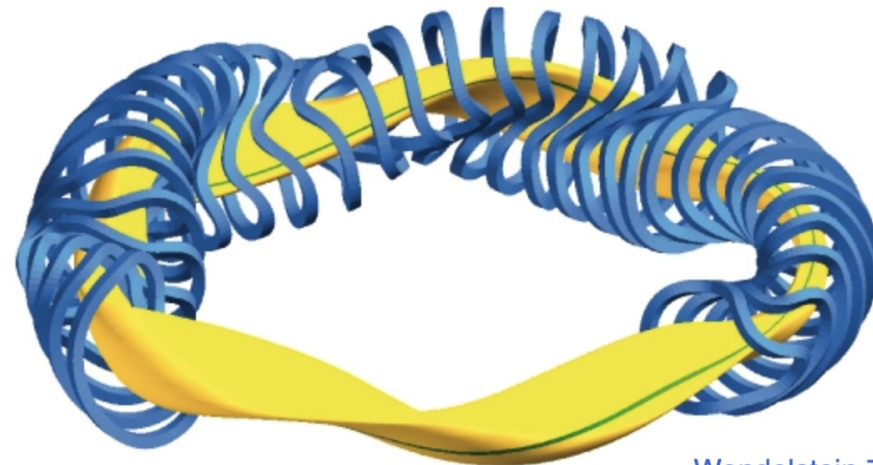
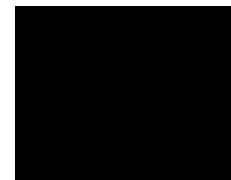


9. Thermonuclear fusion



Wendelstein 7-X

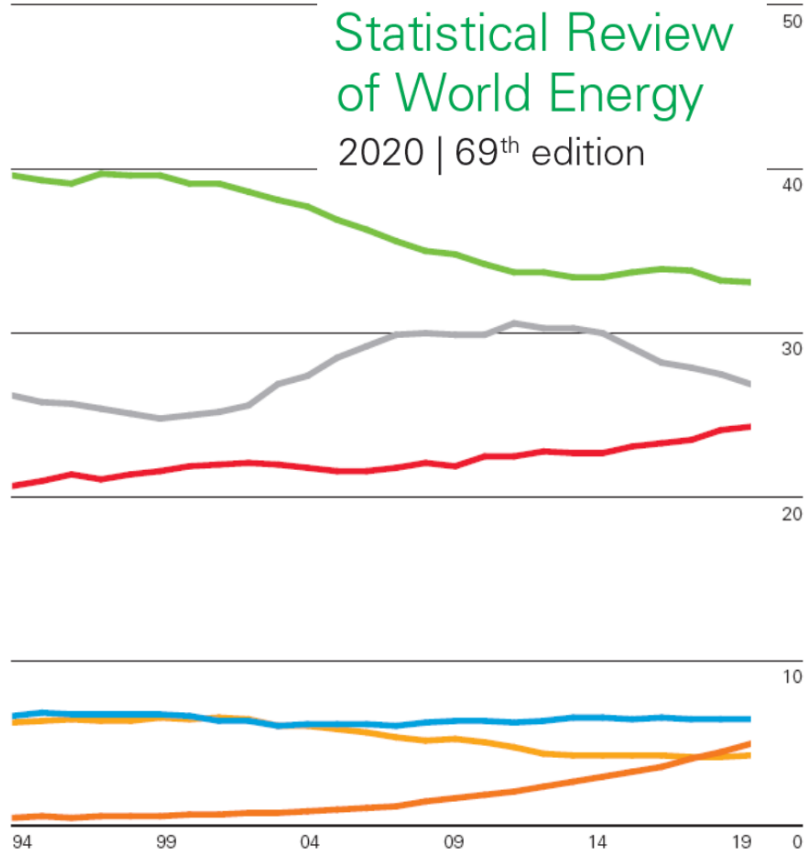


Lecture series contents

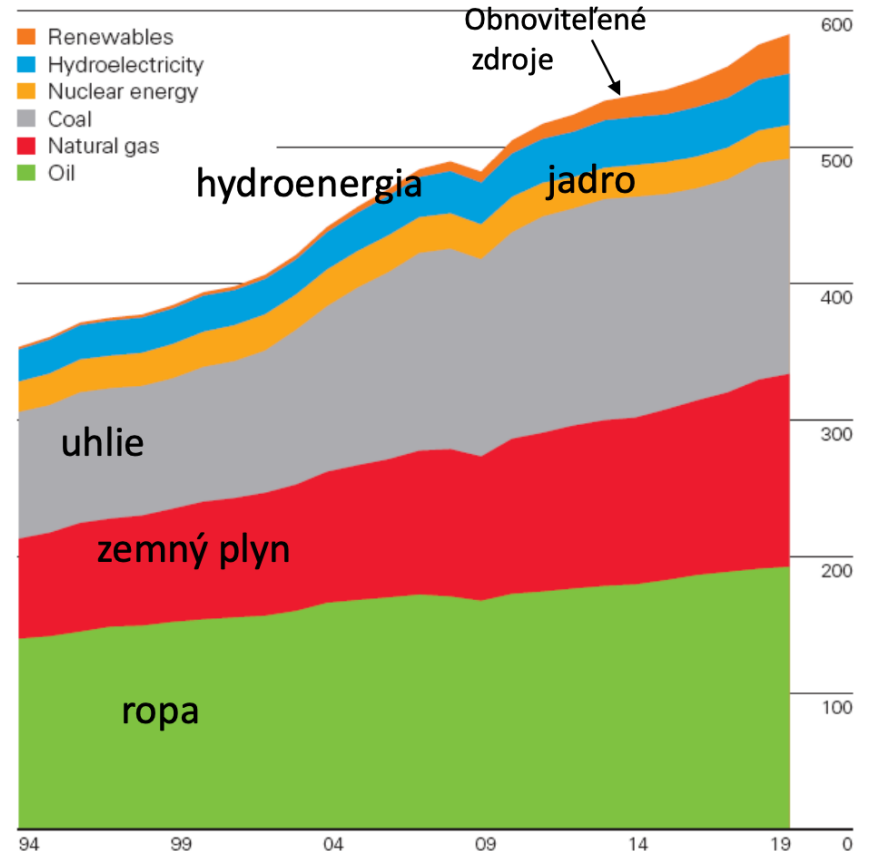
1. Townsend breakdown theory, Paschen's law
2. Glow discharge
3. Electric arc at low and high pressures
4. Magnetized low-pressure plasmas and their role in material deposition methods.
5. Brief introduction to high-frequency discharges
6. Streamer breakdown theory, corona discharge, spark discharge
7. Barrier discharges
8. Leader discharge mechanism, ionization and discharges in planetary atmospheres
9. Discharges in liquids, complex and quantum plasmas
- 10. Thermonuclear fusion, Lawson criterion, magnetic confinement systems, plasma heating and inertial confinement fusion.**

Evolution of world's primary energy

Shares of global primary energy
Percentage



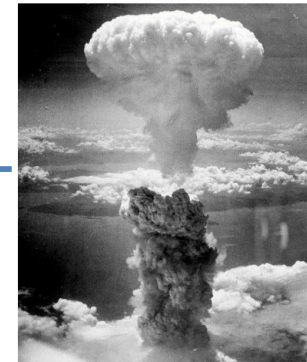
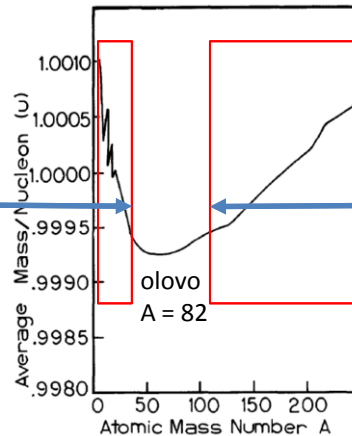
World consumption
Exajoules



- medziročný nárůst spotřeby energie v roce 2019: +7.67E18 J (celkem 2019: 583.9 EJ – exa Joul)
- meziročný nárůst energie z obnovitelných zdrojů 2019: +3.15 EJ (celkem 2019: 28.98 EJ)

Energy consumption by humanity (converted to energy)

- The total consumption from primary energy sources for the year 2019 is 583.9 EJ (exa = 1E18).
- If we convert this to power for the year and with over 40% efficiency of converting thermal energy to electricity, we get 7.4 TW (1E12), which is an annual increase of 0.1 TW.
- For comparison, the power from the Sun hitting Earth is about 40,000 TW, and Earth's thermal radiation is about 32 TW.
- The Temelín power plant has a capacity of 1 GW, meaning that the global energy consumption in 2019 increased by the equivalent of 100 Temelín nuclear power plants.
- ... This is a staggering increase in energy consumption! And of course, thermonuclear fusion is one of the possible solutions, but why?



- The difference between the mass of an atom and the sum of the masses of its nucleons is the source of atomic energy (binding energy), as given by Einstein's equation $E = mc^2$.
- Deuterium from 1 liter of seawater (0.15 per mille) is energetically equivalent to 300 liters of gasoline

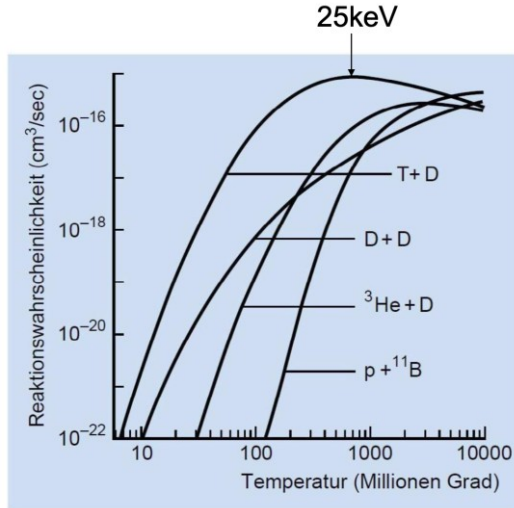
MAIN FUSION PATHWAYS, LAWSON CRITERION, TRIPLE PRODUCT

Main fusion pathways

- Possible fusion reactions for a potential energy source:

DT	$D + T \rightarrow {}^4\text{He} (3,5 \text{ MeV}) + n (14,1 \text{ MeV})$	
DD	$D + D \rightarrow {}^3\text{He} (1,8 \text{ MeV}) + n (2,5 \text{ MeV})$ $\rightarrow T (1,0 \text{ MeV}) + p (3 \text{ MeV})$	
TT	$T + T \rightarrow n + n + {}^4\text{He} \dots\dots\dots 11,3 \text{ MeV}$	
D- ³ He	$D + {}^3\text{He} \rightarrow {}^4\text{He} (3,7 \text{ MeV}) + p (14,6 \text{ MeV})$	
p- ⁶ Li	$p + {}^6\text{Li} \rightarrow {}^4\text{He} + {}^3\text{He} \dots\dots\dots 4,0 \text{ MeV}$	
p- ¹¹ B	$p + {}^{11}\text{B} \rightarrow 3 {}^4\text{He} \dots\dots\dots 8,7 \text{ MeV}$	
DD katalyzovaná	$6 D \rightarrow 2 p + 2 n + 2 {}^4\text{He} + 43,2 \text{ MeV}$	

Rate constants of fusion reactions



D-T

$$n_c \tau_E \geq \frac{12}{E_{3,5\text{MeV}}} k \frac{T}{\langle \sigma v \rangle}$$

$$n_c \tau_E \geq 1,5 \times 10^{20} \text{ sec} \cdot \text{m}^{-3}$$

DT	D + T → ⁴He (3,5 MeV) + n (14,1 MeV)	
DD	D + D → ³He (1,8 MeV) + n (2,5 MeV) → T (1,0 MeV) + p (3 MeV)	
TT	T + T → n + n + ⁴He	11,3 MeV
D-³He	D + ³He → ⁴He (3,7 MeV) + p (14,6 MeV)	
p-⁶Li	p + ⁶Li → ⁴He + ³He	4,0 MeV
p-¹¹B	p + ¹¹B → 3 ⁴He	8,7 MeV
DD katalyzovaná	6 D → 2 p + 2 n + 2 ⁴He + 43,2 MeV	

- Computed **optimal** values

palivo	E_{ch} [MeV]	T [keV]	T [MK]	$T/\langle \sigma v \rangle$ [Km ⁻³ s ⁻¹]	L [m ⁻³ .s]
D-T	3.5	13.6	158	6.88×10^{29}	2.0×10^{20}
D-D	4.2	15	174	6.04×10^{31}	1.5×10^{22}
D-³He	18.3	58	673	8.93×10^{30}	5.0×10^{20}
p-¹¹B	8.7	123	1 427	3.13×10^{31}	3.7×10^{21}

Fuel considerations

The DT reaction has approximately twice the effective cross-section compared to the DD reaction, but:

- It also generates fast neutrons, which initiate fission reactions in the reactor that must then be carefully shielded/stored.
- Only 20% of the energy goes into charged particles.
- Tritium (T) is reactive and must be produced from lithium; lithium supplies are limited and are also used in battery production.
- Tritium production (with a half-life of 12 years) occurs through reactions of slow neutrons with lithium isotopes.



Jméno minerálu	Vzorec
Amblygonit	$\text{LiAlPO}_4 (\text{F}, \text{OH})$
Eukryptit	LiAlSiO_4
Lepidolit	$\text{K}(\text{Li}, \text{Al})_3(\text{Si}, \text{Al})_4\text{O}_{10}(\text{F}, \text{OH})$
Petalit	$\text{LiAlSi}_4\text{O}_{10}$
Spodumen	$\text{LiAlSi}_4\text{O}_{10}$
Zinnwaldit	$\text{KLiFe}^{2+}\text{Al}(\text{AlSi}_3)\text{O}_{10}(\text{F}, \text{OH})_2$

- Reserves: Hydrogen on Earth contains 15 per mille of deuterium, lithium in minerals is about 14 million tons, and in seawater about 0.2 ppm, totaling 230 billion tons.

Main fusion pathways

Necessary physical conditions for the synthesis/fusion of light nuclei:

1. Nuclei repel each other, **therefore approximately 100 keV is needed** for heavy hydrogen isotopes to come within 1 fm
2. To achieve these energies/particle speeds, **accelerators are very inefficient.**
3. Therefore, it is **necessary to heat the matter and use particles from the tail of the Maxwellian distribution** function for speeds ($10 \text{ keV} = 10^8 \text{ K}$) – hence thermonuclear fusion
4. Plasma (ionization) is primarily not needed. The fact that the matter at such temperatures will be in plasma state follows from the Saha equation
5. Controllability of the reaction:
 - a) Is it possible to isolate hot plasma? – by gravity (the Sun), inertially (explosions), electrostatically, and by magnetic fields;
 - b) How to heat the plasma to such a temperature?
 - c) How to design a reactor so that its energy balance is positive?

Lawson criterion

This is a **necessary condition for the occurrence of self-sustaining fusion** in a thermonuclear reactor, derived by John D. Lawson in 1955.

It is based on the following reasoning: the **power input from nuclear synthesis must be greater than the loss power**, which the plasma loses through various paths (radiation, conduction, etc.). Therefore:

$$P^* \geq P_{loss}$$

When considering this equation, it is useful to introduce the concept of **confinement time**.

This time is necessary for the escape of the thermal energy W from the plasma, and therefore:

$$\tau_E = \frac{W}{P_{loss}}$$

Naturally, to obtain quantitative value of the confinement time, we need to know the system as such and be able to quantify P_{loss} based on its geometry, volume, etc...

Lawson criterion – derivation outline

Assuming thermal quasineutral plasma the total energy is:

$$W = \frac{3}{2}n_i kT_i + \frac{3}{2}n_e kT_e = 3n_e kT$$

$$n_e \approx n_i$$

Next, we utilize the condition for positive energy balance of the fusion reaction

number of reactions per unit time (reaction frequency) \rightarrow $P^* \geq P_{loss}$ kinetic energy of charged particles

$$f E_{3,5MeV} \geq P_{loss}$$

$$f = \langle \sigma v \rangle n_D n_T = \frac{1}{2} n_e \frac{1}{2} n_e \langle \sigma v \rangle = \frac{1}{4} n_e^2 \langle \sigma v \rangle$$

$$\frac{1}{4} n_e^2 \langle \sigma v \rangle E_{3,5MeV} \geq \frac{3kT}{\tau_E} n_e$$

Finally, we arrive at the Lawson criterion

$$\boxed{n_e \tau_E \geq \frac{12}{E_{3,5MeV}} k \frac{T}{\langle \sigma v \rangle}} = \mathbf{L}$$

Which for the D-T system gives:

$$n\tau_E \geq 1.5 \cdot 10^{20} \frac{\text{s}}{\text{m}^3}$$

Triple product

- An even more useful figure of merit is the "triple product" of density, temperature, and confinement time $n \cdot T \cdot \tau_E$.
- For most confinement concepts, whether inertial, mirror, or toroidal confinement, the **density and temperature can be varied over a fairly wide range, but the maximum attainable pressure p is a constant given by engineering.**
- When such is the case, the fusion power density is proportional to

$$P \sim \frac{p^2 \langle \sigma v \rangle}{T^2}$$

- So by multiplying the Lawson criterion with T , we obtain the following inequality

$$nT\tau_E \geq \frac{12}{E_{\text{ch}}} \frac{T^2}{\langle \sigma v \rangle}$$

Finally, the Lawson parameter is the ratio $Q = \frac{P^*}{P_{\text{loss}}}$

There are many types of Lawson parameters, depending on what losses are included (transmission loss, etc...). But generally

Lawson parameter

Finally, the **Lawson parameter** is the ratio $Q = \frac{P^*}{P_{\text{loss}}}$

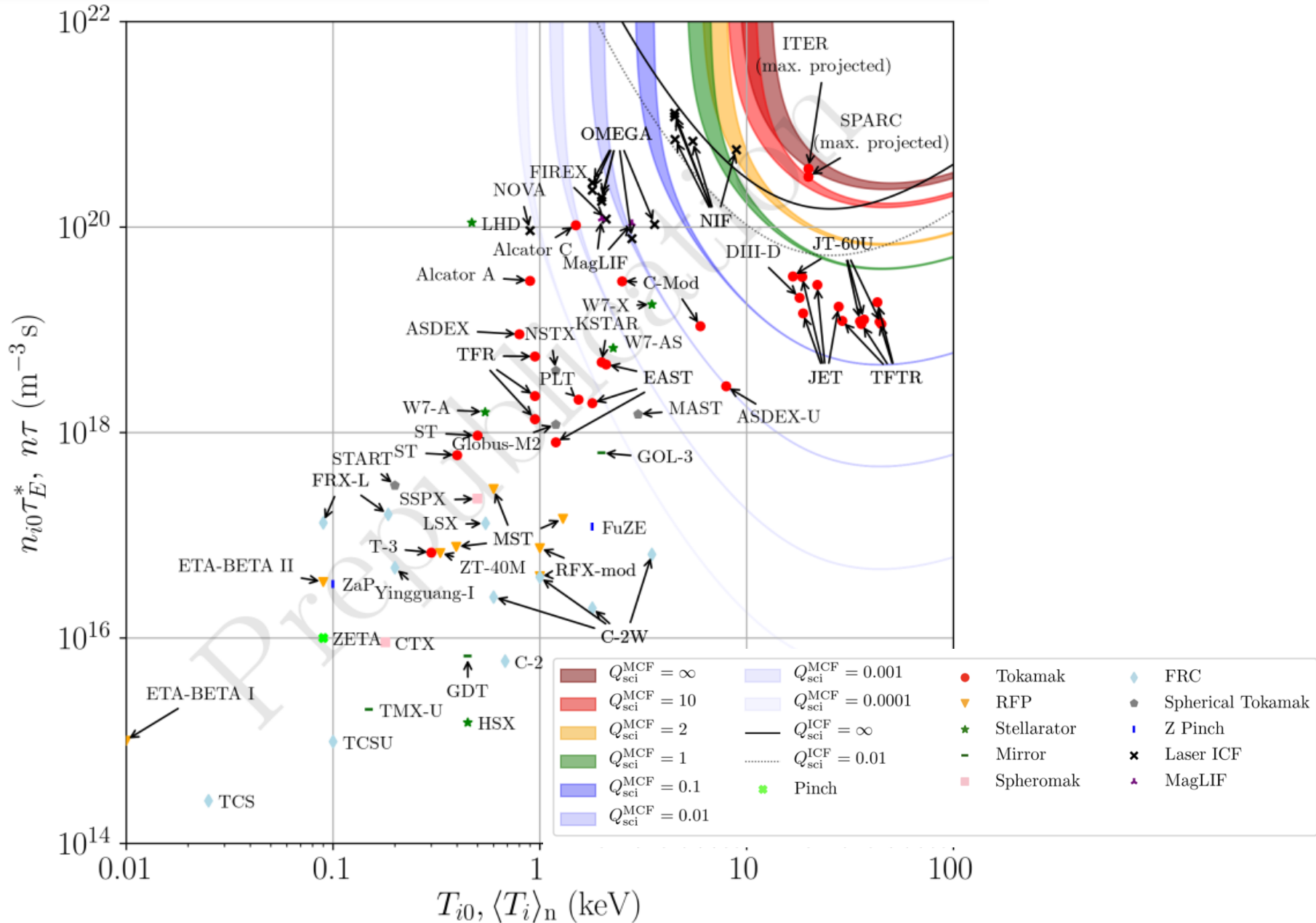
There are **many types of Lawson parameters**, depending on what losses are included (transmission loss, etc...).

Physicist, usually look at the **“scientific gain” Lawson parameter**, which describes whether fusion is producing net energy.

$Q_{\text{SCI}} \geq 1$ denotes useful energy production

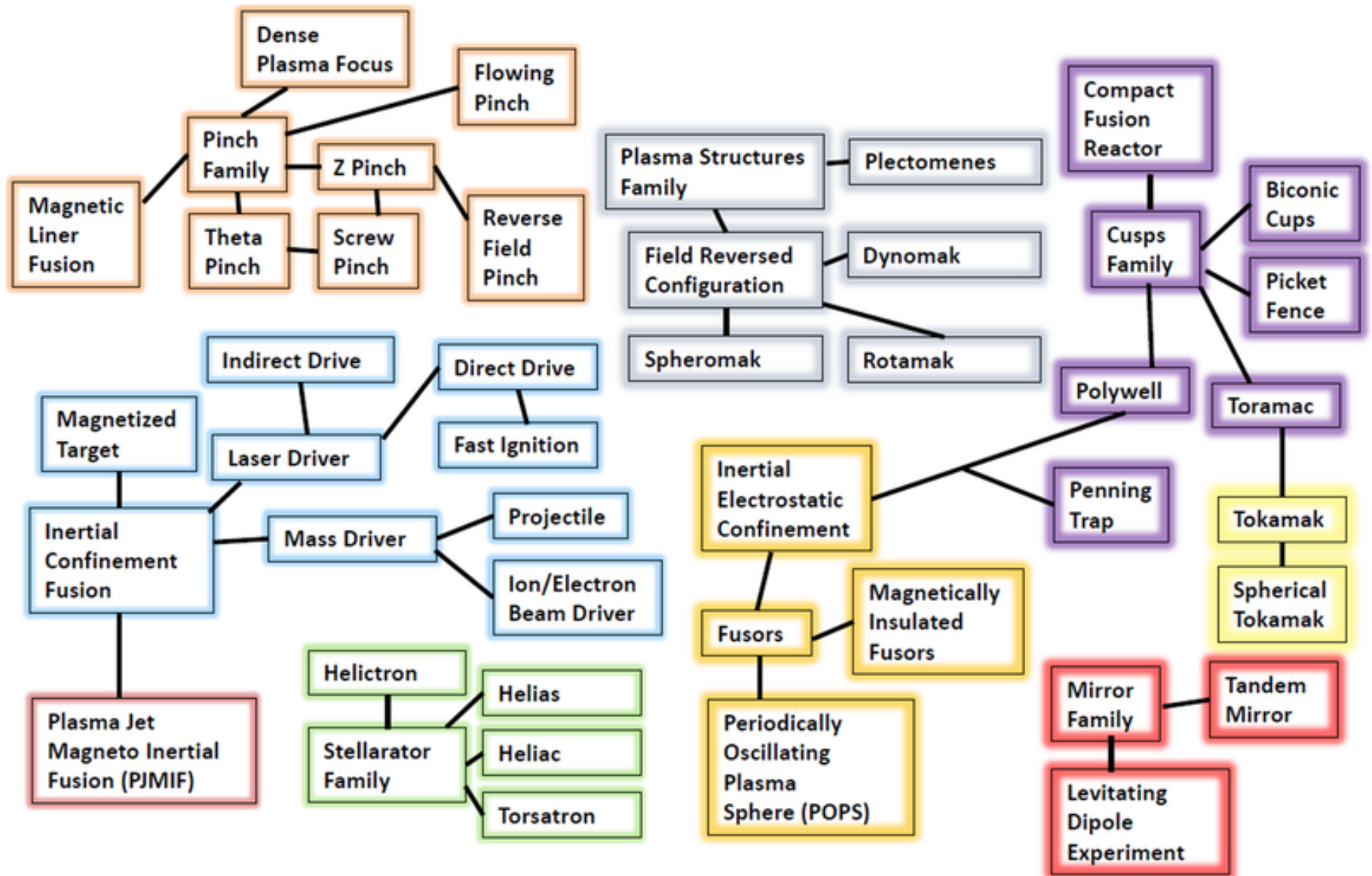
$Q_{\text{SCI}} \rightarrow \infty$ is the ideal state, where the reaction is self-sustained

Fusion devices built to date



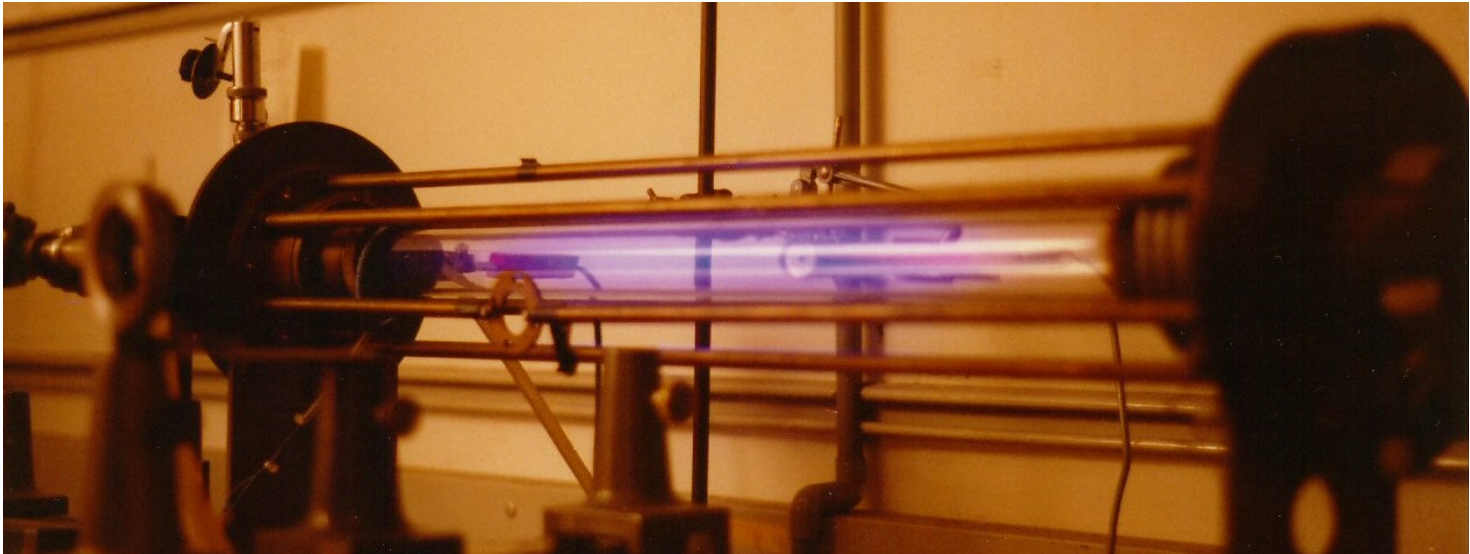
FUSION SETUPS

Fusion setups



FUSION SETUPS – PINCHES

Fusion in pinch systems



In z-pinch and theta-pinch systems (see arc lecture), plasma density can be high enough to observe measurable fusion in H₂ plasmas.

As you can imagine, Q factor here is nearly zero, good for research purposes only.

FUSION SETUPS – MAGNETIC CONFINEMENT FUSION (MCF)

Drifts in B fields

Summary of charged particle drifts you should already know

- **grad B drift**, caused by a change in the density of magnetic field lines, the force is directly proportional to grad B
- **drift caused by a general force and gravitational drift**, perpendicular to the gravitational and magnetic field
- **drift caused by the curvature of the B field lines**, centrifugal force
- **polarization drift**, caused by a slow change in the electric field over time, induced force
- ExB drift

Drifts in B fields

- Curvature drift is a consequence of centrifugal force due to varying B field strength

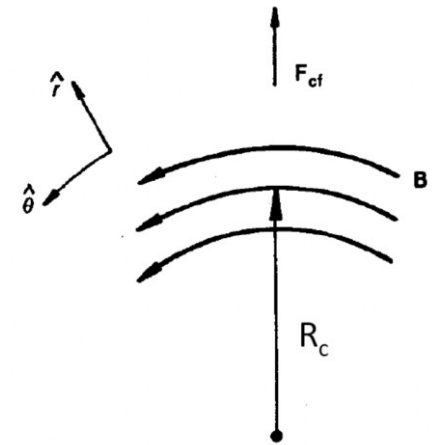
$$\mathbf{F} = \frac{mv_{\parallel}^2}{R_c} \hat{\mathbf{r}} = mv_{\parallel}^2 \frac{\mathbf{R}_c}{R_c^2}$$

- Practically, when you curve magnetic field (e.g. from a solenoid), the magnitude goes down as well, so there is gradB field.

$$\mathbf{V} = \frac{m}{2q} (v_{\parallel}^2 + \frac{1}{2}v_{\perp}^2) \frac{\mathbf{B} \times \nabla B^2}{B^4}$$

- As a consequence, in toroidal B fields, particles always escape via the torus OD because there is always a gradient B there.

$$\mathbf{v}_D = \frac{1}{q} \frac{\mathbf{F} \times \mathbf{B}}{B^2} = \frac{mv_{\parallel}^2}{qB^2} \frac{\mathbf{R}_c \times \mathbf{B}}{R_c^2}$$



Zakřivené magnetické pole.

Isolation through B field

- The condition for the balance between the kinetic pressure of the plasma and the magnetic pressure P_m of the external confinement field, the volumetric density of electromagnetic energy:

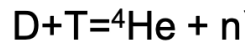
$$w_{elmag} = \frac{\epsilon E^2}{2} + \frac{\mu H^2}{2} \quad P_m = \frac{B^2}{2\mu_0} \quad 1 \text{ Pa} = 1 \frac{\text{N}}{\text{m}^2} = 1 \frac{\text{kg}}{\text{m} \cdot \text{s}^2} = 1 \frac{\text{J}}{\text{m}^3}$$

- However, magnetic confinement systems have a limited mechanical strength at ca $p=10$ Mpa
- For a temperature $T=200$ MK gives a fuel concentration at the level of 10 Pa, or a density of $2 \cdot 10^{21} \text{ m}^{-3}$
- According to the Lawson's criterion, this gives a confinement time of 0.1 s for D-T or 10 s for D-D.
- At the same time, this also means that a magnetic thermonuclear reactor must be constructed as a vacuum vessel, which implicitly ensures resistance to accidents that would disrupt the vacuum and thus automatically stop the reaction.

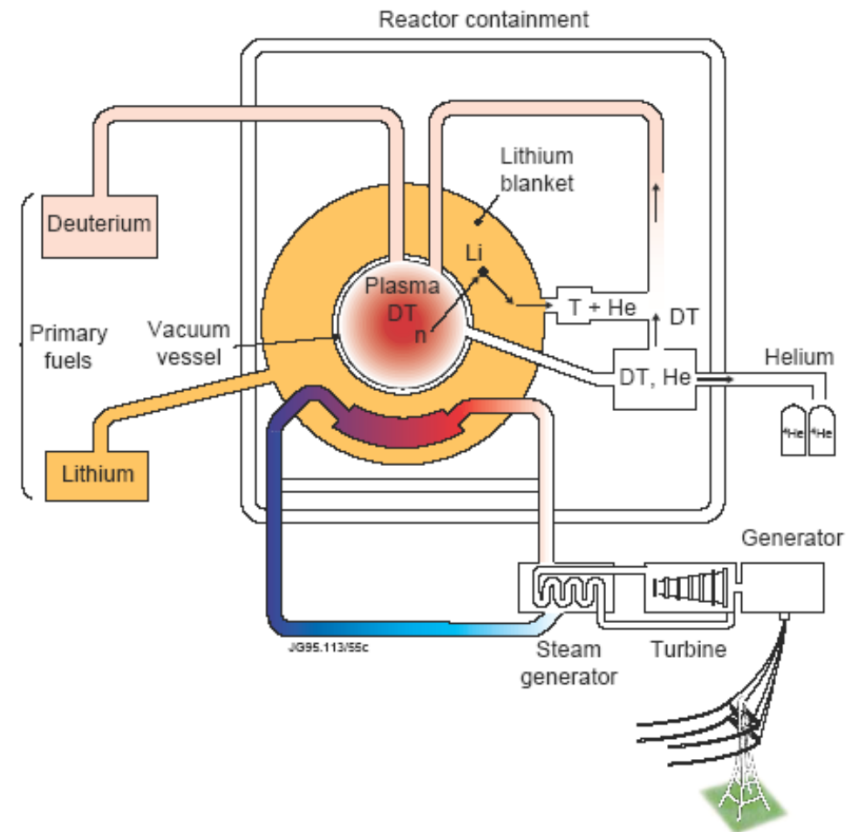
Isolation through B field

The energy production scheme could then look as follows:

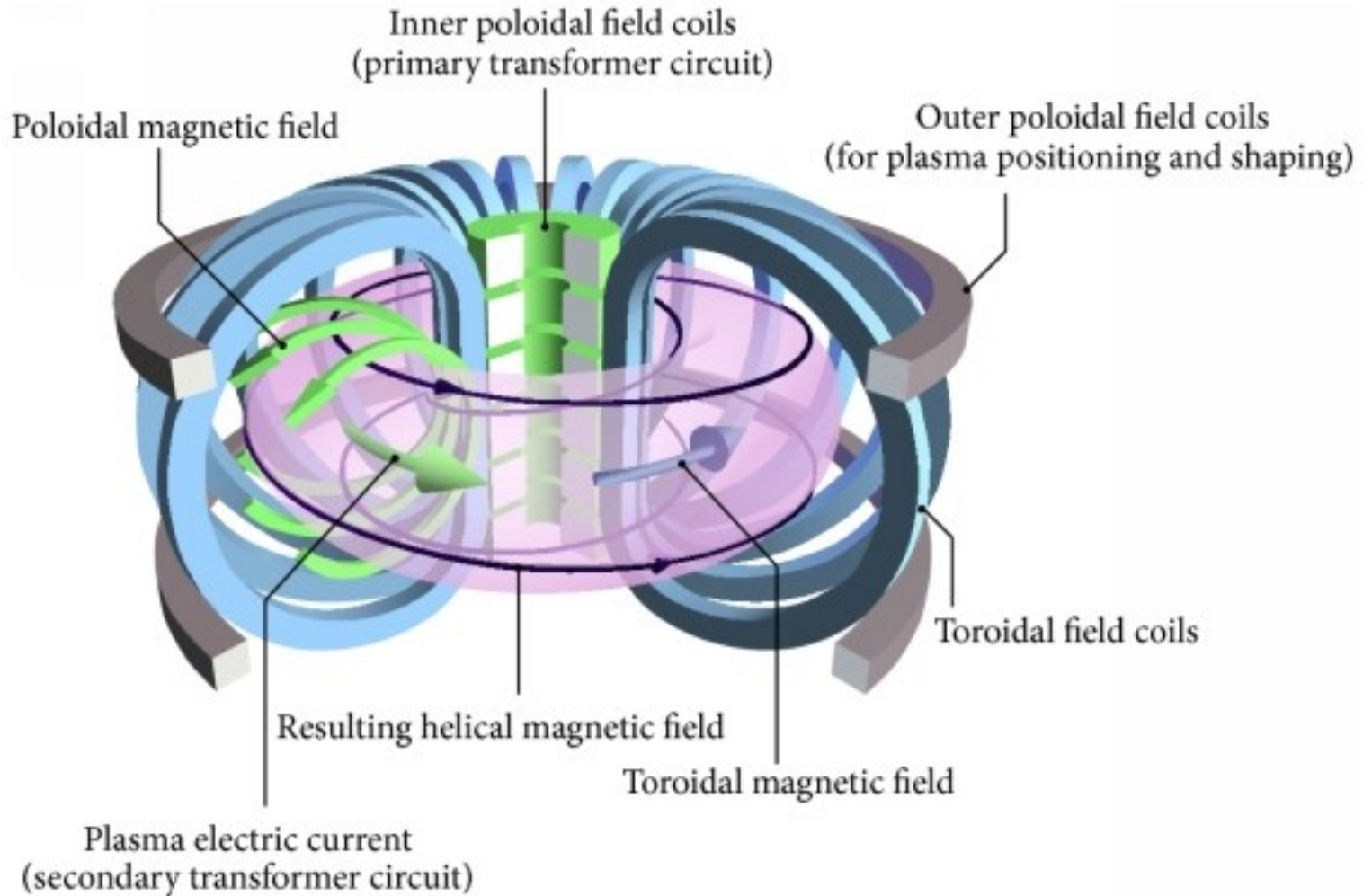
1. A fusion reactor mixes D (deuterium) and T (tritium) isotopes at the center of the reactor and the heat from the released neutrons is captured by a mantle of molten Li (lithium).



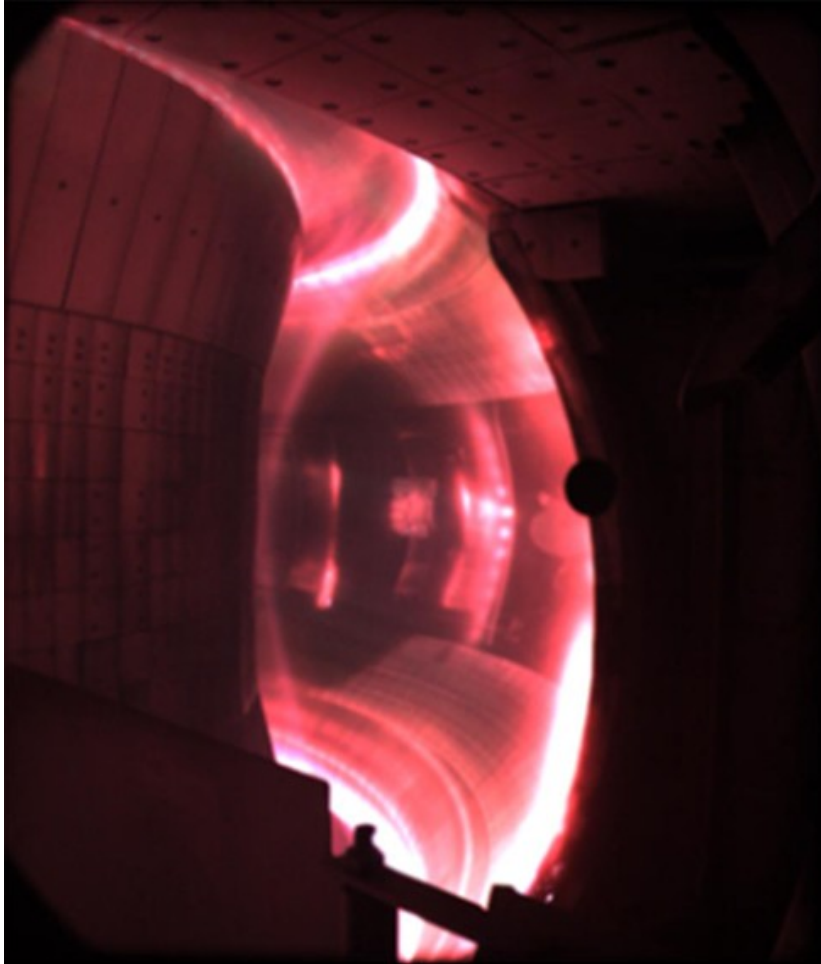
2. This heat then generates steam in a heat exchanger, which will drive a turbine that delivers electrical energy to the grid.
3. Meanwhile, lithium reacts with slow neutrons to produce T (tritium), which can be used as new fuel. $^6\text{Li} + n = T + \text{He}$.
4. The charged alpha particles (He) can be used for direct electricity production through so-called magnetohydrodynamic (MHD) generators.



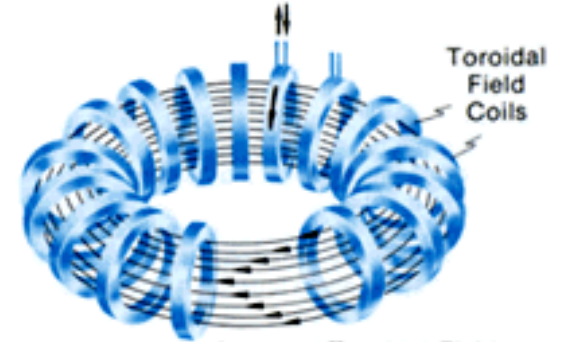
Magnetic confinement fusion - Tokamak



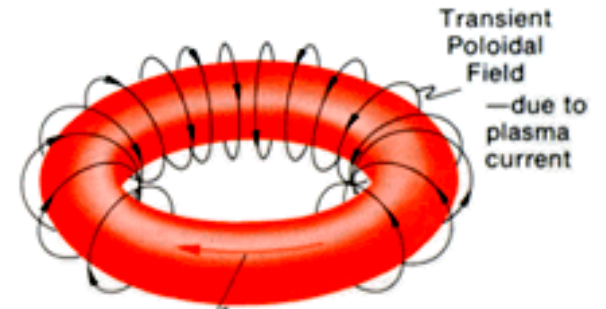
Magnetic confinement fusion - Tokamak



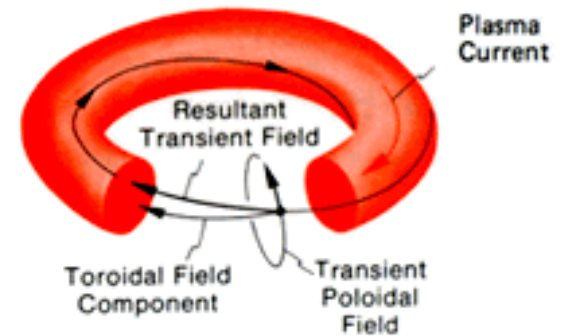
Relatively Constant Electric Current



Constant Toroidal Field



Transient Plasma Current



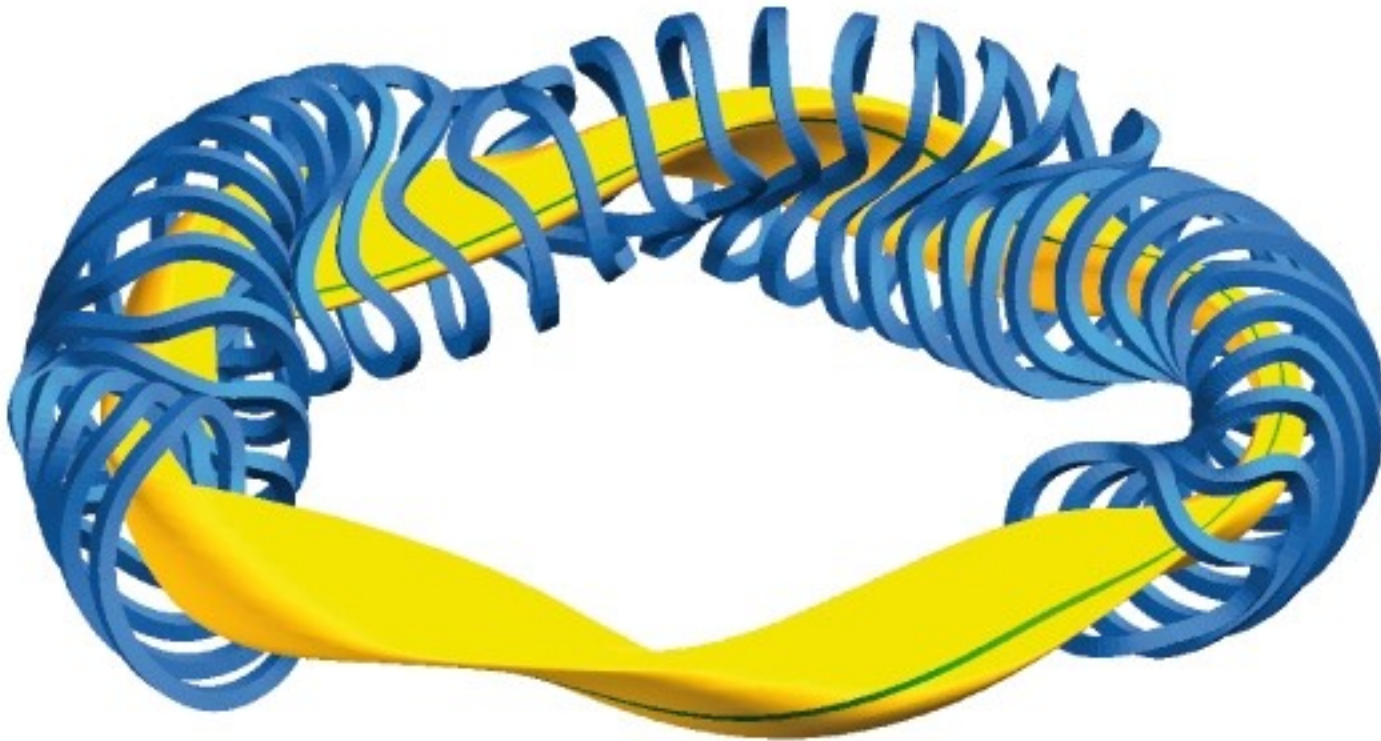
Toroidal Field Component

Transient Poloidal Field

Plasma Current

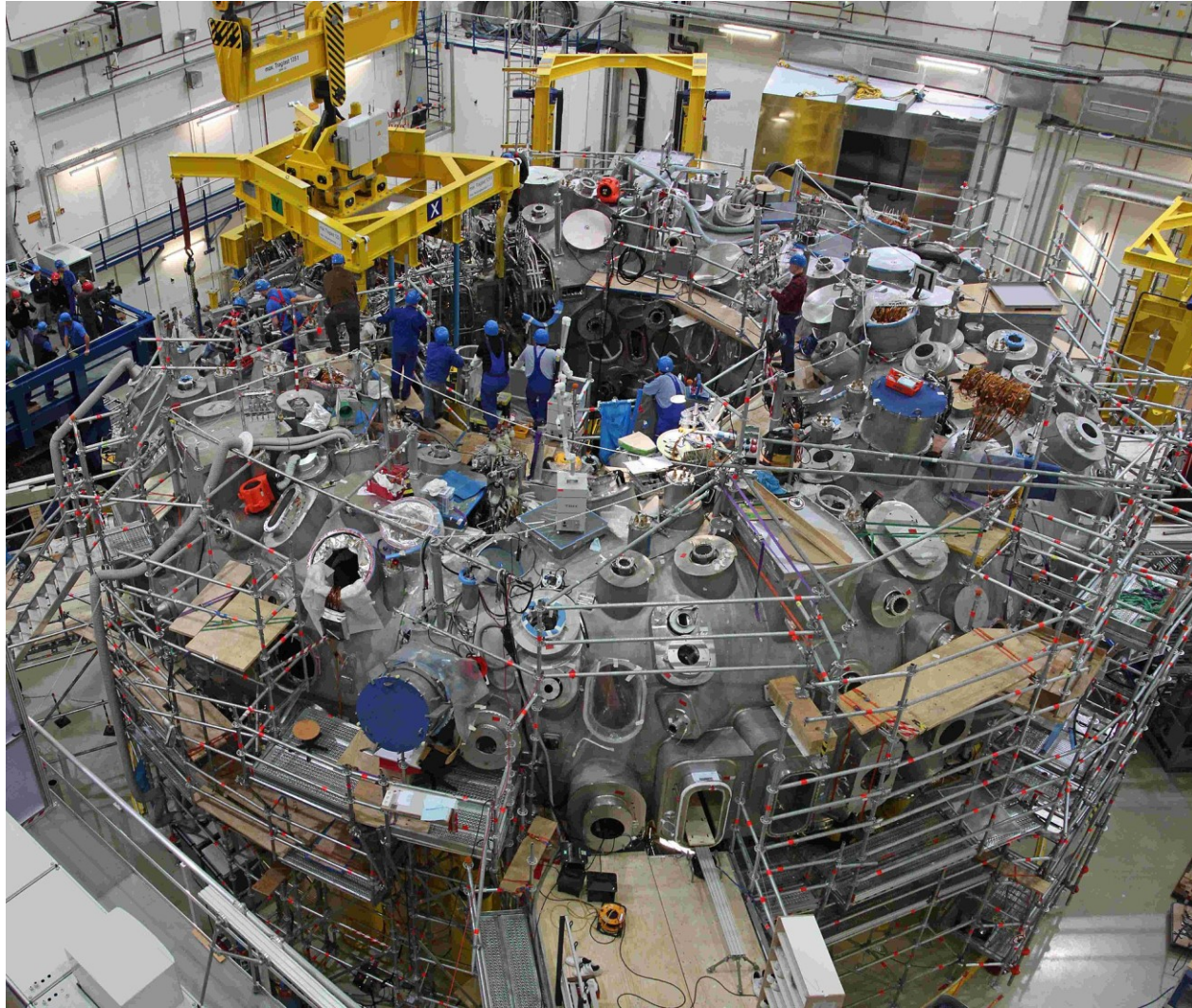
Resultant Transient Field

Magnetic confinement fusion - Stellarator



Highly tailored B field reduces gradB drift losses

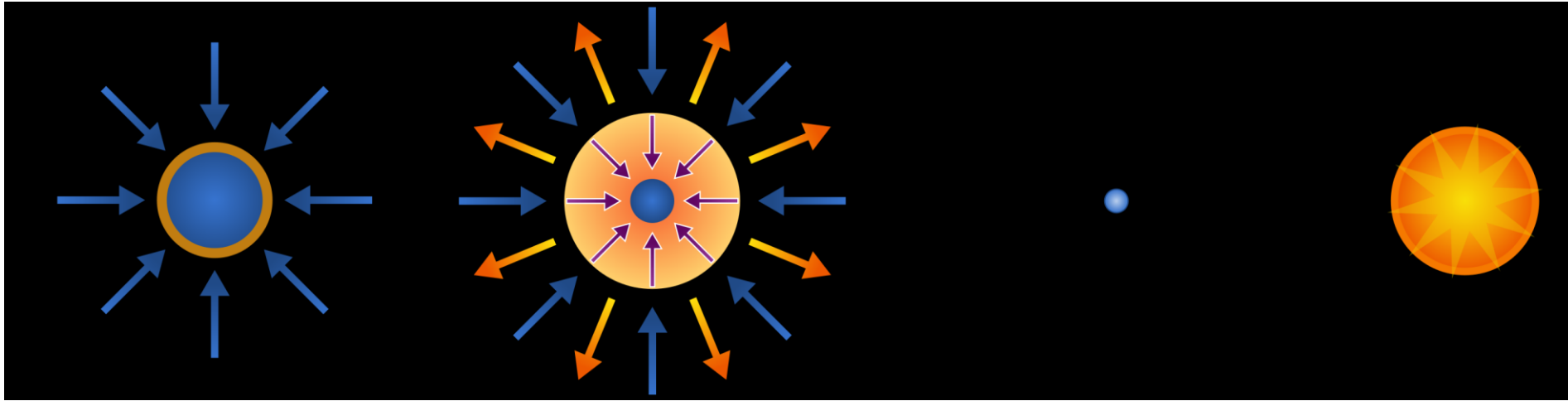
Magnetic confinement fusion - Stellarator



Wendelstein-7X in Greifswald is a very successful testbed for high power levels (even though steady-state demonstrations done with He, no fuel)

FUSION SETUPS – INERTIAL CONFINEMENT FUSION (ICF)

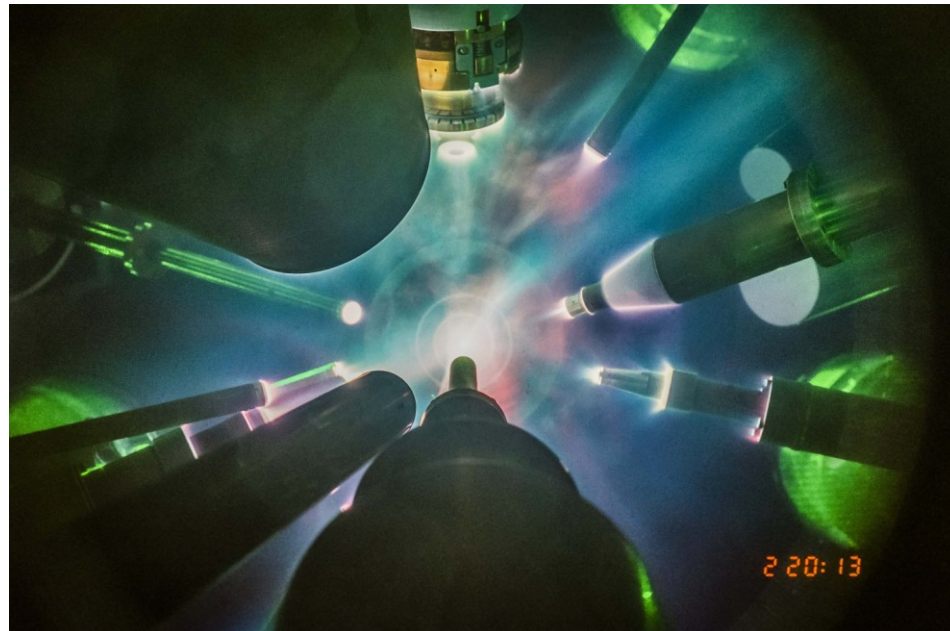
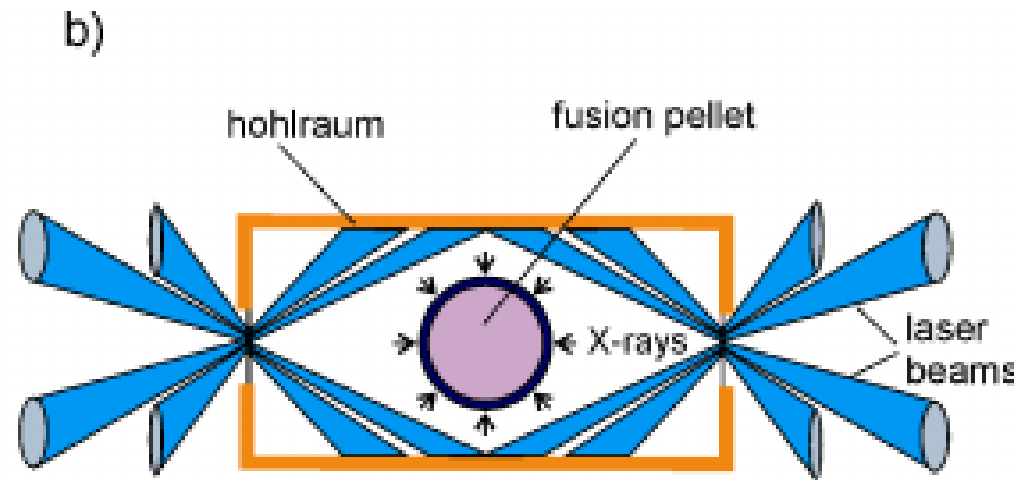
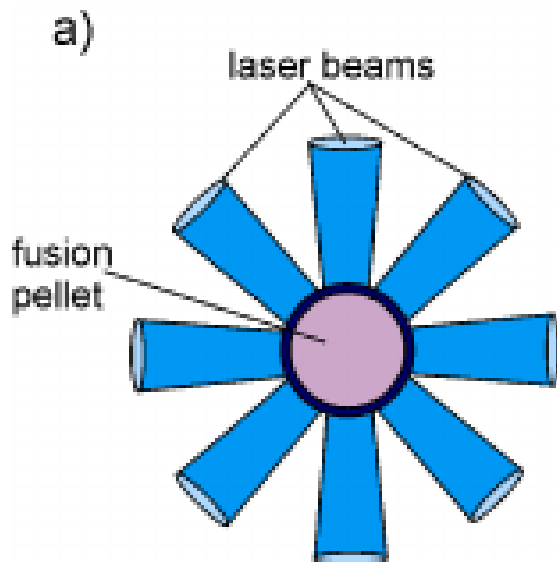
Inertial confinement fusion



Schematic of the stages of inertial confinement fusion using lasers. The blue arrows represent radiation; orange is blowoff; yellow is inwardly transported thermal energy.

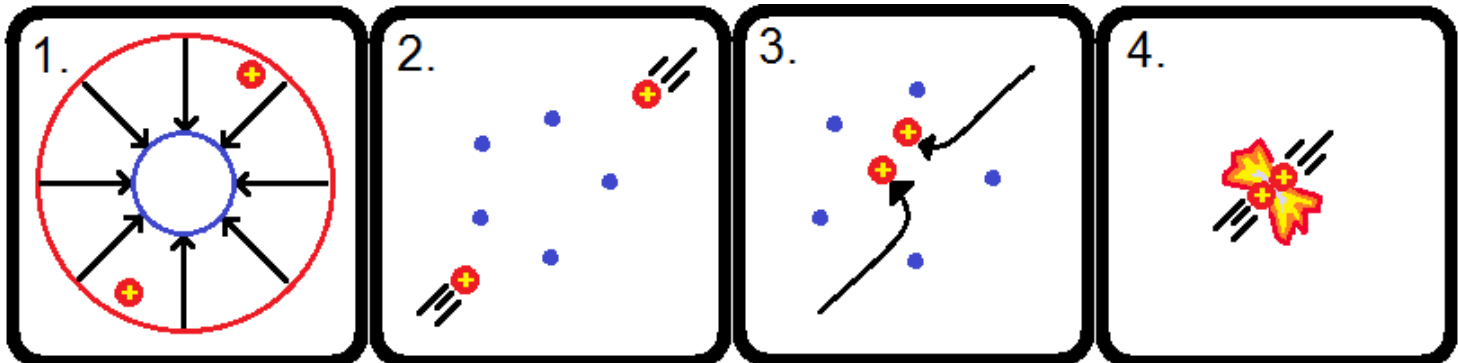
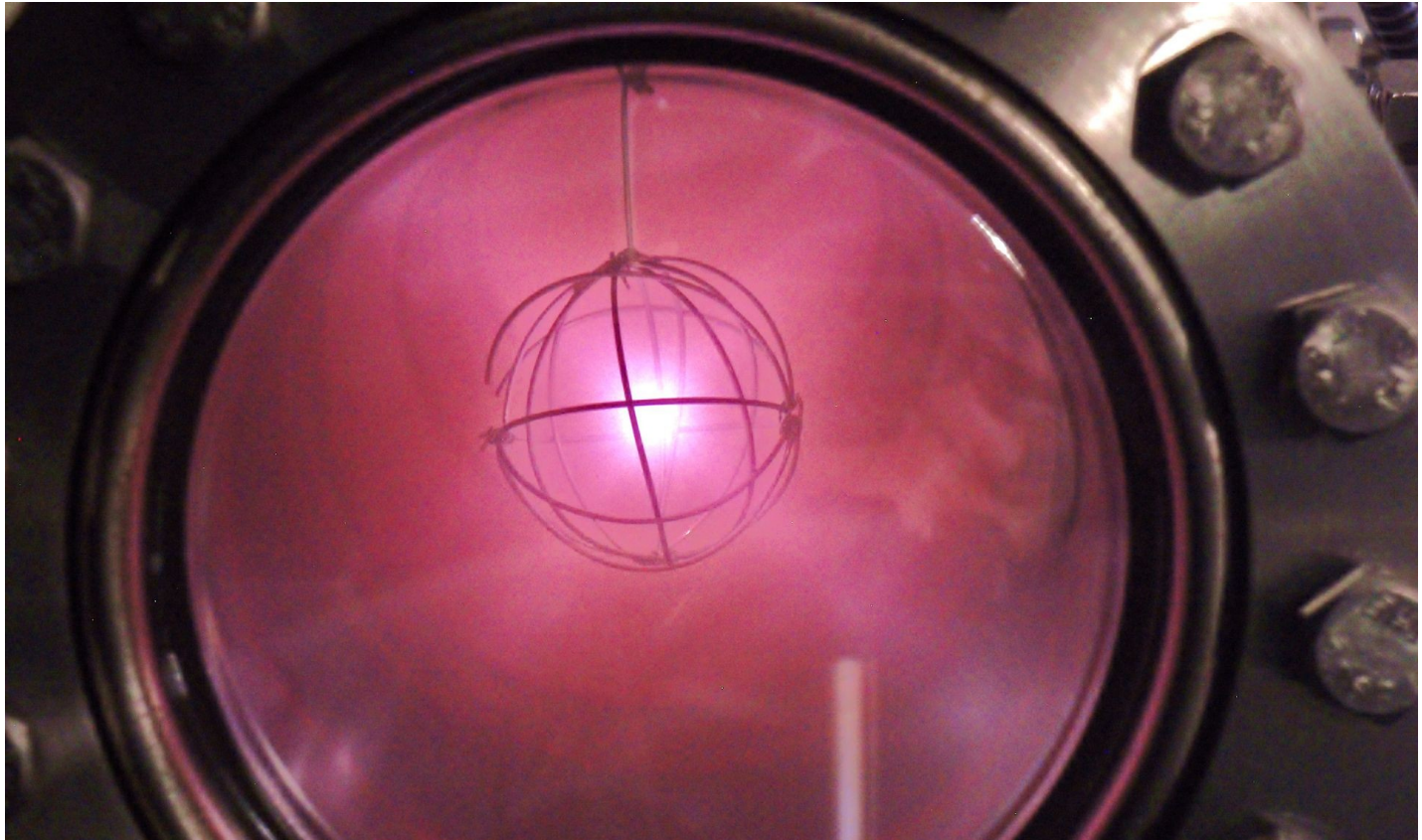
1. Laser beams or laser-produced X-rays rapidly heat the surface of the fusion target, forming a surrounding plasma envelope.
2. Fuel is compressed by the **rocket-like blowoff** of the hot surface material.
3. During the final part of the capsule implosion, the fuel core reaches 20 times the density of lead and ignites at 100 MK.
4. **Thermonuclear burn spreads rapidly** through the compressed fuel, yielding many times the input energy.

Inertial confinement fusion – direct vs indirect drive



FUSION SETUPS – INERTIAL ELECTROSTATIC CONFINEMENT (IEC)

Inertial electrostatic confinement



TAKE AWAYS

Take aways

- Main fusion reactions, optimum in the cross section.
- Lawson criterion and parameter, triple product.
- Different fusion setups with emphasis on MCF as the most widespread concept.