Condensed Matter II

Problem #1

Spring 2023

1 C_{3v} group representation

1.1 Background

The group of symmetry operations of the equilateral triangle, C_{3v}, is isomorphic to the group of permutations of three objects $P(3)$. The elements of the group $P(3)$ are:

E =(123) A =(132) B =(321) C =(213) D =(312), F =(231), in which each parenthesis indicates the final order of the initial elements (123).

The elements of the group C_{3v} are (Schoenflies notation):

- E (identity)
- rotations $C_3(1)$ about the center of the triangle, by angle $2\pi/3$.
- rotations $C_3(2)$ about the center of the triangle, by angle $4\pi/3$.
- reflection $\sigma_v(1)$ with respect to the vertical plane containing vertex 1, and the center of the triangle.
- reflection $\sigma_v(2)$ with respect to the vertical plane containing vertex 2, and the center of the triangle.
- reflection $\sigma_v(3)$ with respect to the vertical plane containing vertex 3, and the center of the triangle.

1.2 Questions

(i) Prove that $P(3)$ and C_{3v} are isomorphic.

(ii) Find the periods of the group (the Abelian subgroups $\{E, A, A^2, \ldots, A^{n-1}\}$ where n is the period of element A).

(iii) Find the subgroups of the group.

(iv) Determine the classes (the set of all elements associated to the others in the set through the relation: $B \sim A \Leftrightarrow \exists X \in G, B = XAX^{-1}$)

(v) Find several representations (groups isomorphic to the group of square matrices).

(vi) Find the irreducible representations and determine the character table.