The binding energy of the atomic nucleus

The general description of a nuclear reaction is

$$I(A_i, Z_i) + J(A_j, Z_j) \leftrightarrow K(A_k, Z_k) + L(A_l, Z_l)$$

Where A_i = *the baryon number,* or *nucleon number (nuclear mass)*

and Z_i = *the nuclear charge*

The nucleus of any element is uniquely defined by the two integers A_i and Z_i

Recall that in any nuclear reaction the following must be conserved:

1. The baryon number – protons, neutrons and their anti-particles
2. The lepton number – light particles, electrons, positrons, neutrinos, and anti-neutrinos
3. Charge

Note also that the anti-particles have the opposite baryon/lepton number to their particles

The binding energy of the atomic nucleus

- The total mass of a nucleus is known to be less than the mass of the constituent nucleons
- Hence there is a decrease in mass if a companion nucleus is formed from nucleons, and from the Einstein mass-energy relation $E = mc^2$ the mass deficit is released as energy
- This difference is known as the *binding energy* of the compound nucleus
- Thus if a nucleus is composed of Z protons and N neutrons, it's binding energy is

$$Q(Z,N) \equiv [Zm_p + Nm_n - m(Z,N)]c^2$$

- For our purposes, a more significant quantity is the total *binding energy per nucleon*
- We can then consider this number relative to the hydrogen nucleus

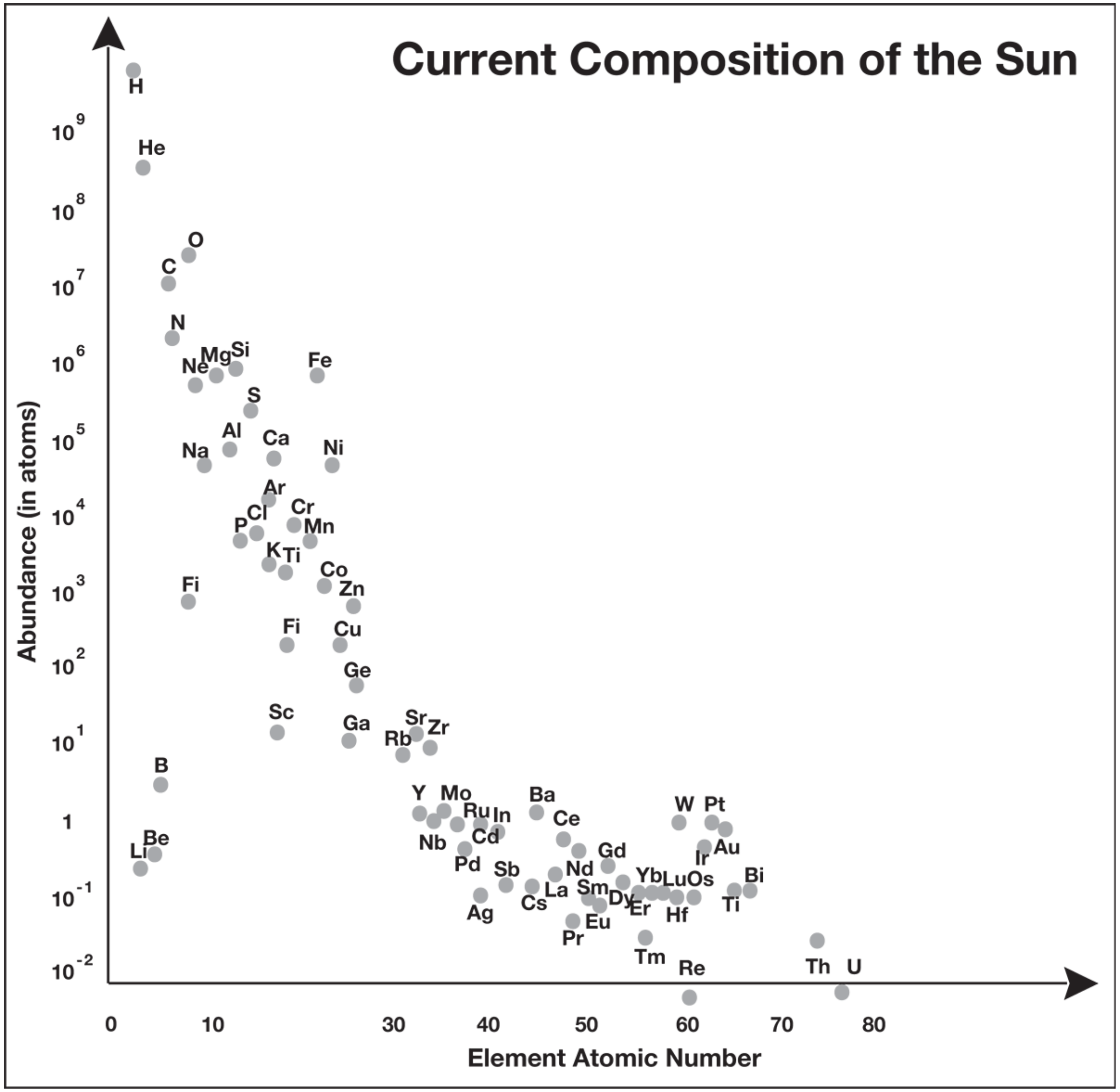
$$\frac{Q(Z,N)}{A}$$

The binding energy per nucleon

The variation of binding energy per nucleon with baryon number *A*

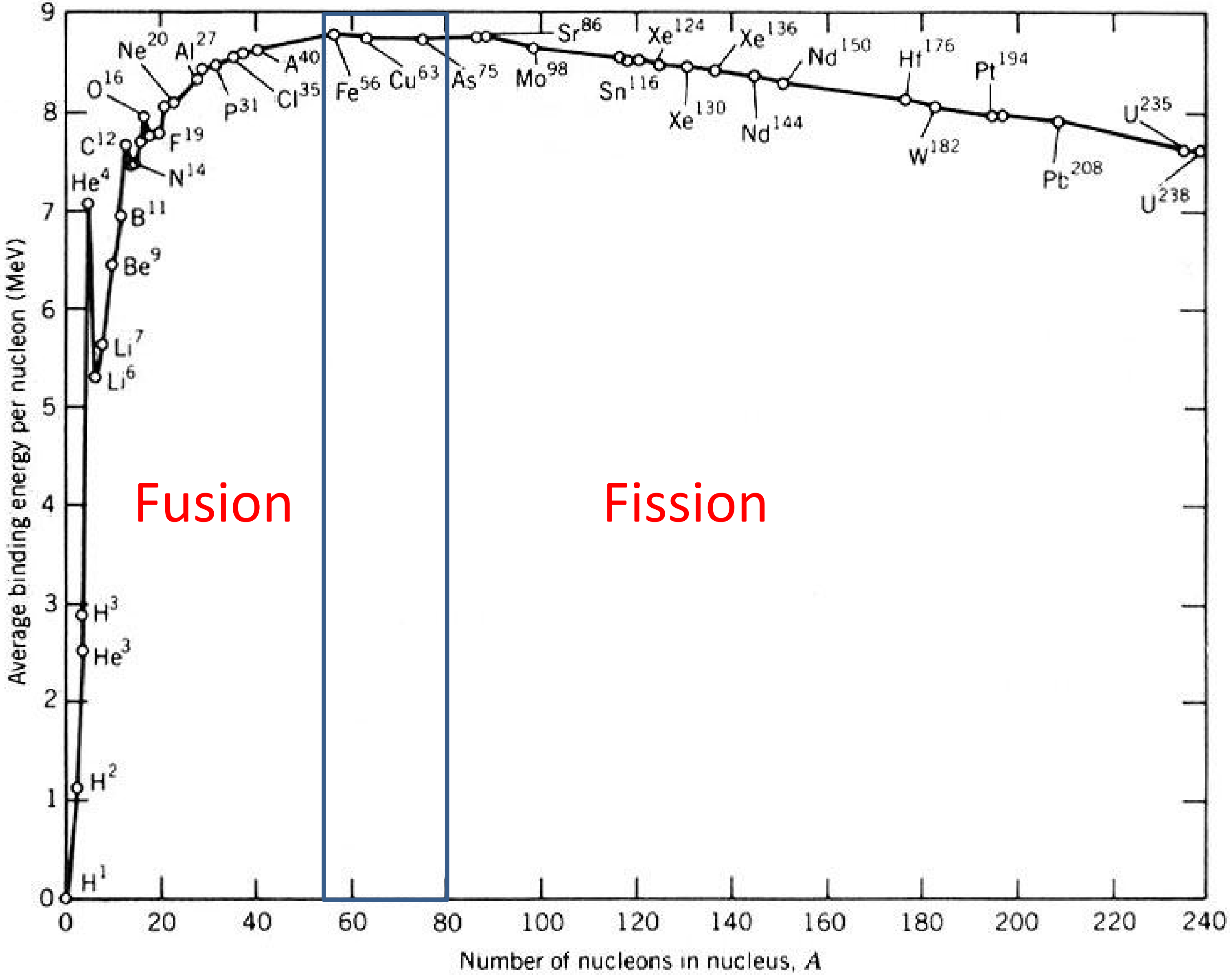
- General trend is an increase of *Q* with atomic mass up to *A*= 56 (Fe), then slow monotonic decline
- There is steep rise from H through ^{2}H, ^{3}He, to ^{4}He => fusion of H to He should release larger amount of energy *per unit mass* than fusion of He to C
- Energy may be gained by *fusion* of light elements to heavier, up to iron
- Or from *fission* of heavy nuclei into lighter ones down to iron

Abundance - Sun

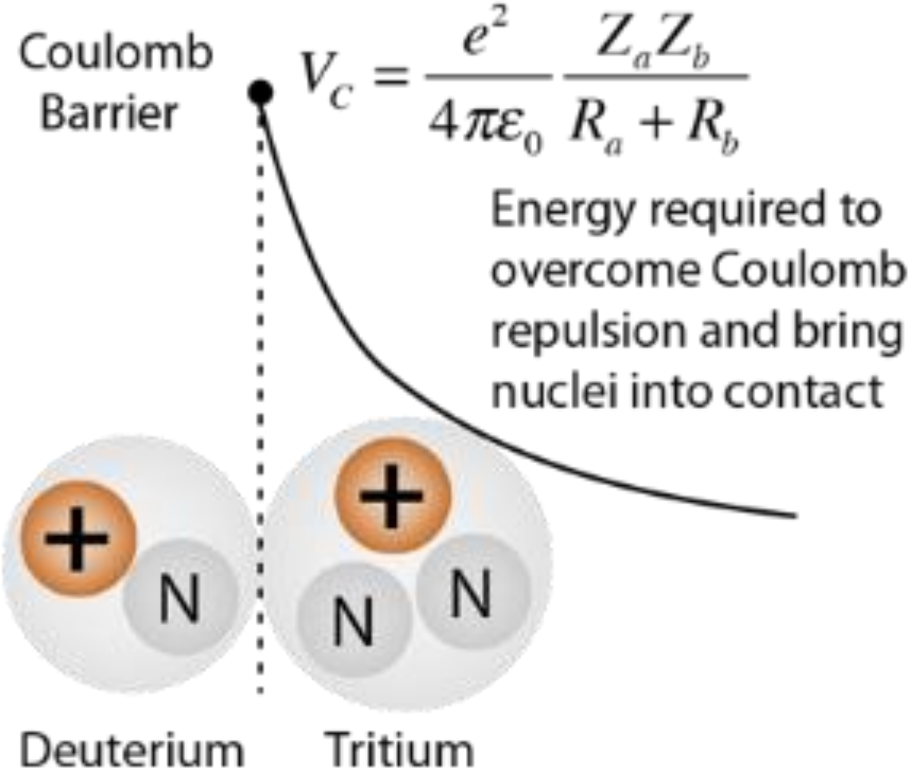


Asplund et al.

The binding energy per nucleon



Coulomb Barrier



$Z_a Z_b$ … number of protons in each nuclei

$R_a R_b$ … interaction radii

ε_0 … permittivity of free space (8.85×10^{-12} C^2N^{-1} m^{-2})

e … charge of electron

D-T reaction: V_C is 0.38 MeV

Gas temperature of

4.4×10^9 K

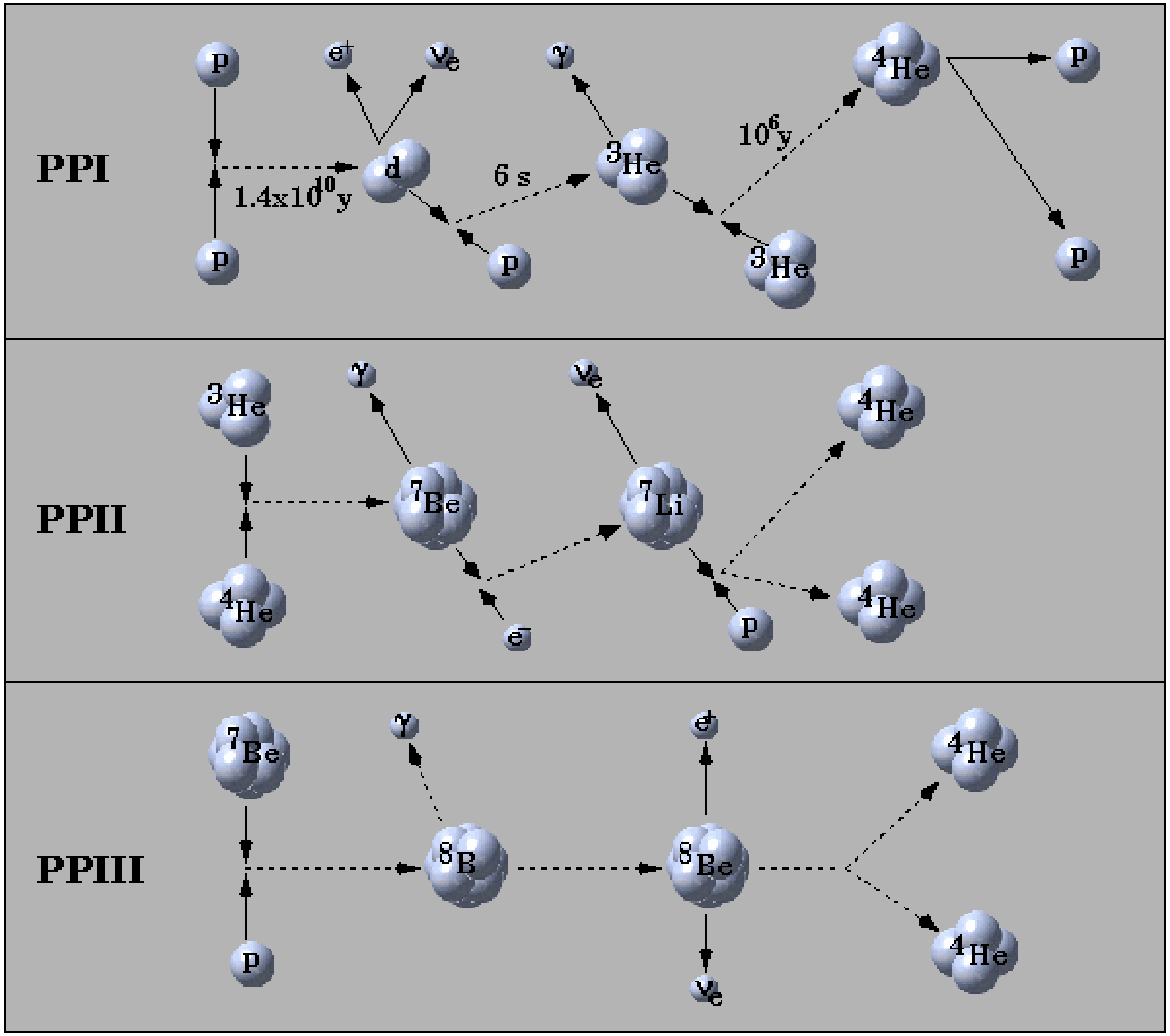
Hydrogen and helium burning

The most important series of fusion reactions are those converting H to He (H-burning). As we shall see this dominates ~90% of lifetime of nearly all stars.

- Fusion of 4 protons to give one ^4He is completely negligible
- Reaction proceeds through steps – involving close encounter of 2 particles
- We will consider the main ones: the **PP - chain** and the **CNO cycle**

The PP - chain has three main branches called the PPI, PPII and PPIII chains.

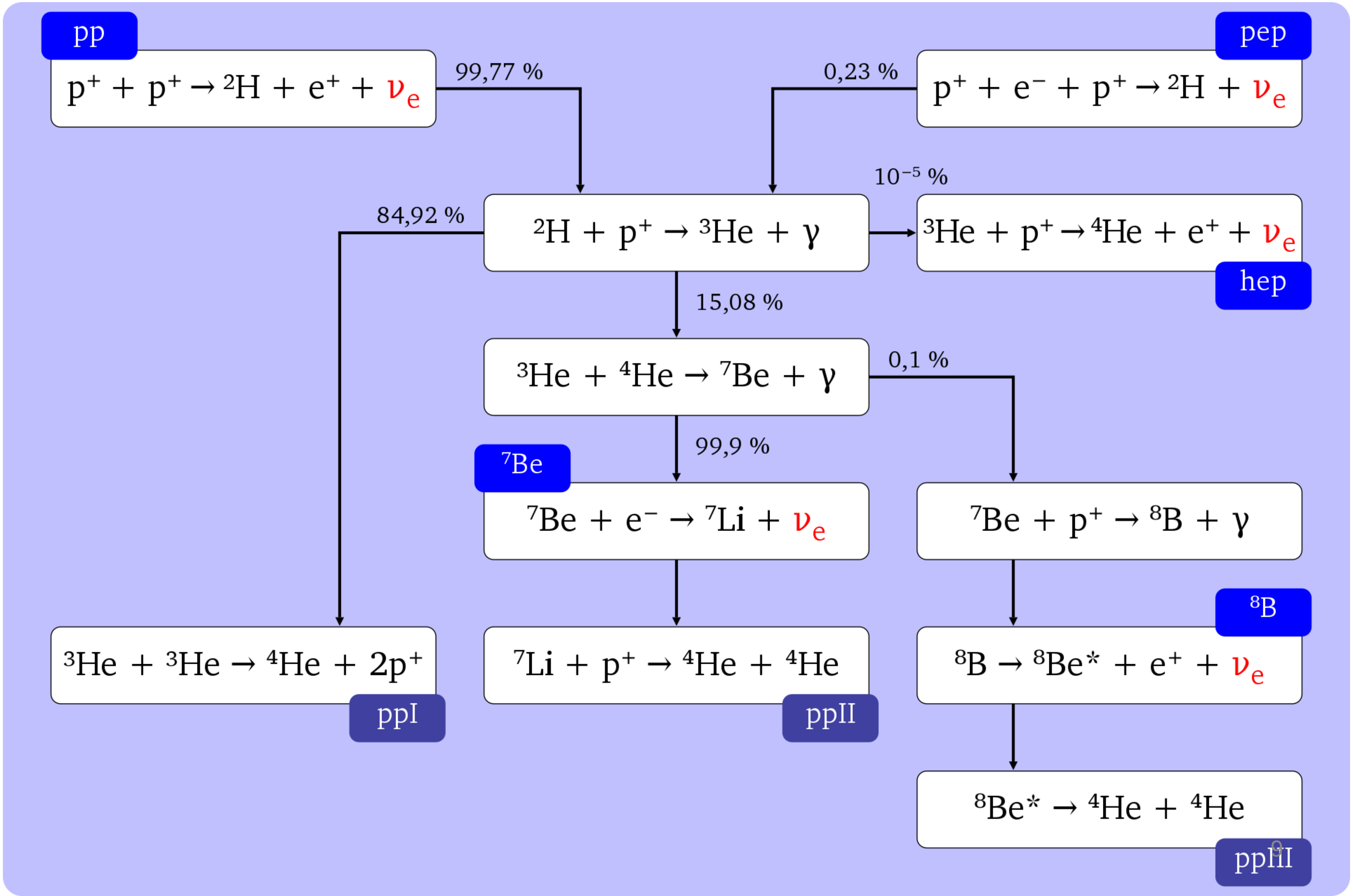
PPI Chain	PPII Chain	PPIII Chain
1 $p + p \rightarrow d + e^+ + \nu_e$	3' $^3He + {}^4He \rightarrow {}^7Be + \gamma$	4'' $^7Be + p \rightarrow {}^8B + \gamma$
2 $d + p \rightarrow {}^3He + \gamma$	4' $^7Be + e^- \rightarrow {}^7Li + \nu_e$	5'' $^8B \rightarrow {}^8Be + e^+ + \nu_e$
3 $^3He + {}^3He \rightarrow {}^4He + 2p$	5' $^7Li + p \rightarrow {}^4He + {}^4He$	6'' $^8Be \rightarrow 2\,{}^4He$



Relative importance of PPI and PPII chains (*branching ratios*) depend on conditions of H-burning (*T*,ρ, abundances). The transition from PPI to PPII occurs at temperatures in excess of 1.3×10^7 K.

Above 3×10^7 K the PPIII chain dominates over the other two, but another process takes over in this case.

Neutrinos – Solar Fusion



pp

$p^+ + p^+ \rightarrow {}^2H + e^+ + \nu_e$

99,77 %

pep

$p^+ + e^- + p^+ \rightarrow {}^2H + \nu_e$

0,23 %

84,92 %

${}^2H + p^+ \rightarrow {}^3He + \gamma$

10^{-5} %

${}^3He + p^+ \rightarrow {}^4He + e^+ + \nu_e$

hep

15,08 %

${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

0,1 %

99,9 %

${}^7Be$

${}^7Be + e^- \rightarrow {}^7Li + \nu_e$

${}^7Be + p^+ \rightarrow {}^8B + \gamma$

${}^8B$

${}^3He + {}^3He \rightarrow {}^4He + 2p^+$

ppI

${}^7Li + p^+ \rightarrow {}^4He + {}^4He$

ppII

${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$

${}^8Be^* \rightarrow {}^4He + {}^4He$

ppIII

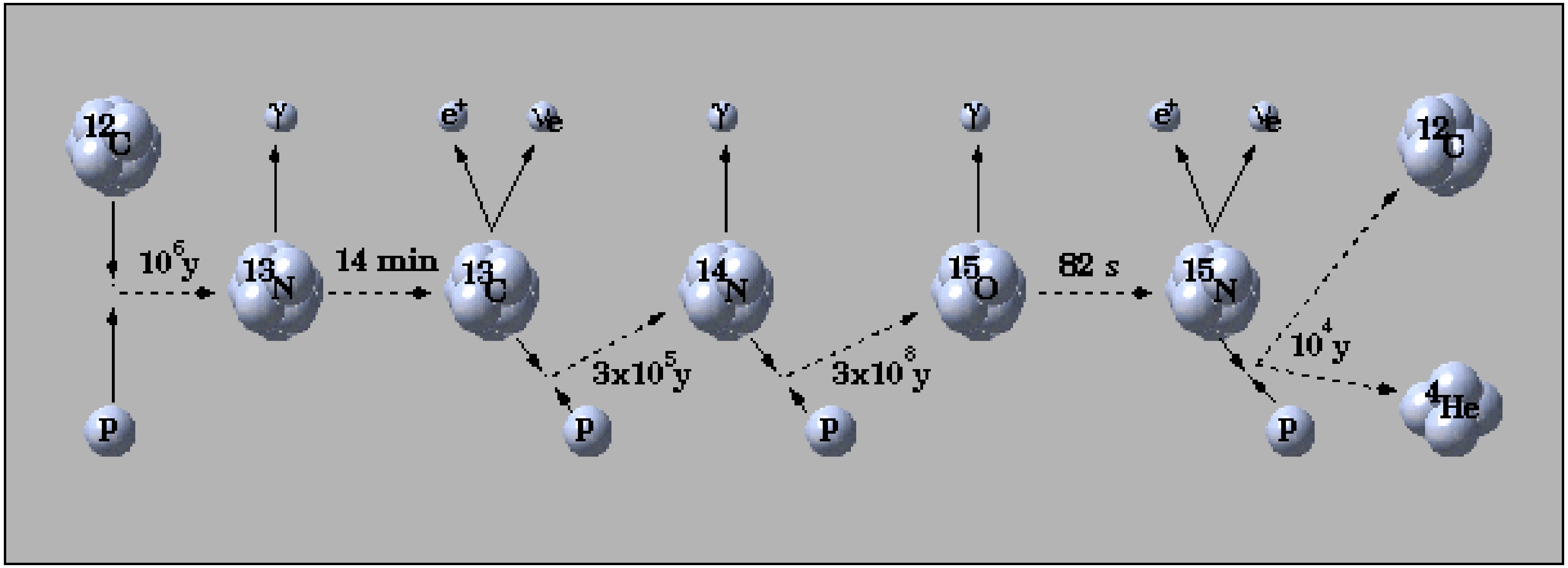
The CNO Cycle

- Remember: [Z] < 2%, most abundant CNO
- CNO induce a chain of H-burning reactions in which they act as catalysts
- The process is known as the CNO Cycle. There are alternative names in the literature:

1. The CNO bi-cycle
2. The CNOF cycle
3. The CN and NO cycles
4. The CN and NO bi-cycles

- In this course we will just refer to it all as the CNO cycle

The main CNO branch



1 $^{12}C + p \rightarrow {}^{13}N + \gamma$

2 $^{13}N \rightarrow {}^{13}C + e^+ + \nu_e$

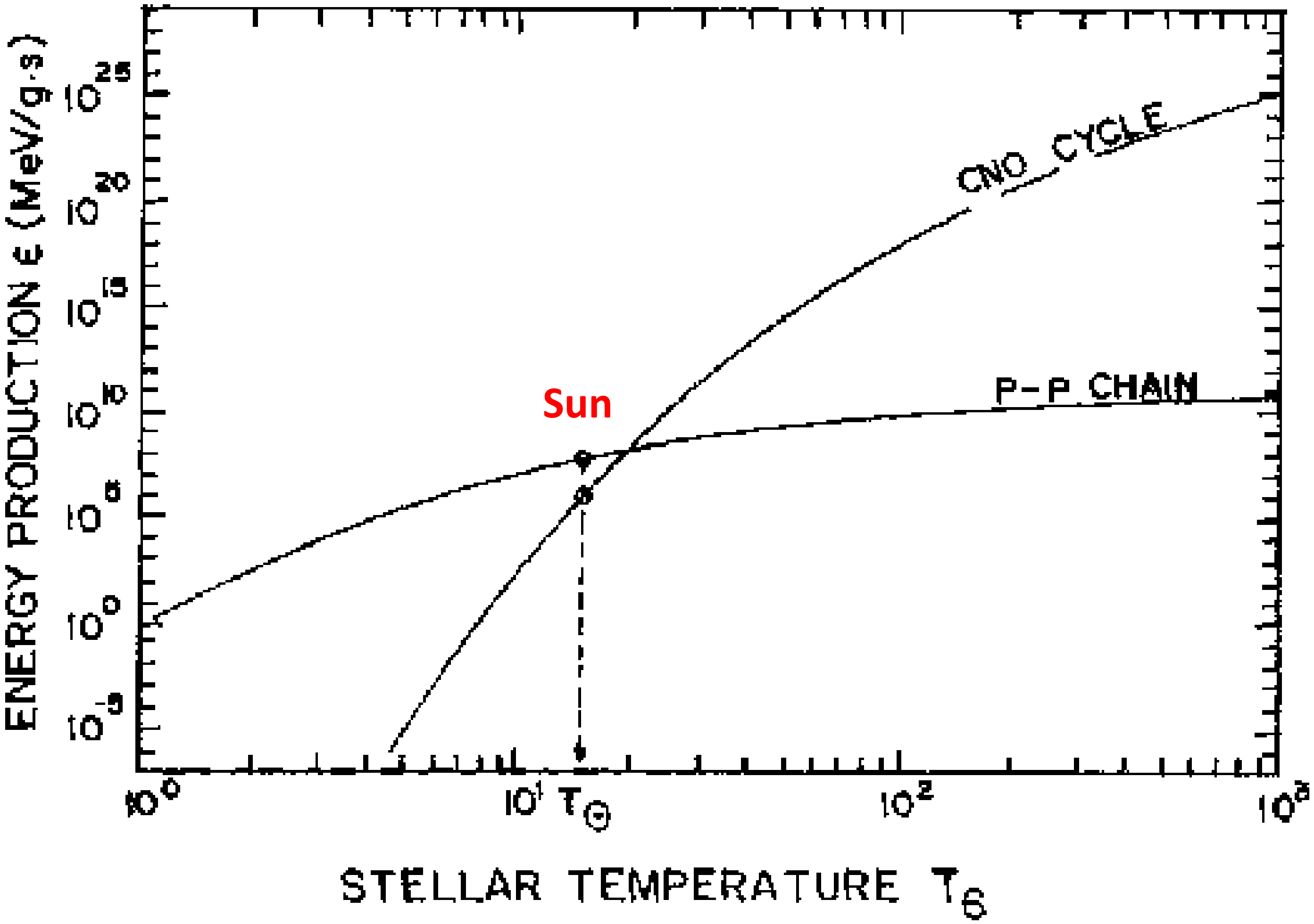
3 $^{13}C + p \rightarrow {}^{14}N + \gamma$

4 $^{14}N + p \rightarrow {}^{15}O + \gamma$

5 $^{15}O \rightarrow {}^{15}N + e^+ + \nu_e$

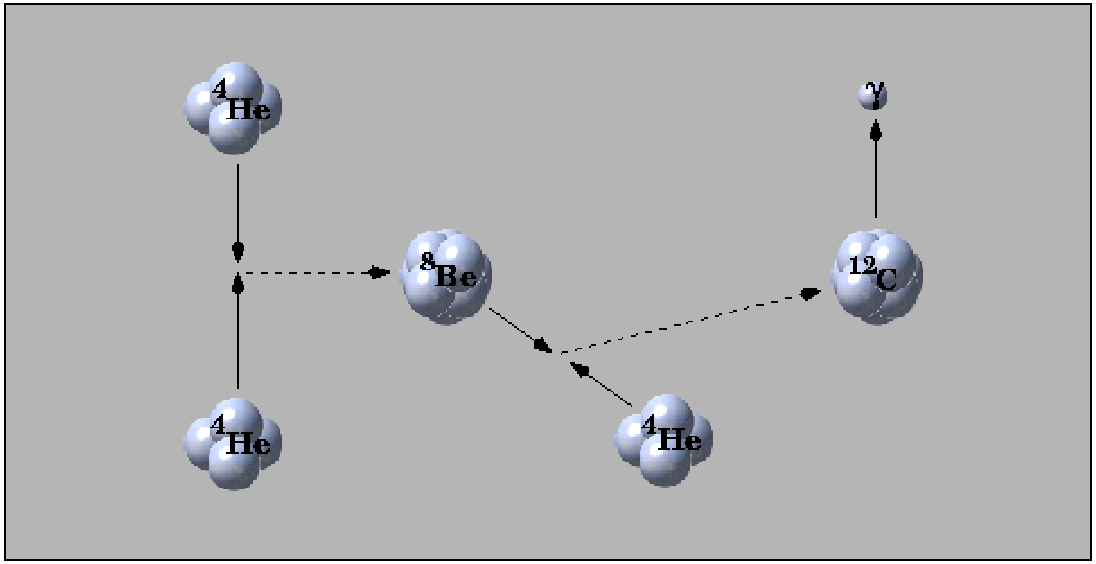
6 $^{15}N + p \rightarrow {}^{12}C + {}^{4}He$

In the steady state case, the abundances of isotopes must take values such that the isotopes which react more slowly have higher abundance. The slowest reaction is p capture by ^{14}N. Hence most of ^{12}C is converted to ^{14}N.



CNO cycle for stars with $M > 1.2\ M_{\odot}$ dominant

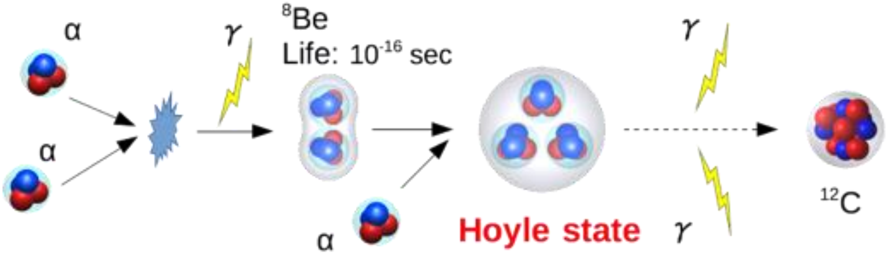
Helium Burning: the triple - α reaction



- Simplest reaction should be the fusion of two helium nuclei
- $^4He + ^4He \rightarrow ^8Be$
- There is no stable configuration with *A = 8.*
- Beryllium isotope ^{8}Be has a lifetime of only 2.6×10^{-16} s
- But a third helium nucleus can be added to ^{8}Be before decay, forming ^{12}C by the "triple-alpha" reaction

  $^4He + ^4He \rightarrow ^8Be$

  $^8Be + ^4He \rightarrow ^{12}C + \gamma$



Carbon and oxygen burning

Carbon burning (fusion of two C nuclei) requires temperatures above 5 $\times 10^8$ K, and oxygen burning in excess of 10^9 K.

Interactions of C and O nuclei are negligible – as at the intermediate temperatures required by the coulomb barrier the C nuclei are quickly destroyed by interacting with themselves

$$\begin{array}{lcl@{\qquad}lcl}
{}^{12}C + {}^{12}C & \rightarrow & {}^{16}O + 2\,{}^{4}He & {}^{16}O + {}^{16}O & \rightarrow & {}^{24}Mg + 2\,{}^{4}He \\
 & \rightarrow & {}^{20}Ne + {}^{4}He & & \rightarrow & {}^{28}Si + {}^{4}He \\
 & \rightarrow & {}^{23}Na + p^{+} & & \rightarrow & {}^{31}P + p^{+} \\
 & \rightarrow & {}^{23}Mg + n & & \rightarrow & {}^{31}S + n \\
 & \rightarrow & {}^{24}Mg + \gamma & & \rightarrow & {}^{32}S + \gamma
\end{array}$$

The branching ratios for these reactions are temperature dependent probabilities.

${}^{12}C + {}^{12}C \rightarrow$ ~13 MeV

${}^{16}O + {}^{16}O \rightarrow$ ~16 MeV

These reactions produce p, n, He, which are immediately captured by heavy nuclei, thus many isotopes created by secondary reactions.

Silicon burning

Two Si nuclei could fuse to create ^{56}Fe – the end of the fusion chain

But now very high Coulomb barrier, at *T* above O burning, but below that required for Si burning, ***photodisintegration*** takes place

$$^{28}Si + {}^{4}He \rightarrow {}^{32}S + \gamma$$

$$^{32}S + {}^{4}He \rightarrow {}^{36}Ar + \gamma$$

$$\vdots$$

$$^{52}Cr + {}^{4}He \rightarrow {}^{56}Fe + \gamma$$

Si disintegration occurs around 3×10^{9} K, and the light particles emitted are recaptured by other Si nuclei. Although the reactions tend to a state of equilibrium, a leakage occurs towards the stable iron group nuclei (Fe, Co, Ni), which resist photodisintegration up to 7×10^{9} K.

Summary - nuclear burning processes

- Release of energy by consumption of nuclear fuel
- Rates of energy release vary enormously
- Nuclear processes can also absorb energy from radiation field

Nuclear Fuel	Process	$T_{threshold}$ 10^6K	Products	Energy per nucleon (MeV)
H	PP	~4	He	6.55
H	CNO	15	He	6.25
He	3α	100	C, O	0.61
C	C+C	600	O, Ne, Na, Mg	0.54
O	O+O	1000	Mg, S, P, Si	~0.3
Si	Nuc eq.	3000	Co, Fe, Ni	<0.18

The *s*-process and *r*-process

Interaction between nuclei and free neutrons (neutron capture) – the neutrons are produced during C, O and Si burning.

Neutrons capture by heavy nuclei is not limited by the Coulomb barrier – so could proceed at relatively low temperatures. The obstacle is the scarcity of free neutrons. If enough neutrons available, chain of reactions possible:

$I(A, Z) + n \rightarrow I_1(A+1, Z)$

$I_1(A+1, Z) + n \rightarrow I_2(A+2, Z)$

$I_2(A+2, Z) + n \rightarrow I_3(A+3, Z)$

If a radioactive isotope is formed it will undergo β – decay, creating new element.

$I_N(A+N, Z) \rightarrow J(A+N, Z+1) + e^- + \overline{\nu}$

If new element stable, it will resume neutron capture, otherwise undergo series of β-decays

$J(A+N, Z+1) \rightarrow K(A+N, Z+2) + e^- + \overline{\nu}$

$K(A+N, Z+2) \rightarrow L(A+N, Z+3) + e^- + \overline{\nu}$

The *s*-process and *r*-process

- Two types of reactions and two types of nuclei
  1. Neutron captures and β-decays
  2. Stable and unstable nuclei
- Stable nuclei may undergo only neutron captures
- Unstable ones my undergo both
- Outcome depending on the timescales for the two processes
- Neutron capture reactions may proceed more ***slowly*** or more ***rapidly*** than the competing β – decays
- The different chains of reactions and products are called the ***s – process*** and ***r – process***

The *s*-process and *r*-process