

Condensed Matter II

Problem set #4

Spring 2023

1 Free electrons in Na

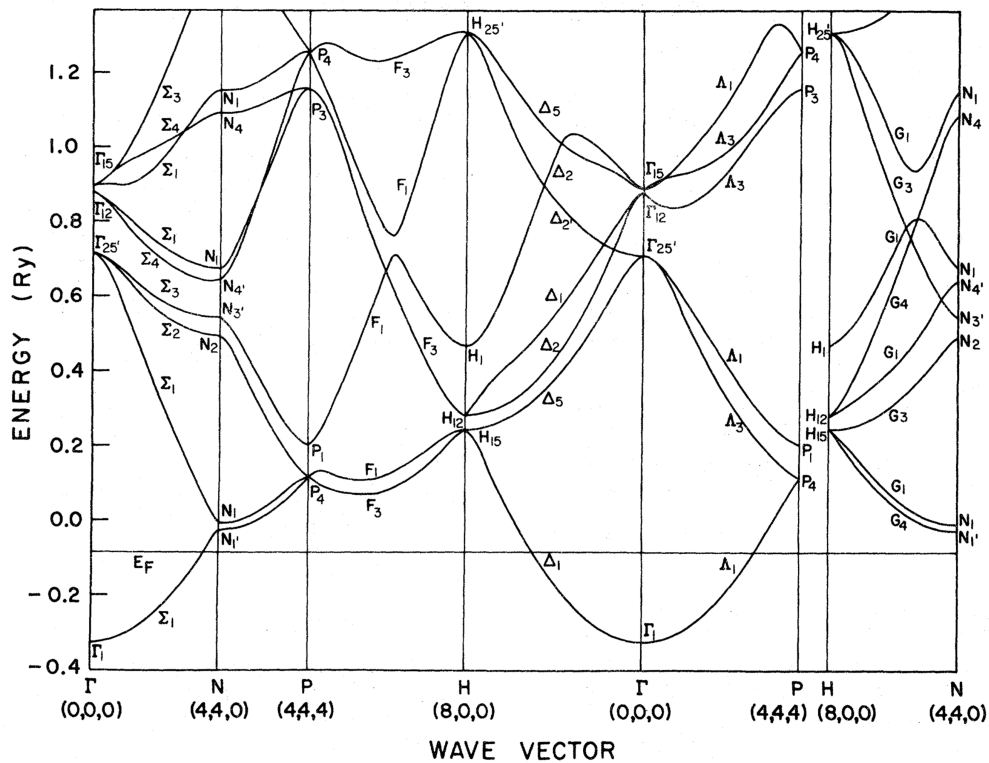


Figure 1: Energy bands in sodium along symmetry directions. Chang and Callaway, PRB 11, 1324 (1975)

1.1 Qualitative study

- (i) With the help of the computed band structure of Na in Fig. 1, suggest what the structure of the empty BCC lattice must look like (assuming isotropic effective mass m_0).
- (ii) How high above the bottom of the conduction band is the Fermi energy located in this model?
- (iii) Justify that Na may be considered “the simplest of the simple metals”.

1.2 Quantitative study

- (i) Use Bloch's theorem to give the expression of the energy $E_l(k)$ in a generic 3D crystal, where $l \equiv (l_1, l_2, l_3)$ is the band index, and k is a wave vector.
- (ii) A schematic of the Brillouin zone of the BCC lattice is provided in Fig. 2. Use the previously obtained result, and derive the expression of the energy levels at the special points Γ , H , P , and N in the empty BCC lattice.

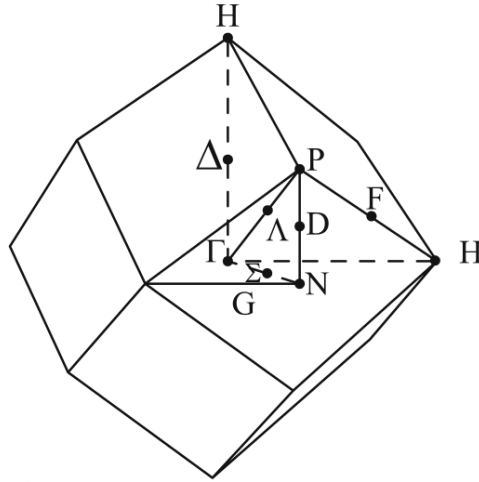


Figure 2: Brillouin zone for the body-centered cubic lattice. M. Dresselhaus, Group theory - Application to the Physics of Condensed Matter, Fig. C.5.b, p 511.

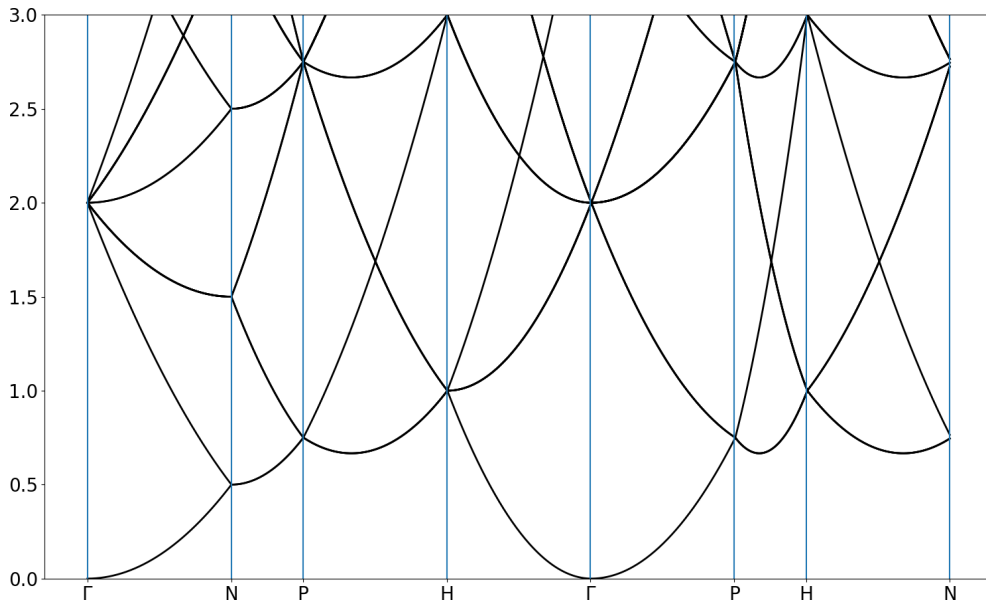


Figure 3: Empty lattice.