Paschen law

Contents

1 Part I: Paschen law

1.1 Derivation of the Paschen law

The gas in the plasma state is characterised by a large density of free electrons. While this is not the case of gas in less extreme conditions, it would be wrong to assume, that there are none - always a small number of "random" free electrons (and other exotic species) is present. These are produced by natural processes such as a stray high energy particle, radioactivity background or cosmic radiation. When the electric field is applied in a diluted gas, these random free electrons are accelerated to so high energy, that during collisions of these electrons with gas molecules, electron impact ionization occurs and the number of free electrons begins to grow exponentially. This is due to the Townsend avalanche process, depending on the path d, these electrons travelled.

$$n = n_0 \exp(\alpha d) \tag{1}$$

The linear coefficient α is denoted as the "first Townsend ionization coefficient" (it is the mean number of ionization collisions the electron goes through over a unit distance) and n_0 is the number of electrons at the position $d = 0$. Let the electric field be a product of voltage U applied between two parallel flat electrodes in a diluted gas, their distance being d. At the beginning of the electron avalanche, $n - n_0$, or better $n_0[\exp(\alpha d) - 1]$ new ions are created. These ions drift in the electric field towards the cathode and fall to its surface. At their impact, the secondary emission of electrons from the cathode surface occurs. The coefficient of the secondary emission γ indicates the mean number of electrons emitted per one ion impact. The emission by virtue of electron-positive ion pair creation is also added to the value of γ by some authors.

The condition for the discharge ignition can be written as

$$\gamma(\exp(\alpha d) - 1) = 1 \tag{2}$$

meaning that in the avalanche one primary electron must produce enough ions, to yield one new electron from their impact to the cathode (by secondary emission). The Townsend ionization coefficient α is a function of electric field intensity E as follows (see also the "Measurement of the first Townsend coefficient")

$$\frac{\alpha}{p} = A \exp\left(-\frac{Bp}{E}\right) \tag{3}$$

where p is the gas pressure, $A = 1/p\lambda_\mathrm{e}$ and $B = U_\mathrm{i}/p\lambda_\mathrm{e}$ are the linear coefficients, that dependent on the choice of gas, λ_e is the mean free path of electrons and U_i is the gas ionization potential.

Substituting (3) into (2) leads to

$$\gamma \left[\exp\left(Apd \exp\left(-\frac{Bpd}{U_\mathrm{z}}\right)\right) - 1\right] = 1 \tag{4}$$

Which after some rearranging and applying logarithm

$$\exp\left[Apd \exp\left(-\frac{Bpd}{U_\mathrm{z}}\right)\right] = \frac{1}{\gamma} + 1 \tag{5}$$

$$Apd \exp\left(-\frac{Bpd}{U_\mathrm{z}}\right) = \ln\left(\frac{1}{\gamma} + 1\right) \tag{6}$$

Now let's define a constant C (for specific cathode material and gas)

$$\ln\left(\frac{1}{\gamma}+1\right)=C \tag{7}$$

The equation (6) then yields

$$\exp\left(-\frac{Bpd}{U_\mathrm{z}}\right)=\frac{C}{Apd} \tag{8}$$

$$-\frac{Bpd}{U_\mathrm{z}}=\ln\left(\frac{C}{Apd}\right) \tag{9}$$

$$U_\mathrm{z}=\frac{Bpd}{\ln\left(Apd/C\right)} \tag{10}$$

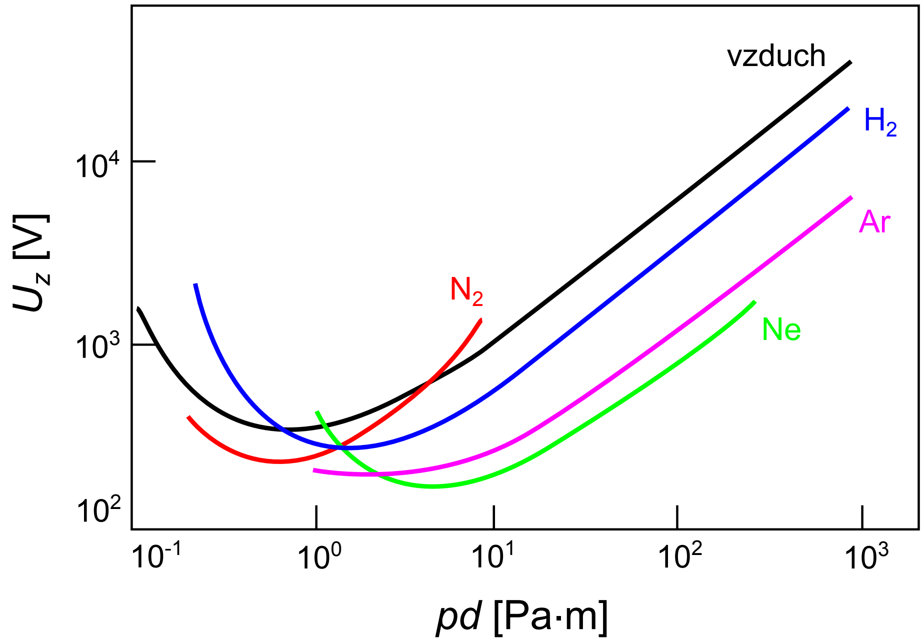

Figure 1: Paschen curves for different gases.

The U_z dependency on the product (pd) is called Paschen law. This function is characterised by the presence of minimum. (see Fig. 1). Decreasing the pressure leads to increase in the mean free path and therefore, should the electron perform the same number of ionizing collisions on the path between electrodes, the electrode distance d must be increased. On the other hand, with growing pressure and dropping mean free path, the path of an electron between collisions is short. In order to gain enough energy to ionize gas molecules, the electric field $E=U_\mathrm{z}/d$ has to be increased - therefore either we change the distance d or the discharge will be ignited at higher U_z. The idea of changing d is a bit more intuitive, so we focus on that. With increasing d the electric field intensity E decreases and, should the electron reach the threshold of the ionization energy, the U_z has to increase. With decreasing d the total interaction length of the electron is decreased, resulting in lower number of collisions and ionizations. This again leads to increase in U_z.

1.2 Measurement

The discharge tube with adjustable electrodes (d=1-50 mm) is used for the Paschen law verification. The gas pressure varies in the range $10 - 500\,\mathrm{Pa}$ and is maintained by the dynamic equilibrium between rotary vane pumping speed and the flow of the gas (atmospheric pressure air) through the needle valve. Pirani gauge is used to measure the pressure and the ignition voltage is determined from the voltmeter with high internal resistance. The circuit is shown in the figure 2.

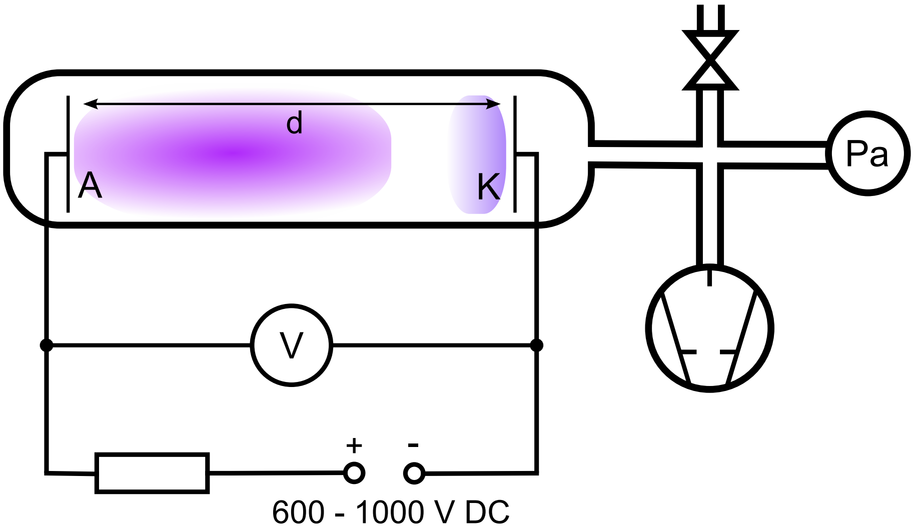

Figure 2: Diagram of the circuit sued for the measurement of Paschen curve.

The voltage applied across the discharge tube is increased until the discharge is ignited. The ignition itself is apparent from the steep drop in the discharge voltage, caused by the shunt resistance. The voltage value immediately before the point of ignition is the relevant one/is recorded. Afterwards the voltage needs to be lowered to zero value and next measurement is to be performed, but only after a short while (1 minute) so that the charges in the discharge tube volume completely recombine. In first series of measurements the function $U_\mathrm{z} = f(pd)$ for constant pressure p is investigated and plotted , while in the other measurement series the distance d is fixed and the pressure is varied varies.

2 Part II: Current-voltage characteristics of the glow discharge

2.1 Current–voltage characteristic of the self-sustained discharge

The electric discharges always contain a certain number of free electrons and ions, with the electric conductivity of the gas is not constant but rather a function of the discharge current. The classification of different discharges can be based on more factors - for example their light emission properties but also their electric properties and their current–voltage characteristic. The current–voltage characteristic of a discharge is determined by the measurement of the voltage across the discharge tube as a function of discharge current, $U = f(I)$. The current-voltage characteristic for the self-sustaining discharge is shown in the fig. 3. Changing the discharge current greatly influences the nature of an electric discharge transiting from one type to another, starting with the dark discharge and ending with an arc discharge. It is apparent, that the characteristic is strongly non-linear with some regions, where the current grows at constant or even decreasing voltage.

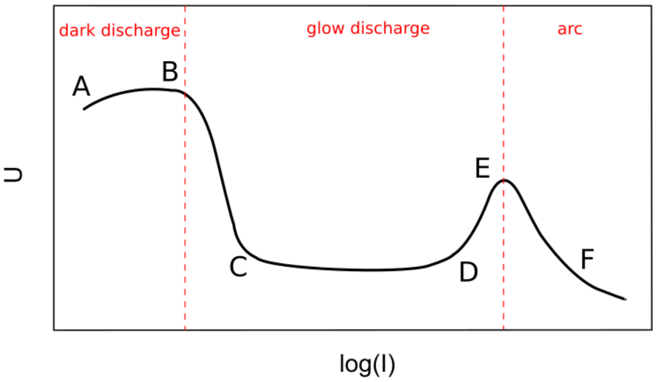

Figure 3: Current-voltage characteristic of the self-sustaining discharge (logarithmic scale is used for the current axis).

The first region AB belongs to the dark discharge, which is very weak, and therefore virtually invisible for a human eye. In this region, the Townsend electron avalanches are already present, however, the glow discharge can't be ignited until the ions bombarding the cathode reach the secondary emission threshold energy.

The next region characterises the normal glow discharge. In the BC part the current grows despite the decreasing voltage. This phenomenon can be explained by the creation of space charge and the alteration of the original electric field produced by the voltage applied on the electrodes. If the pattern of the space charge induced electric field favours the ionization in the discharge, the discharge current will increase spontaneously. During this process, the voltage on the electrodes may drop, thus yielding the decreasing part of the current-voltage characteristic. In the opposite case, where the space charge induced electric field inhibits the ionization in the discharge (worse conditions), the discharge current can be only increased by increasing the applied external voltage. This is the case of the increasing part of the current-voltage characteristic curve.

The voltage in the region CD is almost independent of current, with the current range over several orders of magnitude. The increase of the current in this region cannot be explained by the growth of drift velocity for charged particles, since the voltage and therefore the electric field remains constant. Therefore the growth of the discharge current has to be caused by increasing the number of charged particles moving through the discharge tube cross-section. In the positive column, this effect is ensured by the growth of the charged particle concentration, but this is not the case of cathode region. At first the only a small fraction of the cathode surface is in contact with plasma. However the discharge current increases, the contact area becomes larger until at last the whole area of cathode is covered with plasma - this is the case of point D. beyond this, the discharge enters the region of anomalous glow discharge. The voltage in the cathode layer sharply increases, faster than the voltage drop in positive column. The result is the increasing curve in the current-voltage characteristic region DE.

the current in the anomalous glow discharge is so high, that the incident ions heat the cathode to the point, where the thermoemission of electrons starts an the discharge transits into an arc, represented by region F in the figure 3.

2.2 Measurement

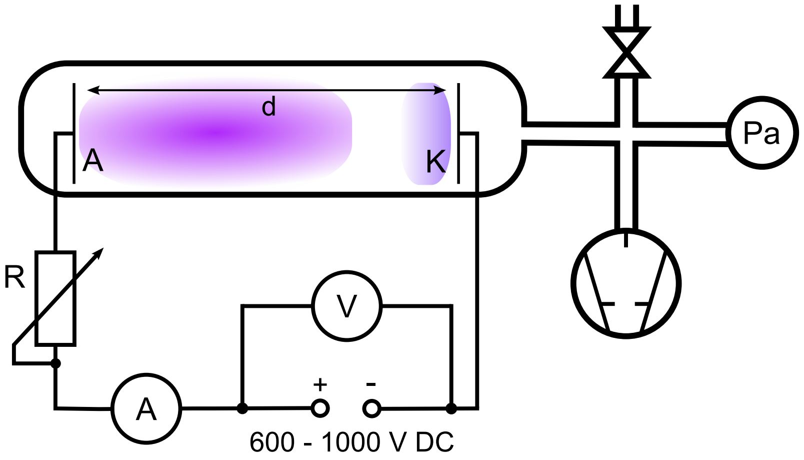

Figure 4: The circuit diagram for the current-voltage characteristic measurement

In this part of the laboratory work, the current-charge characteristic of the glow discharge will be measured. The discharge tube is pumped by the rotary vane, while at the same time, the atmospheric air is admitted into the system through the needle valve. The pressure, measured by the Pirani gauge, adjustment is therefore achieved simply by changing the mass flow rate of the air. Furthermore the current in the circuit I and source output voltage U_z are measured. The voltage applied across the discharge tube U_v is determined from the relation $U_\mathrm{v} = U_\mathrm{z} - RI$, where R is the preset value of an adjustable resistor. For the characteristic measurement, the current will be varied over a few orders of magnitude (μA - mA), which means that the ampermeter ranges have to be switched. Given that there is a definite error for each of them, it may happen that the values at the point of switching do not match properly. Considering this, the measurement procedure will be as follows - at a given range the characteristic is determined from maximum current all the way down to the zero value. This is repeated for all relevant ranges. When the data will be processed, one reference range is chosen and the data from other ranges are compared and scaled to fit the reference data.

The measurement in the configuration seen on fig. 4 is repeated for three different electrode distances. The current is set by the switchable resistor R. All three curves $U_\mathrm{v} = f(I)$ should be ultimately plotted in one graph.

3 Part III: Cathode fall in the glow discharge

3.1 Glow discharge

The discharge, which has its produced particles or photons impact the cathode so that the emission of electrons occurs is called a glow discharge. The field at the cathode is defined by the volume charge, with and thermal effects being not essential for sustainment of such discharge. In the 10-100 Pa range of pressures, the alternating bright and dark regions can be observed in the glow discharge, see the figure 5.

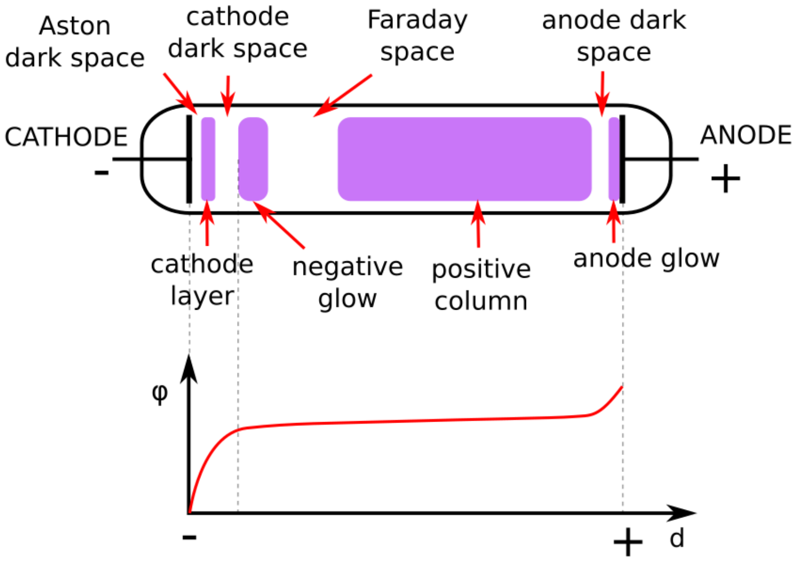

Figure 5: The glow discharge with its typical distribution of bright and dark regions and the potential between the electrodes.

The formation of individual regions can be explained, by investigating the movement of electrons from cathode to anode. The electron starts at the cathode with low energy. Until its energy reaches the lowest excitation potential (5-10 eV), it cannot excite the molecules. This region corresponds to Ashton dark space. In the cathode glow, the electron energy has reached maximum of the excitation function. Then in the cathode dark space, the majority of electrons has higher energy than, the the optimum for excitation, resulting in the light intensity decrease. Eventually the electron energy reaches ionization threshold, so a large number of new electron-ion pairs with low energy is created in this region. The slow electrons travel to the anode, gaining energy and exciting molecules in the negative glow. This time the electron energy is more efficiently depleted and again drops bellow the excitation threshold, which explains the Faraday dark space. Near the end of this region the electric field intensity is slowly increasing. The electrons are slowly gaining energy and the rate of recombination with ions decreases. Over some distance the electrons reach the excitation threshold and the positive column appears. The magnitude of the electric field inside the positive column is much lower (by a few orders of magnitude) than in the cathode region. In this region excitation and ionization processes are mostly caused by the chaotic motion of electrons. At the anode end of the positive column, the anode drop of potential occurs due to the presence of a space charge. The electrons exit the positive column with low energy, but are accelerated in the anode dark space, which is also the reason anode glow.

3.2 The normal cathode fall

The term cathode fall is usually understood as the potential drop between the sharp edge of the negative glow and the cathode. The magnitude of the cathode fall can be determined from the stationary state of the glow discharge. The stationary state condition is, that each electron emitted by the cathode must produce as many ions, metastables, photons etc., as it takes to emit next electron. As we are already aware, γ is the number of electrons emitted by cathode per one ion impact (and in this case, also the contribution from other particles is included). The coefficient α has also been defined - it is the number of electron-ion pairs created by an electron over the unit length along the field. The total number of electrons, that arise as a result of single one electron emitted from the cathode in the distance d_k from the cathode (i.e. at the boundary of the cathode dark space and the negative glow) is

$$\exp\left(\int_0^{d_\mathrm{k}} \alpha \,\mathrm{d}x\right) \tag{11}$$

The ion count is naturally lower by one. Therefore the stationary state condition implies

$$\gamma\left[\exp\left(\int_0^{d_\mathrm{k}} \alpha \,\mathrm{d}x\right)\right] = 1 \qquad \text{and further} \qquad \int_0^{d_\mathrm{k}} \alpha \,\mathrm{d}x = \ln\left(1+\frac{1}{\gamma}\right) \tag{12}$$

The integral indicates the number of ionizations in the interval d_k. It can be approximated as

$$\int_0^{d_\mathrm{k}} \alpha \,\mathrm{d}x = \bar{\alpha} d_\mathrm{k} \doteq \frac{U_\mathrm{k}}{\eta} \tag{13}$$

Where $\bar{\alpha}$ is the mean value of the ionization coefficient in the related interval, U_k is the cathode fall of potential and η is the potential difference elapsed by an electron per one electron-ion pair created. Combining the equations (12) and (13) yields

$$U_\mathrm{k} = \eta \ln\left(1+\frac{1}{\gamma}\right) \tag{14}$$

Looking at (14), it becomes apparent, that the cathode fall of potential U_k is dependent on the electrode material (hidden in γ) and on the gas (hidden in η)

3.3 Measurement

The circuit for this measurement in figure 6. The discharge tube is again pumped while admitting certain flow rate of air. The discharge current is maintained at a constant value. The dependence of the voltage across the discharge wrt. the electrode distance is measured, while the anode is moving towards the cathode. As soon as the anode reaches the negative glow, the anode glow disappears. Further decreasing the electrode distance leads to increase of the voltage across the discharge. This can be explained by the process of ionization being more difficult, due to the radiation, which plays an important role in the mechanism of cathode fall formation, is suppressed. If we plot $U = f(d)$ at the constant discharge current, then the voltage minimum U_k is equal to the cathode fall value. The measurement should be performed for three different discharge currents in the "normal glow" range.

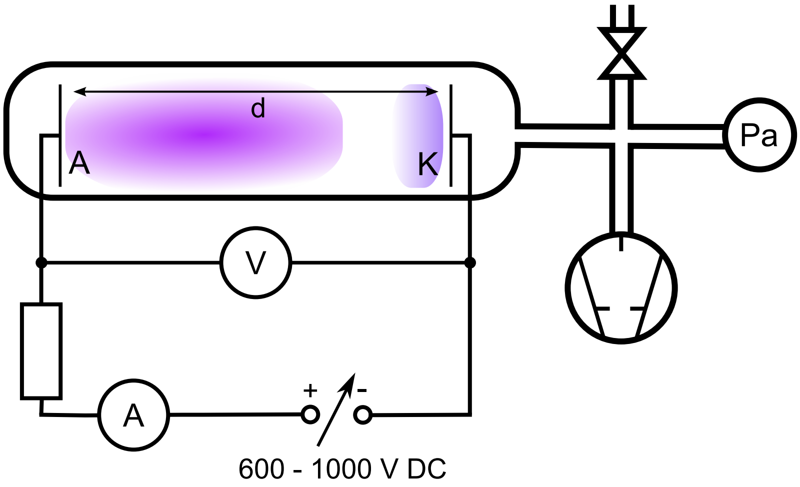

Figure 6: Diagram of the circuit for cathode fall measurement.

4 Assignments

Observe the visual appearance and the changes of the discharge during the whole laboratory measurement and describe them in your laboratory report.

1. Paschen law
    - Measure the Paschen curve for different electrode distances d and fixed pressure p. Plot these curves into single graph and fit them using the equation (10).
    - Measure the Paschen curve for different pressures p and fixed electrode distance d. Plot these curves into single graph and fit them using the equation (10).
2. Current-voltage characteristic of the glow discharge
    - Determine the current-voltage characteristic of a self-sustaining discharge for three different electrode distances.
3. Cathode fall of the glow discharge
    - Measure the normal cathode fall in the glow discharge for three different discharge currents

While the experimental set-up is not particularly complicated, its parts look slightly confusing. For orientation throughout the experiment, please see the figure 7

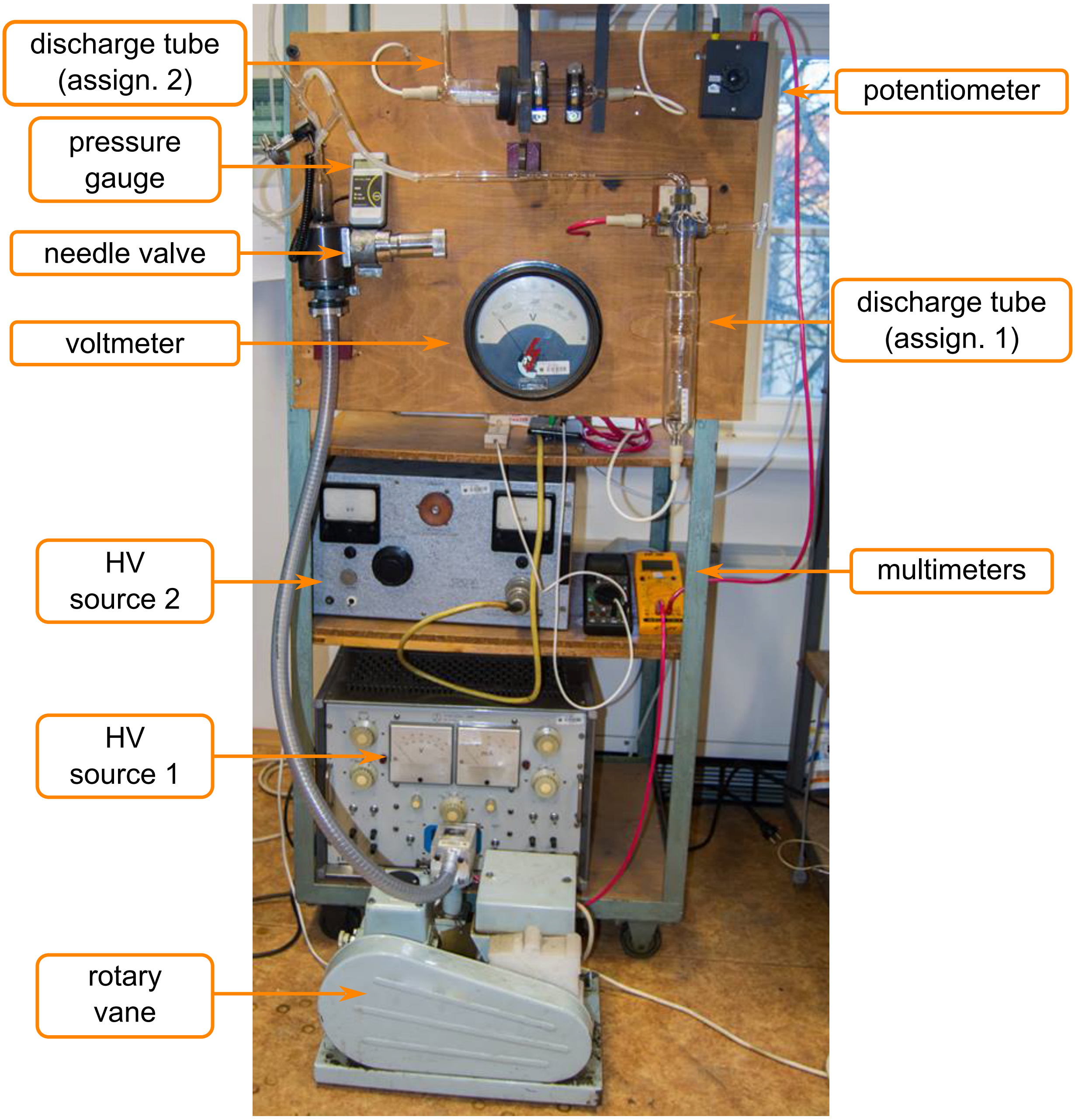

Figure 7: Photo of the experimental set-up. 1 - discharge tube with adjustable anode, 2 - voltage source, 3 - ampermeter, 4 - voltmeter, 5 - rotary vane, 6 - Pirani gauge, 7 - needle valve for the gas inlet