

MUNI  
SCI

# Plasma Physics 2

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# Introduction and context (CZ)

## Předcházející povinné a volitelné kurzy

- F5170 Úvod do fyziky plazmatu / dr. Z. Bonaventura  
(definice plazmatu, pohyb částic v el-mag poli, Transportní rovnice v plazmatu)
- F7241 Fyzika plazmatu 1 / doc. Zajíčková  
(rozdělovací funkce, teorie stěnové vrstvy)
- F3180 Výboje v plynech / prof. Cernák + Dr. Krumpolec  
(klasifikace výbojů a teorie formování jednotlivých výbojů)
- F4280 Technologie depozice tenkých vrstev a povrchových úprav / prof. Vašina + doc. Zajíčková  
(plazmochemie, plazmové zdroje pro PVD a PECVD)

# Course requirements

- Understanding the presented topics, especially:
  - Definitions of different discharge modes and types.
  - Transition mechanisms between discharge modes.
  - Conditions and properties of different discharge types.
  - Analytical calculations, estimating plasma properties at various conditions.
  - ... (see details below)
- Oral exam

Individual consultations available:

- Adam Obrusník
- Tomáš Hoder

# Literature

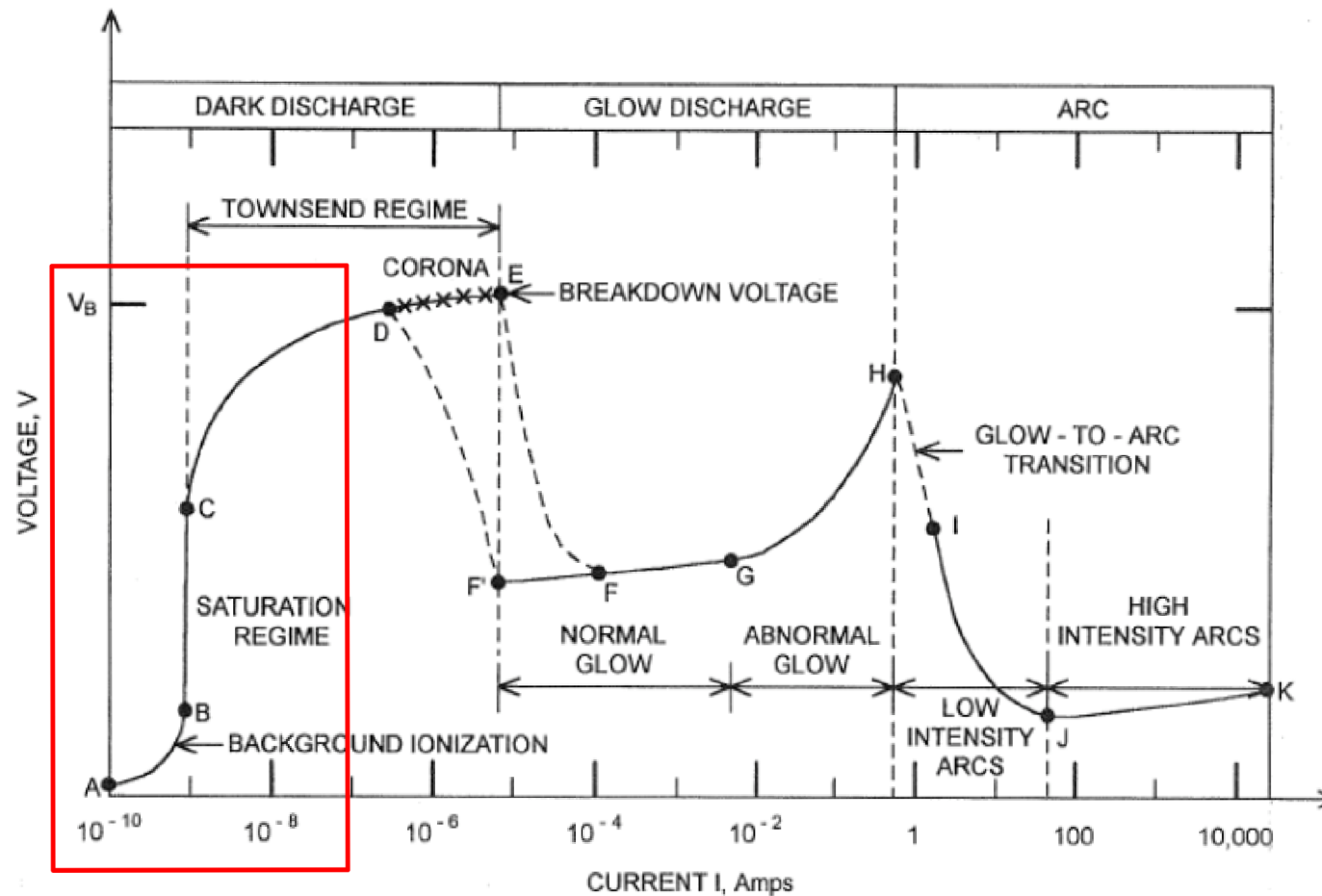
- RAIZER, Y.P. Gas discharge physics, Springer, 1991
- COBINE, J.D. Gaseous conductors, Dover Publications, New York, 1958
- CHEN, Francis F. *Introduction to plasma physics and controlled fusion*. 2nd ed. New York: Plenum Press, 1984. xv, 421. ISBN 0306413329.
- BITTENCOURT, J. A. *Fundamentals of plasma physics*. 3rd ed. Sao José dos Campos: National Institute for Space Research, 2003. xxiii, 678. ISBN 85-900100-3-1.
- BAZELYAN E.M., Raizer Y.P. Spark discharge, CRC Press, Taylor&Francis, 1998



# Lecture series contents

1. Townsend breakdown theory, Paschen's law
2. Glow discharge
3. Electric arc at low and high pressures
4. Magnetized low-pressure plasmas and their role in material deposition methods.
5. Brief introduction to high-frequency discharges
6. Streamer breakdown theory, corona discharge, spark discharge
7. Barrier discharges
8. Leader discharge mechanism, ionization and discharges in planetary atmospheres
9. Discharges in liquids, complex and quantum plasmas
10. Thermonuclear fusion, Lawson criterion, magnetic confinement systems, plasma heating and inertial confinement fusion.

# Discharges – what this Lesson covers?



# Electron avalanche, Townsend criterion, discharge breakdown

# Electron avalanche

- At the turn of 19th and 20th century: Townsend and Paschen formulate the basics of gaseous discharges / gas discharges
- 1930s: Experimental observation of electron avalanches in vapor chambers and later in vacuum chambers
- Luminous structures between the cathode and the anode – more luminous for higher voltages.

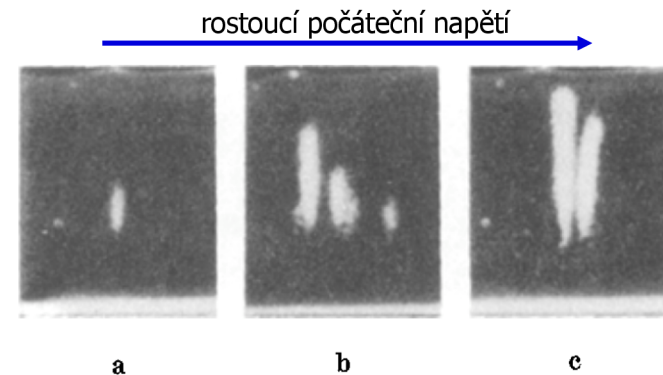
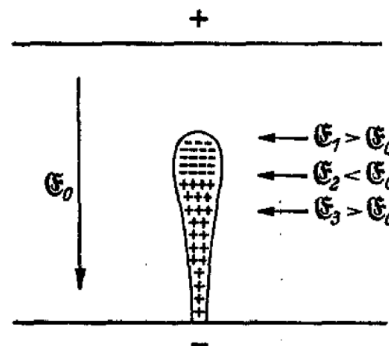


John S. Townsend



Friedrich Paschen

**Zeitschrift für Physik.**  
**Die Entwicklung der Elektronenlawine in den Funkenkanal.**  
 (Nach Beobachtungen in der Nebelkammer.)  
 Von H. Raether in Jena.  
 Mit 8 Abbildungen. (Eingegangen am 28. Februar 1939.)



# Electron avalanche – the principle

**Q:** What energy do you need to ionize an atom?

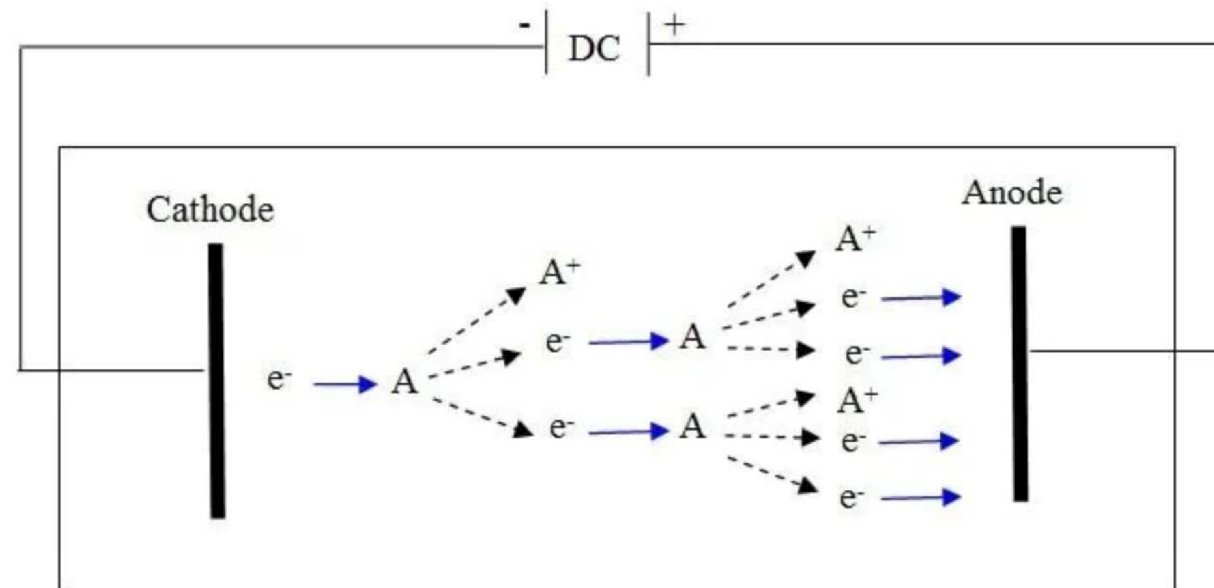
**A:** 10-15 eV for gases, 5-10 eV for metal vapors

**Q:** Is that energy higher or lower than e.g. dissociation of  $\text{CO}_2 \Rightarrow \text{CO} + \text{O}$ ?

**A:** Ionization is higher, chemical bond energy typically 1 – 8 eV

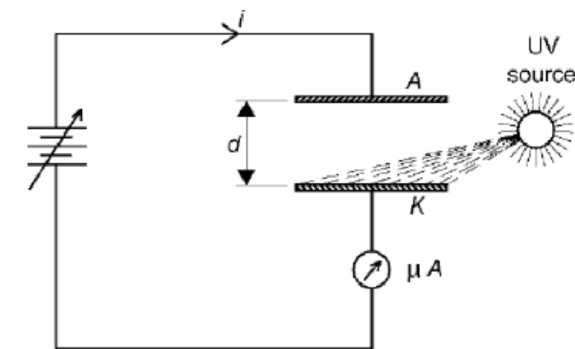
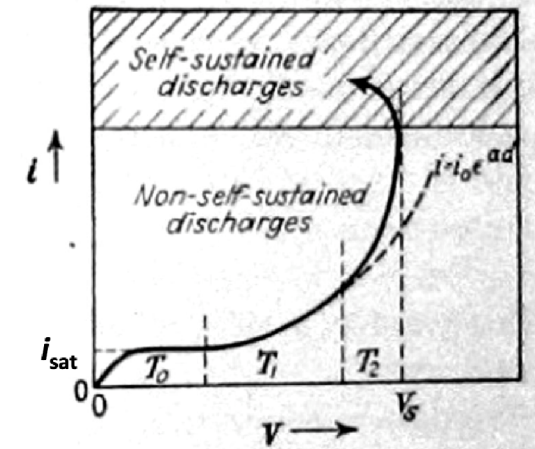
**Q:** Where did the first electron come from?

**A:** There was a “plenty” – background ionization



# Electron avalanche – empirical summary

- **Plane-parallel electrodes with constant E field**
- Volume filled with primary electrons (cosmic radiation, radioactivity or external UV)
- When voltage is applied, charged particles move towards the electrodes where they are lost. Volume recombination can be neglected in the first approximation.
- **Electrons are accelerated to energies over the ionization threshold**
- In such an experiment, one can observe 3 scenarios:
  1. After turning off the external source of ionization, current disappears = non self-sustaining discharge, see range  $T_0$
  2. Further voltage increase leads to ionization and creation of avalanches, external ionization source not needed = Townsend discharge (range  $T_1$ ), described very well by the electron avalanche theory
  3. Further voltage increase leads to range  $T_2$  = additional phenomena (e.g. recombination), no longer described by the simple electron avalanche theory.

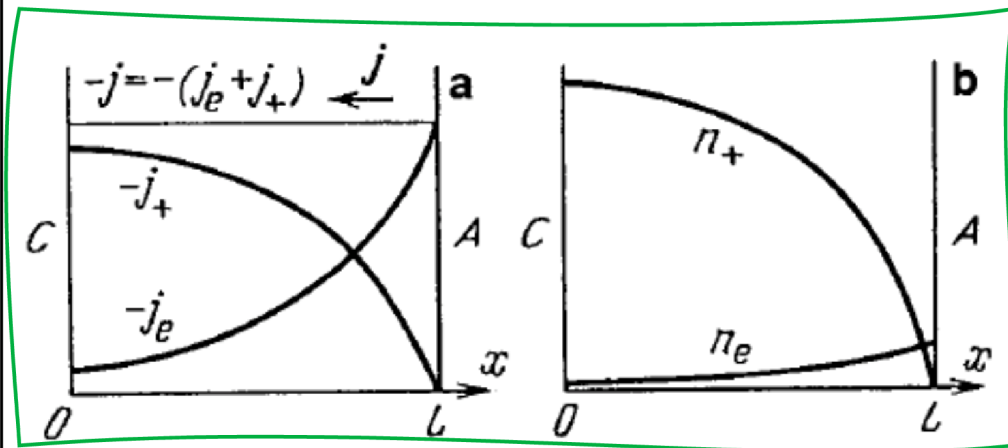


# Important assumption of further slides

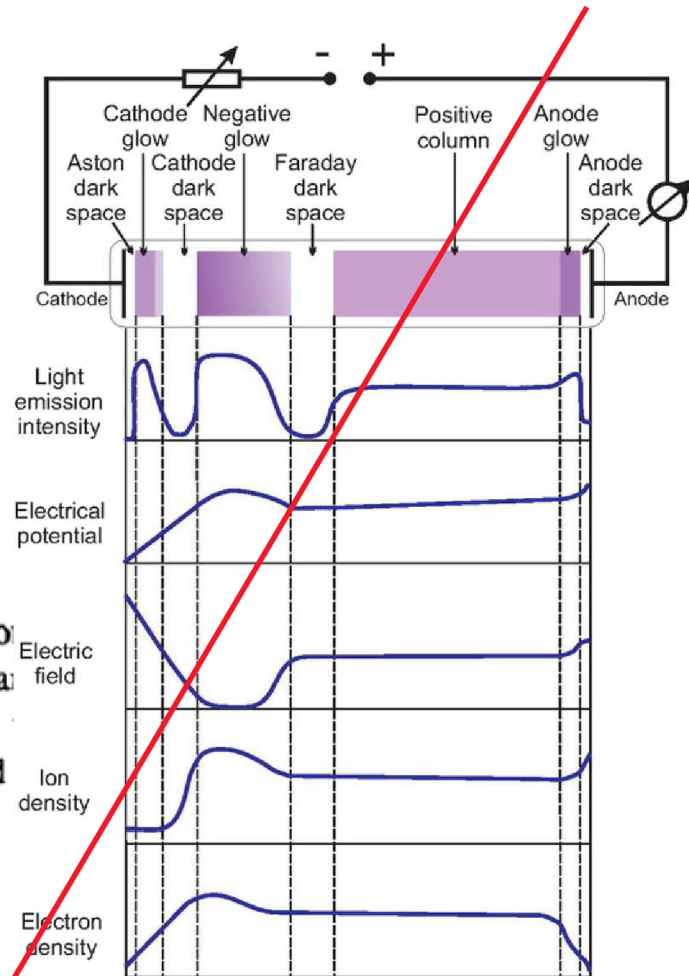
- In the derivations below, we assume **Townsend dark discharge**, i.e. we maintain plasma densities low enough that they do not affect the electric field

$$\Delta V = -\frac{\rho}{\epsilon_0} \approx 0$$

- This holds only before discharge ignition, so the slides below describe how discharges are initiated but not how they operate once they are ignited.



**Fig. 8.6.** Distributions of electro and ionic currents (a), and charge density distribution (b) when field in the gap is not distorted space charge



# The first Townsend coefficient $\alpha$

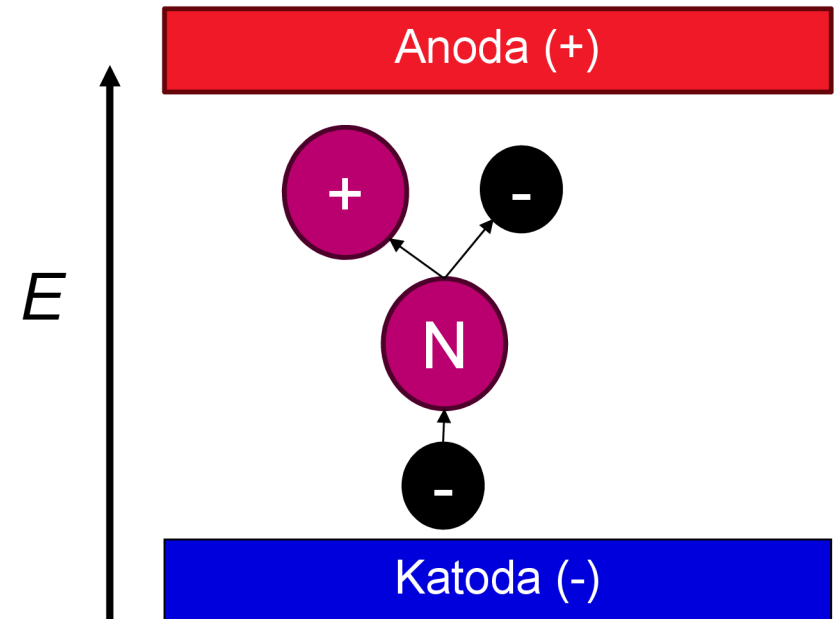
Q: How do we mathematically describe the growth of electron density in an electric field?

$$dn_e = \alpha n_e dx$$

$$n_e = n_{e,0} e^{\alpha x}$$

Q: What does the coefficient depend on?

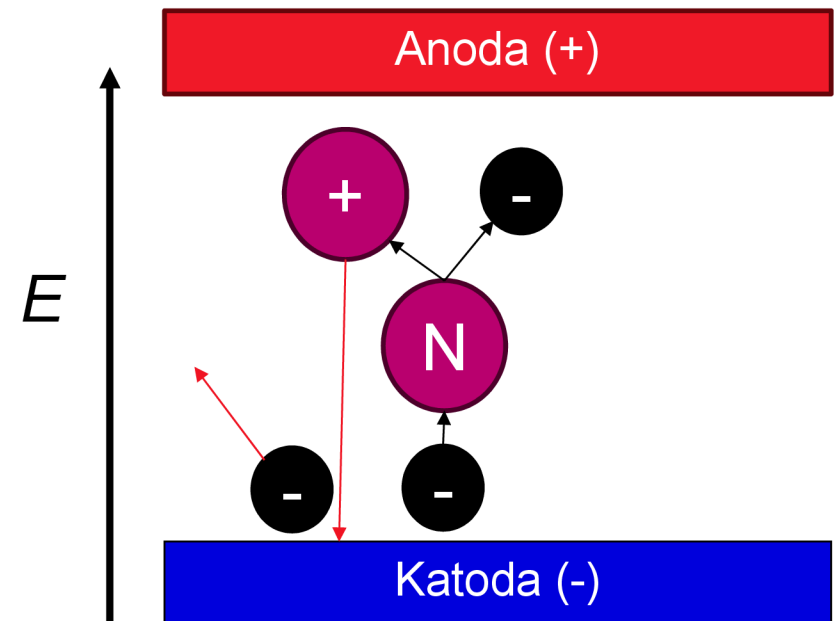
- Type of gas
- Pressure
- E field magnitude
- Energy distribution of electrons





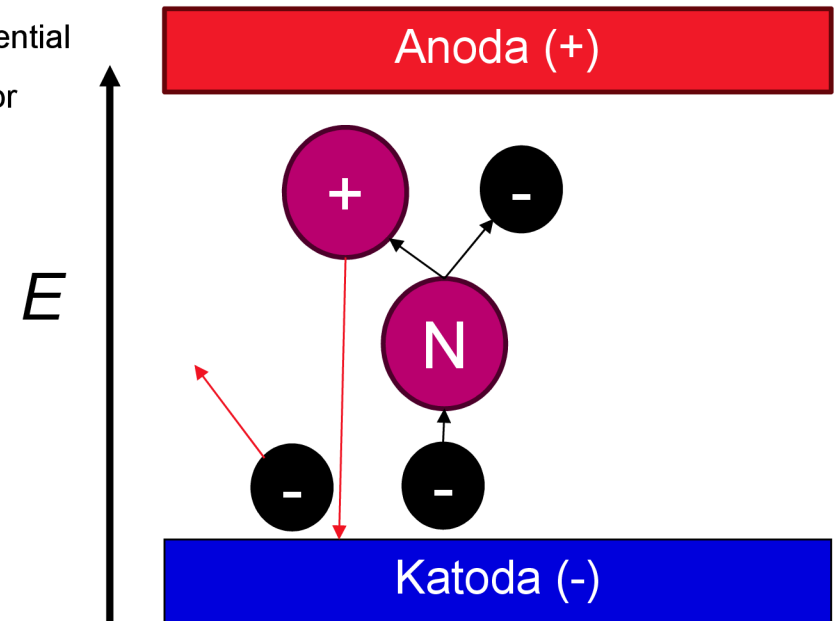
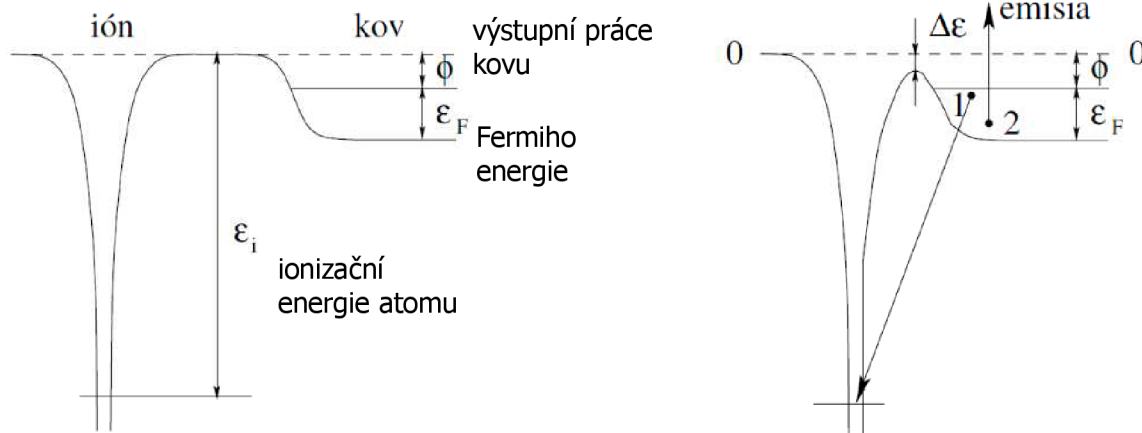
# The third Townsend coefficient $\gamma$

- When an ion hits the surface, so-called secondary electron emission can occur.
- The probability of such effect is described by a factor  $\gamma$  (or ISEE = ion-induced secondary electron emission).
- Values of  $\gamma$ :
  - 0.1 – 0.2 in laboratory plasmas (ion energy 100s of eV)
  - 0.1 – 10 by ion milling (ion energy 1 – 10 keV)



# The third Townsend coefficient $\gamma$

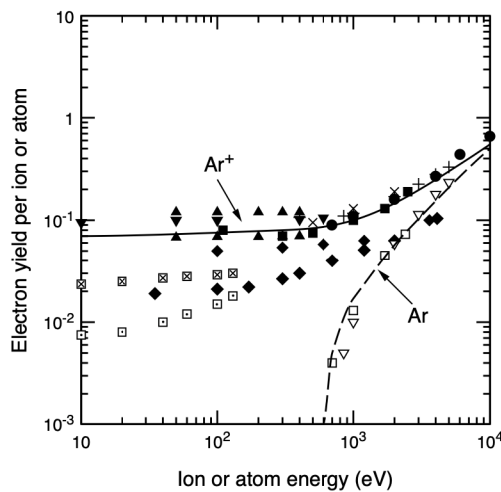
- At voltages below 500 eV, **potential emission is the main mechanism** of ISEE
- Electron no.1 from the conduction band of the metal tunnels through the potential barrier and recombines with the positive ion. The energy gain can be used for releasing electron 2
- Above ca 500 eV, kinetic energy of the ion starts to make a contribution



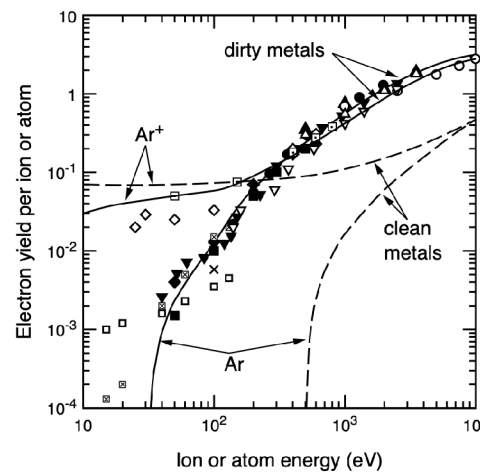
# The third Townsend coefficient $\gamma$

- In “clean metals”, which are very smooth, ISEE depends on the material work function  $W_f$  – ion energy must be at least  $2W_f$  for ISEE to occur.
- In real life, ISEE is **surprisingly quite material-independent** and depends more on surface micro-roughness

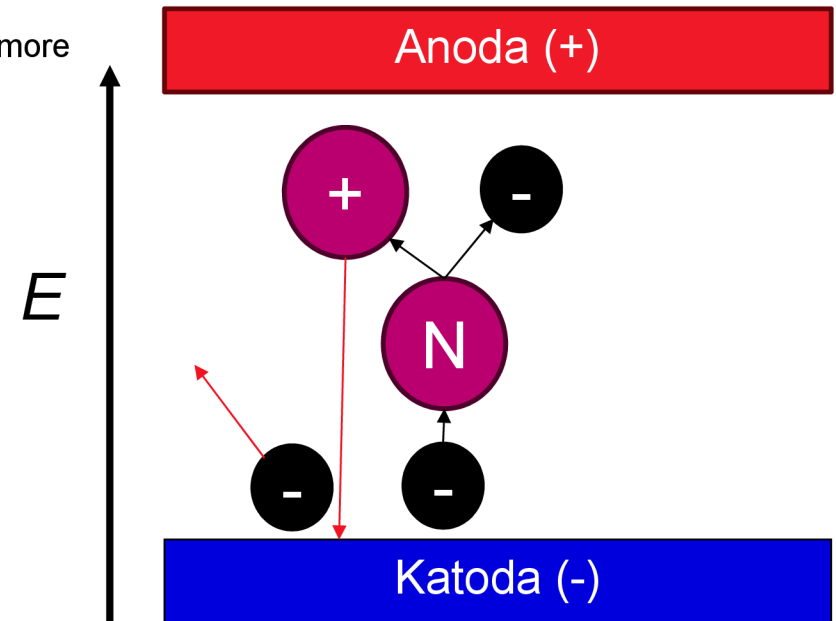
[Phelps, A. V., & Petrovic, Z. L. (1999). PSST, 8(3), R21–R44.doi:10.1088/0963-0252/8/3/201]



**Figure 1.** Electron yields for  $\text{Ar}^+$  and Ar beams incident on various clean metal surfaces versus particle energy. The solid symbols are for  $\text{Ar}^+$  and the open symbols are for Ar. The symbols, metals and references are:  $\nabla$ , W, [68];  $+\nabla$ , Mo, [46];  $\square$ ,  $\blacksquare$ , Mo, [47];  $\blacktriangle$ , Mo, [70];  $\bullet$ , Mo, [66];  $\blacklozenge$ , Au, [71];  $\times$ , Cu, [67];  $\boxtimes$ , Pt, [69] and  $\square$ , Ta, [69]. The curves drawn through representative values will be used in our model.



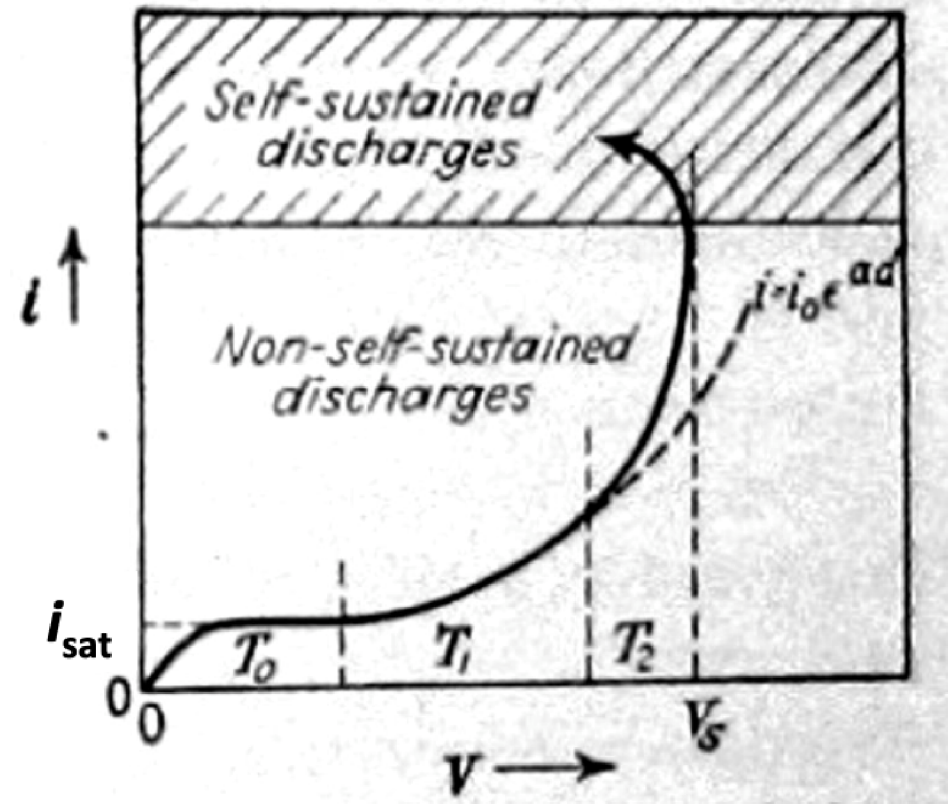
**Figure 2.** Electron yields for  $\text{Ar}^+$  and Ar beams incident on various dirty metal surfaces versus particle energy. The open symbols are for  $\text{Ar}^+$  and the solid symbols are for Ar. The symbols, metals and references are:  $\square$ , Pt, [69];  $\boxtimes$ , Ta, [69];  $\nabla$ ,  $\blacktriangledown$ , Au, [49];  $\circ$ , Cu, [75];  $\diamond$ ,  $\blacklozenge$ , Cu, [45];  $\triangle$ ,  $\blacktriangle$ , Ta, [80];  $\times$ , W, [79];  $\square$ , brass, [81];  $\bullet$ , unknown, [50] and  $\blacksquare$ , CuBe, [51]. The solid curves are plots of the analytical yield expressions for dirty surfaces for  $\text{Ar}^+$  and Ar, while the dashed curves are the representative yield curves for clean surfaces for  $\text{Ar}^+$  ions and Ar atoms from figure 1.



# What about the second coefficient?

- The second Townsend coefficient  $\beta$  was supposed to better describe the region T2, where glow discharge starts to form.
- It never really caught on because it was way too simplistic a treatment.

**As a plasma-physicist, you can sometimes be wrong. The world will still appreciate that you are trying to tackle this beast ☺**



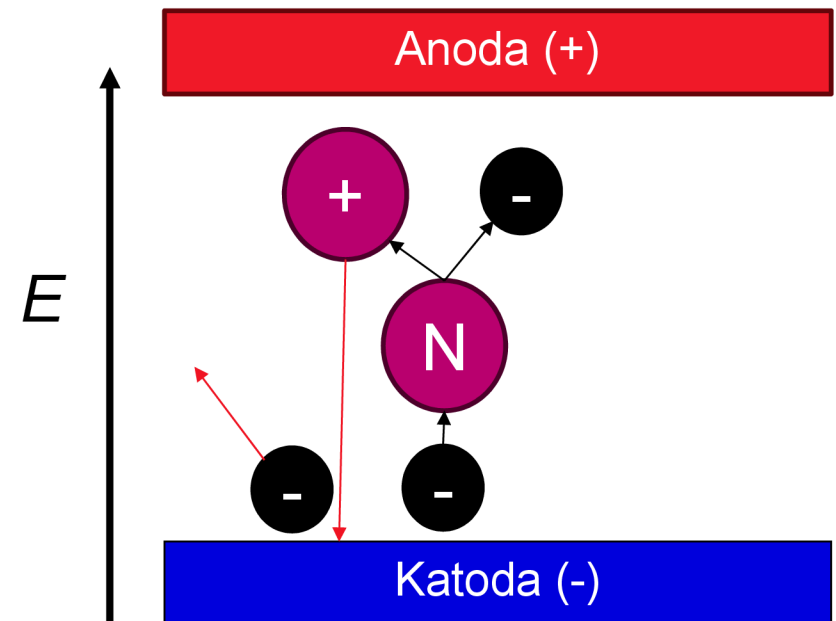
# Discharge breakdown condition

- To define the breakdown condition, we will use the concept of “current density”

$$j_e = q_e n_e \mathbf{v}_{\text{drift},e} = q_e \mu_e n_e \mathbf{E}$$

$$j_i = q_i n_i \mathbf{v}_{\text{drift},i} = q_i \mu_i n_i \mathbf{E}$$

- Where  $\mathbf{v}_{\text{drift},x}$  is the drift velocity of a particle and  $\mu_x$  is the charge carrier mobility. Other symbols usual meaning.
- Thinking in terms of current density is practical **because current is always a conserved quantity.**



# Discharge breakdown condition

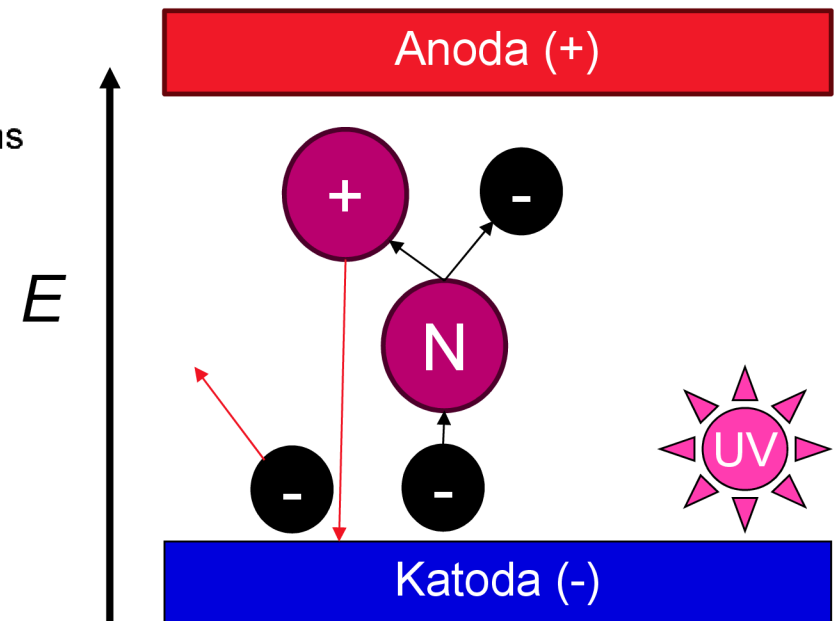
- Choose the cathode at  $x=0$  and anode at  $x=d$
- The cathode emits electrons through ISEE and through a constant external source (e.g UV). Furthermore, we can link the current of ions to electron current density.

$$j_e(0) = j_0 + \gamma j_i(0) = j_0 + \gamma j_e(0)(e^{\alpha d} - 1)$$

- Electron current from the cathode is then:

$$j_e(0) = \frac{j_0}{1 - \gamma(e^{\alpha d} - 1)}$$

- And before they reach the anode, they multiply through volume ionization

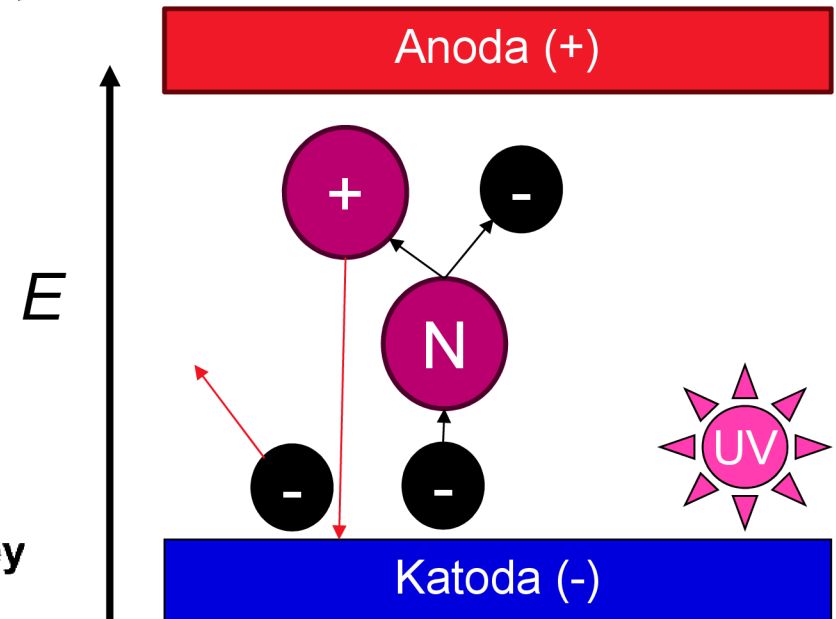


# Discharge breakdown condition

- We derived the total discharge current to be  $j_e(d) = \frac{j_0 e^{\alpha d}}{1 - \gamma(e^{\alpha d} - 1)}$
- If we turn off the external source of electrons, the current stops => non self-sustaining Townsend discharge
- However, the current grows towards infinity if

$$\gamma(e^{\alpha d} - 1) = 1$$

- This is what we call the discharge breakdown criterion: **The amount of ions created by one electron during its passage between the electrodes has to be such, that they create another electron by ISEE.**



# Discharge breakdown condition

= The ignition condition

$$\gamma(e^{\alpha d} - 1) = 1$$

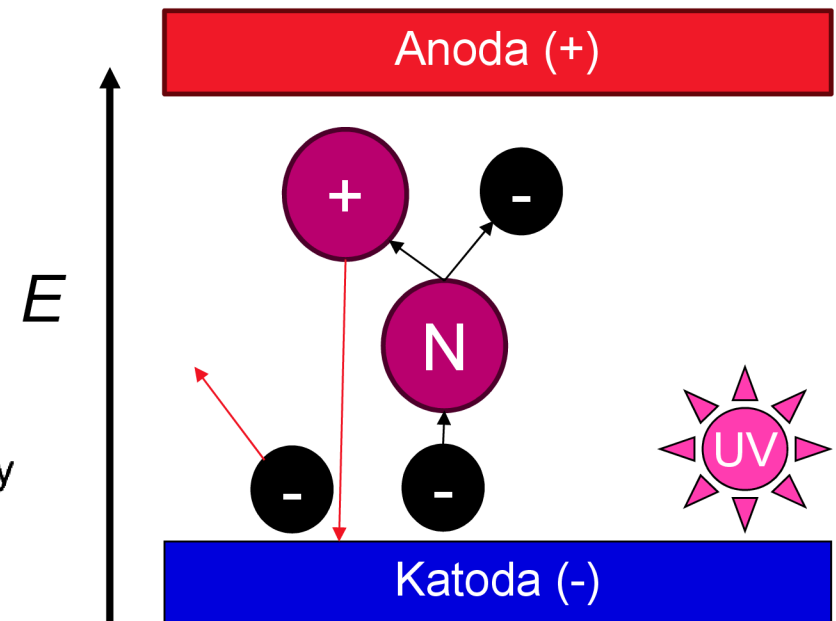
Is often written as

$$\gamma(e^{\alpha d} - 1) > 1$$

or even

$$\gamma(e^{\alpha d} - 1) \gg 1$$

- This is because maintaining a Townsend discharge is usually not the goal in the applications. **What we usually want is to ignite a stable self-sustained discharge and multiply the**





# Breakdown voltage, Paschen law

# Breakdown voltage

- Practical observation: **Voltage between the cathode and the anode has to be higher than a certain value so that a discharge is ignited.**
- Typically denoted  $V_b$

**Q:** What do you think affects the value of  $V_b$

**A:** Cathode-anode distance (the actual “constant” is break down E field more than breakdown voltage)

**A:** Type of gas and electrode condition => ionization energy affects  $\alpha$ , electrode affects  $\gamma$

**A:** On the collision mean free path (so pressure) => more collisions imply higher  $\alpha$

# Breakdown voltage

— Generally, we can approximate the Townsend coefficient as

$$\alpha = \frac{f(E, \lambda)}{\lambda}$$

— Since the mean free path depends on  $\lambda \sim \frac{1}{N} \sim \frac{1}{p}$  we can define the **reduced Townsend coefficient**

$$\frac{\alpha}{p} = F\left(\frac{E}{p}\right) \text{ nebo } \frac{\alpha}{N} = G\left(\frac{E}{N}\right)$$

where  $N$  is gas density in  $\text{m}^{-3}$ .

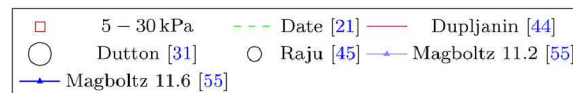
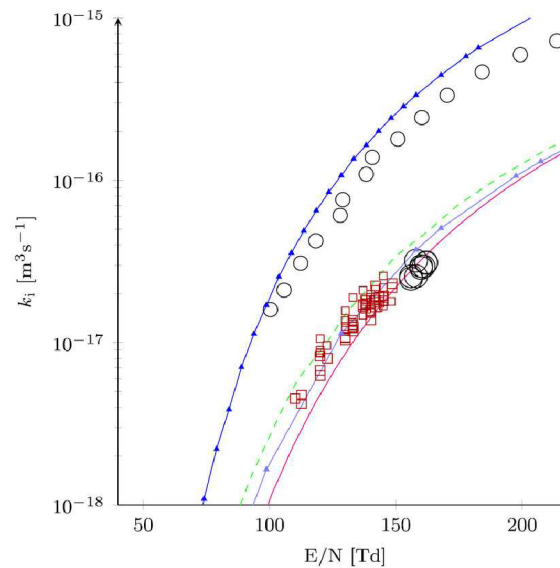
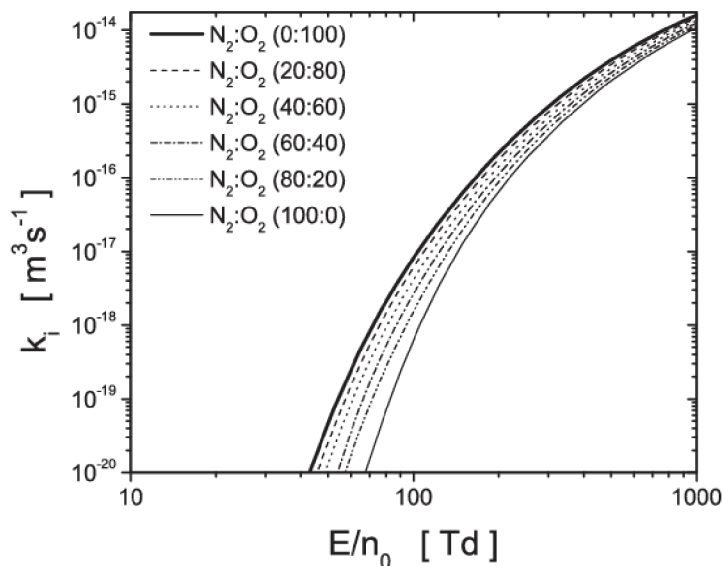
— This hints to a **special role of the quantity  $\frac{E}{N}$  or  $\frac{E}{p}$  in Plasma physics.**

— It turns out that **most transport and ionization coefficients depends, in very good approximation, only on  $\frac{E}{N}$  but not on gas density or electric field alone.**

— Note: People use either  $\frac{E}{N}$  or  $\frac{E}{p}$ . The former is closer to the physics truth while the latter kinda sorta

# E/N in plasma science

= SI unit for E/N is  $V \cdot m^2$ . Unfortunately, this reaches values of  $10^{20} - 10^{25}$  and because physicists are not yet comfortable with saying “zettavolts” or “yottavolts”, people use  $1 \text{ Td} = 10^{-21} V \cdot m^2$



# Paschen law

– From the above, we can derive the analytical formula for discharge breakdown condition

$$\frac{\alpha}{p} = F\left(\frac{E}{p}\right)$$

$$\frac{E}{p} = F_{\text{inv}}\left(\frac{\alpha}{p}\right)$$

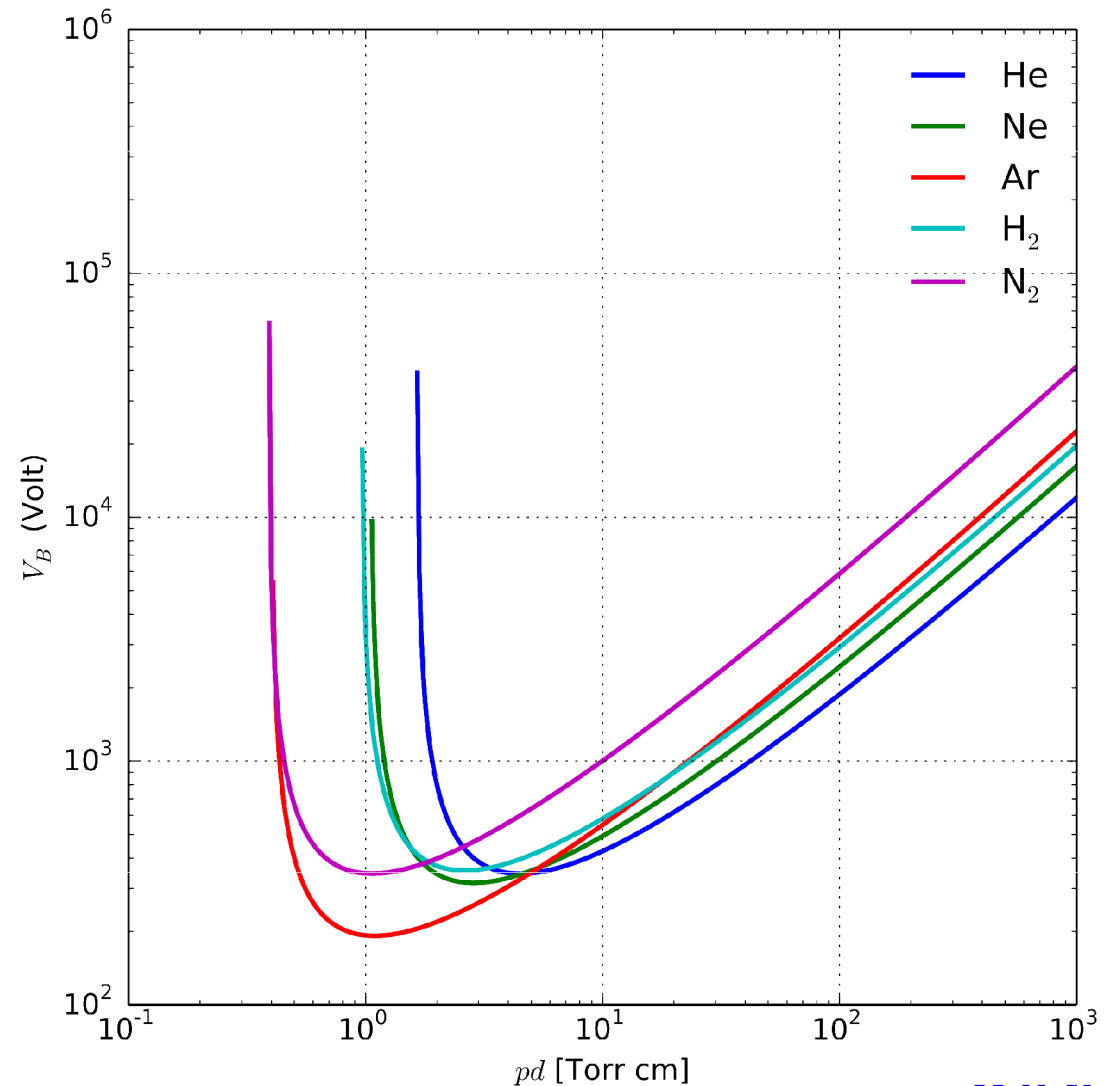
– Combining that with  $\gamma(e^{\alpha d} - 1) = 1$  yields  $\alpha = \frac{1}{d} \ln\left(\frac{1}{\gamma} + 1\right)$

– And if  $V_b = E \cdot d$  before the plasma is ignited, it also has to hold that

$$V_b = pd \cdot F_{\text{inv}}\left(\frac{1}{pd} \ln\left(\frac{1}{\gamma} + 1\right)\right)$$

# Paschen law

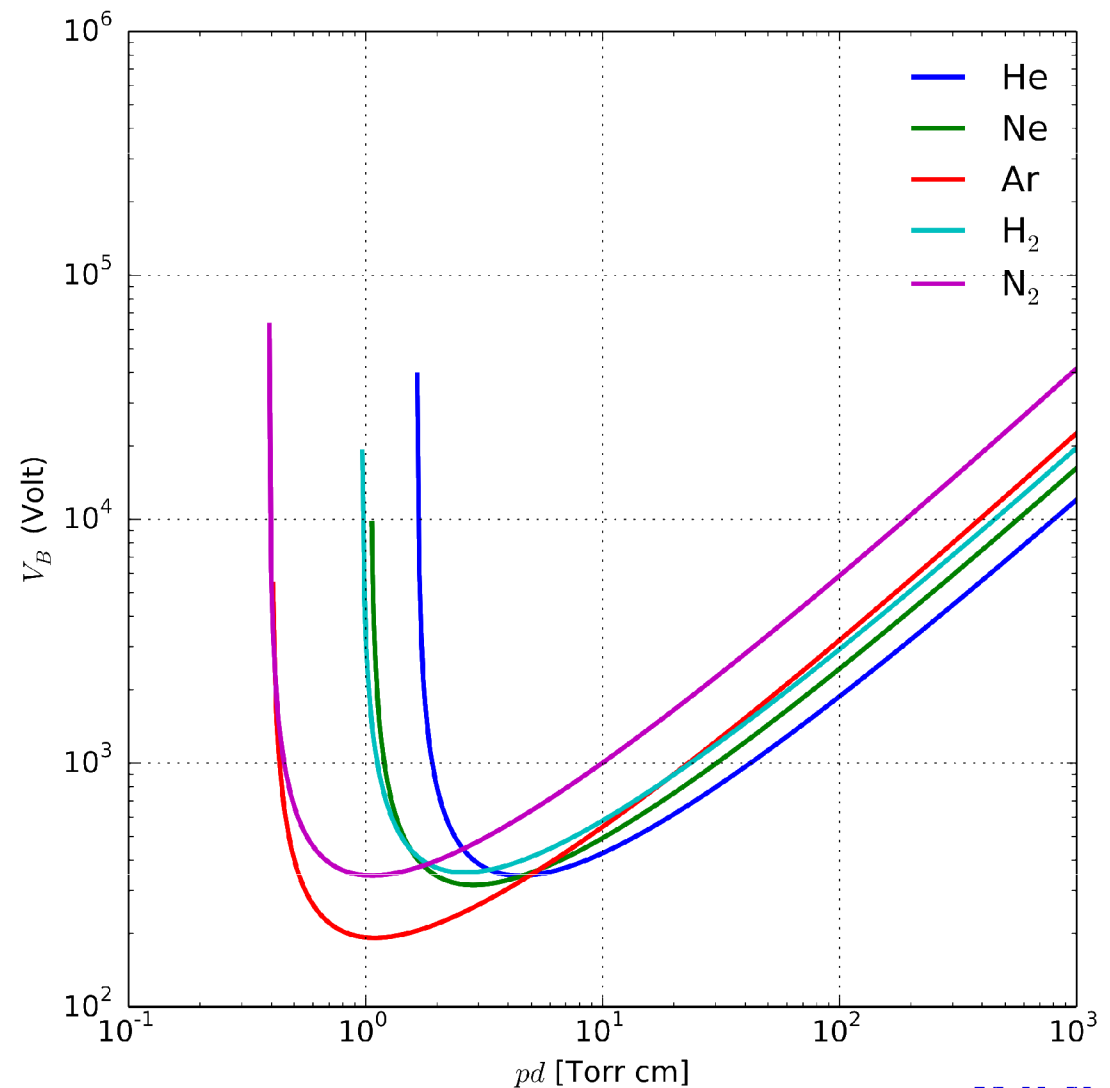
- $V_b = pd \cdot F_{\text{inv}} \left( \frac{1}{pd} \ln \left( \frac{1}{y} + 1 \right) \right)$
- Paschen law states that: **For a given gas, the breakdown voltage is a function of the product of pressure and distance.**
- The function minimum is called the **Stoletow point.**



# Paschen law

—**Q:** Try to interpret the Paschen law. Why does the curve grow in both directions?

**A:** Low  $pd$  implies either low pressure or distance – so there are not enough collisions to meet the Townsend criterion. At higher  $pd$  and fixed voltage, the value of  $\alpha$  decreases because  $E$  decreases and it is difficult to meet the Townsend criterion.



# Paschen law - quantitatively

— We can obtain  $\alpha$  by solving BKE and somehow fit  $\frac{\alpha}{p} \approx A \cdot \exp\left(-\frac{Bp}{E}\right)$

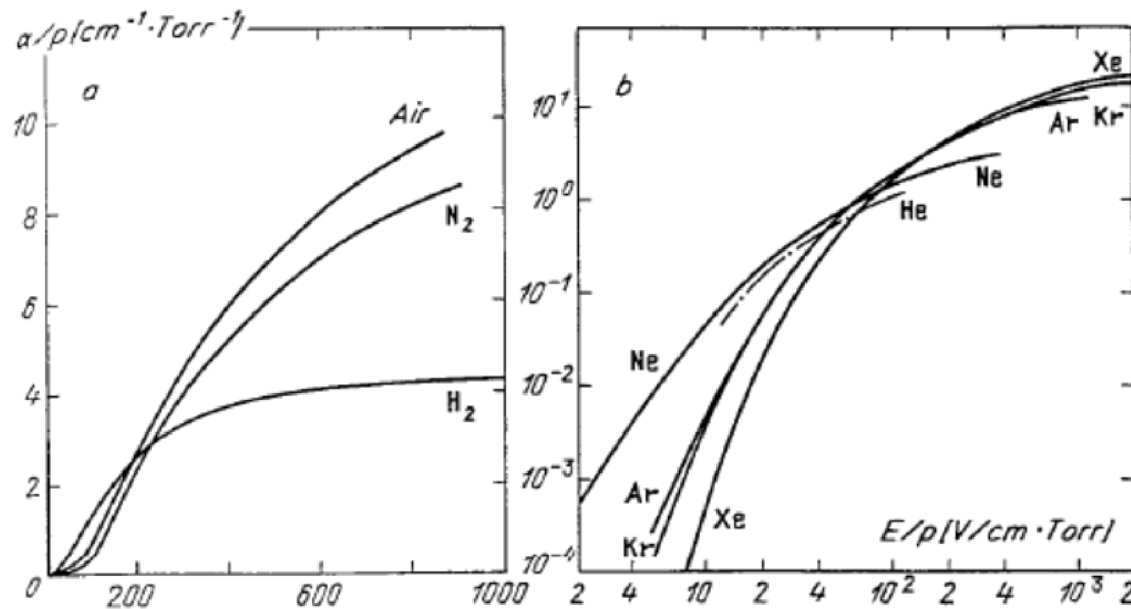


Fig. 4.3. Ionization coefficients for a wide range of  $E/p$  values (a) in molecular gases, (b) in inert gases. From [4.3]

Plyn	A [ $\text{cm}^{-1} \text{Torr}^{-1}$ ]	B [ $\text{Vcm}^{-1} \text{Torr}^{-1}$ ]	oblast' $ E /p_0$ [ $\text{Vcm}^{-1} \text{Torr}^{-1}$ ]
He	3	34	20 – 150
Ne	4	100	100 – 400
Ar	14	180	100 – 600
Kr	17	240	100 – 1000
Xe	26	350	200 – 800
vzduch	15	365	100 – 800
$\text{H}_2$	5	130	150 – 600
$\text{N}_2$	12	342	100 – 600
$\text{CO}_2$	20	466	500 – 1000
$\text{H}_2\text{O}$	13	290	150 – 1000
Hg	20	370	200 – 600



# Paschen law - quantitatively

- We can obtain  $\alpha$  by solving BKE and somehow fit  $\frac{\alpha}{p} \approx A \cdot \exp\left(-\frac{Bp}{E}\right)$
- By substituting into the Paschen law and differentiating, we can get

$$V_b = \frac{Bpd}{\ln(Apd) - \ln\left[\ln\left(1 + \frac{1}{\gamma}\right)\right]}$$

$$V_{b,\min} = e \frac{B}{A} \ln\left(1 + \frac{1}{\gamma}\right)$$

- Due to various real-life phenomena (finite electrodes, recombinations, other gas-phase collisions), this rarely corresponds to reality. **But it is a decent first estimation of the discharge voltage for different gases and gas mixtures.**

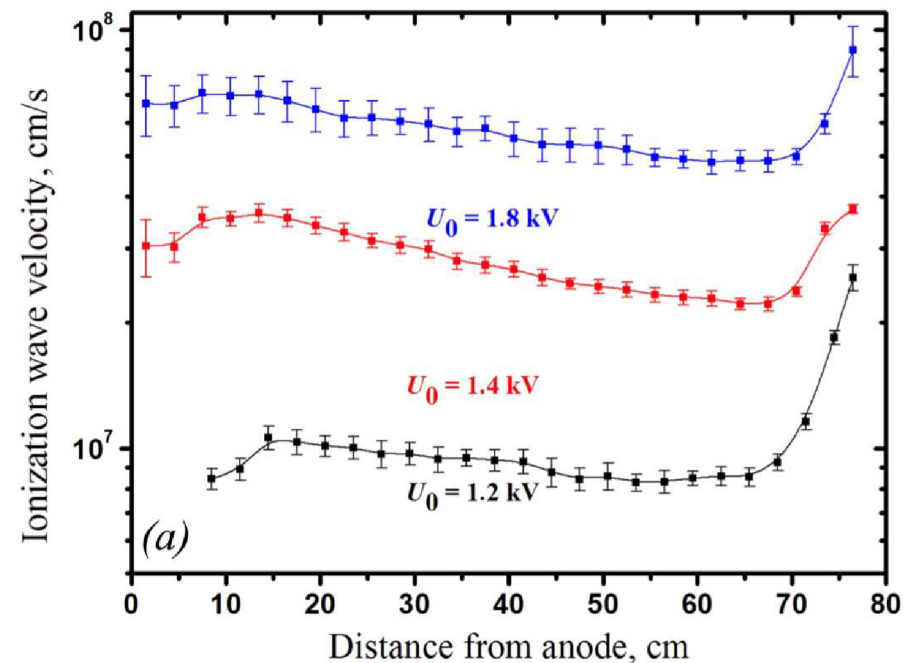
# Actual discharge ignition

- Nice overview of the problematics:

[Shishpanov, A. I., Meshchanov, A. V., Kalinin, S. A., & Ionikh, Y. Z. (2017). Processes of discharge ignition in long tubes at low gas pressure. Plasma Sources Science and Technology, 26(6), 065017. doi:10.1088/1361-6595/aa6f7c ]

The ionization does not happen instantaneously, it proceeds with a certain **ionization wave velocity**.

This causes a **delay in discharge breakdown w.r.t voltage application**.



# A short note on actual discharge ignition

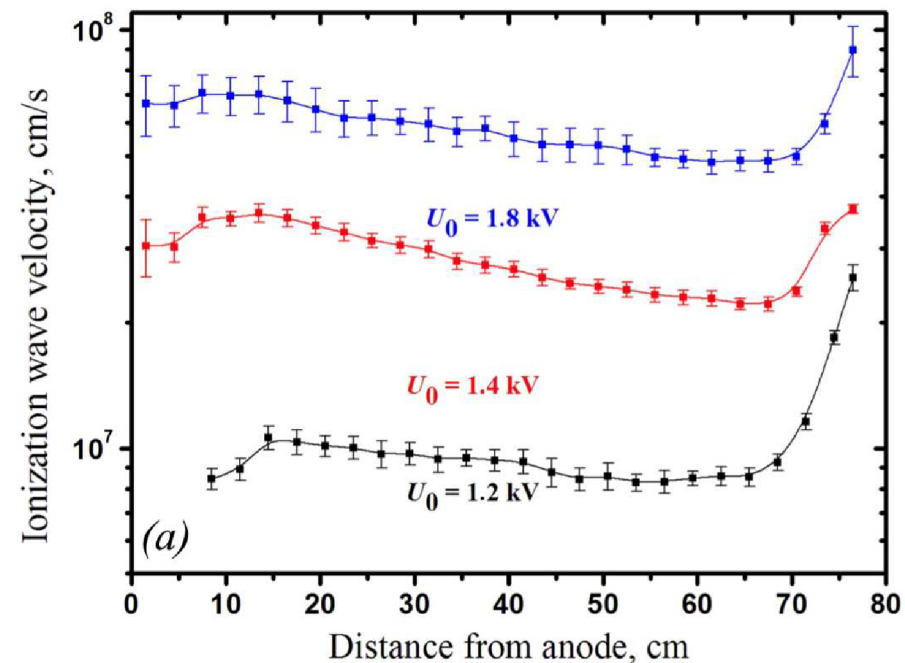
# Actual discharge ignition

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The ionization does not happen instantaneously, it proceeds with a certain **ionization wave velocity**.

This causes a **delay in discharge breakdown w.r.t voltage application**.



# Actual discharge ignition

- The time after which the discharge is ignited is expressed as  $t_d = t_s + t_f + t_w$ ,

where the symbols  $t_s$  and  $t_f$  correspond to random phenomena.

- Based on that, we can express the Laue distribution

number of breakdowns with  
larger breakdown time than  $t_d$

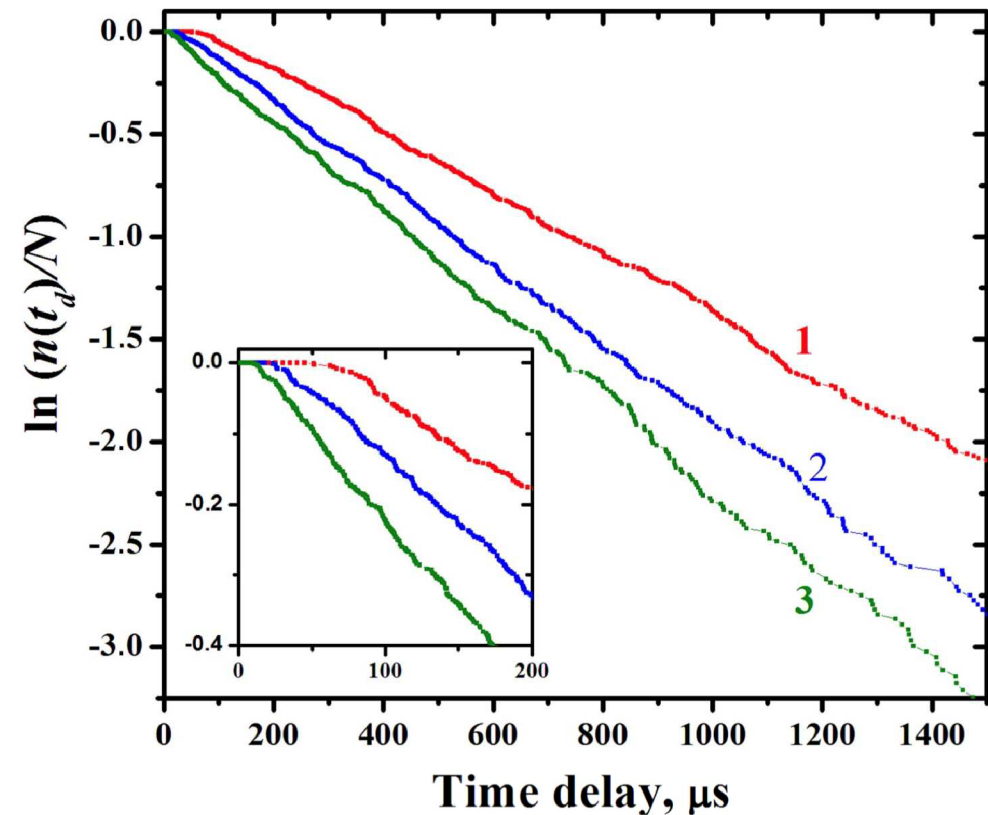
total number of breakdowns

$$\frac{n(t_d)}{N} = \exp\left[-\frac{t_d - (t_f + t_w)}{\bar{t}_s}\right]$$

# Actual discharge ignition

- This expression produces “Lauegrams”, from which we can read out the probability of discharge ignition and time delay at various voltages

$$\frac{n(t_d)}{N} = \exp\left[-\frac{t_d - (t_f + t_w)}{\bar{t}_s}\right]$$



**Figure 3.** Lauegrams for different pulse amplitudes:  $U_0 = 1048$  V (1), 1197 V (2), 1290 V (3). Tube T1, neon, 1 Torr.

# Main take-aways

# Take-aways from lesson 1

1. Be able to formulate and derive Townsend breakdown criterion, be aware of the underlying assumptions and limitations.
2. Be able to accurately and exactly describe Paschen law.
3. Be aware of the special role of  $E/N$  in plasma science and where it comes from.
4. General awareness of actual breakdown mechanisms – there is a time delay, Lauegrams exist.