | m \rangle \langle m | \hat{a}_\alpha | n \rangle \delta(\omega + E_m - E_n)$$

$$= \frac{1}{Z} \sum_{n,m,\alpha} e^{-\beta E_m} |\xi_{i\alpha}|^2 \langle n | \hat{a}_\alpha | m \rangle \langle m | \hat{a}_\alpha^+ | n \rangle \delta(\omega - E_m + E_n)$$

$$= \frac{1}{Z} \sum_{m,\alpha} e^{-\beta E_m} |\xi_{i\alpha}|^2 \delta(\omega - \epsilon_\alpha)$$

orbital α is full

$$A_{ii}(\omega) + B_{ii}(\omega) = \Big(\frac{1}{Z} \sum_n e^{-\beta E_n} \Big) \sum_\alpha |\xi_{i\alpha}|^2 \delta(\omega - \epsilon_\alpha) =$$

all states

$$= \sum_\alpha |\xi_{i\alpha}|^2 \delta(\omega - \epsilon_\alpha)$$