Condensed Matter II

Problem set #3

Spring 2023

1 T_d group representation

1.1 Background

The group of symmetry operations of the regular tetrahedron T_d is isomorphic to the group of permutations of four objects $P(4)$.

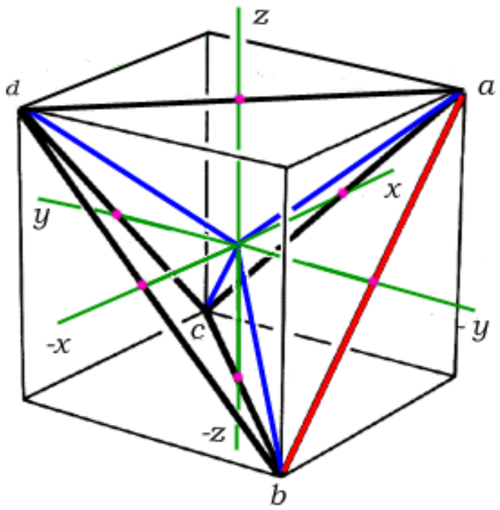

Figure 1: Regular tetrahedron with vertices abcd.

The elements of the group T_d are (Schoenflies notation):

- E (identity)
- 8 rotations C_3 about the diagonals of a cube.
- 3 rotations C_2 about axes x, y, z.
- 6 improper rotations S_4 about axis x, y, z (rotations of angle $\pi/2$ followed by a reflection in a plane perpendicular to the axis of rotation).
- 6 reflections σ_d in planes containing one edge and the center of the tetrahedron.

The elements of the group $P(4)$ are:

- $E =$(abcd) $A =$(acbd) $B =$(cbad) $C =$(bacd) $D =$(cabd) $F =$(bcad) (perm. abc;d)
- $G =$(abdc) $H =$(adbc) $J =$(dbac) $K =$(badc) $L =$(dabc) $M =$(bdac) (perm. abd;c)
- $N =$(adcb) $O =$(acdb) $P =$(cdab) $Q =$(dacb) $R =$(cadb) $S =$(dcab) (perm. acd;b)
- $T =$(dbca) $U =$(dcba) $V =$(cbda) $W =$(bdca) $X =$(cdba) $Y =$(bcda) (perm. bcd;a)

1.2 Questions

(i) Show that $P(4)$ and T_d are isomorphic.

(ii) Partition the elements of $P(4)$ so that the elements of each subset have the same order (the order n of element X is the smallest $n \in \mathbb{N}$ such that $X^n = E$).

(iii) Determine at least 10 subgroups. Among those, determine the subgroups isomorphic to $P(3)$.

(iv) Explain why $\{E, G, N, T\}$ is or is not a subgroup.

(v) Determine the classes (sets of equivalent elements, through the relation $A \sim B \Leftrightarrow \exists X \in G, B = XAX^{-1}$)

(vi) Find several representations.

(vii) Determine the irreducible representations, and the character table of the group.