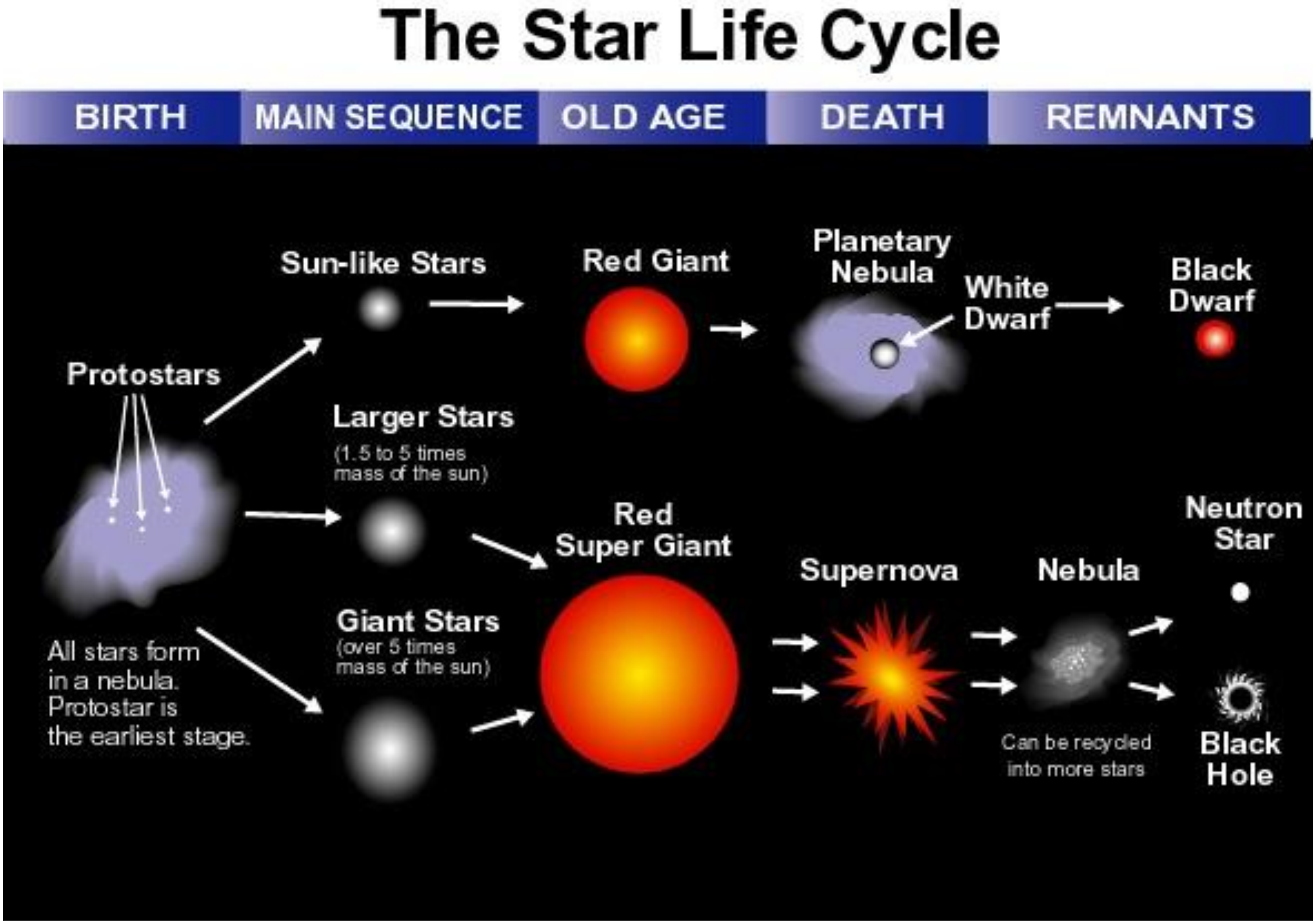
Fundamental physical constants required in this course

a	radiation density constant	7.55×10^{-16} J m^{-3} K^{-4}
c	velocity of light	3.00×10^{8} m s^{-1}
G	gravitational constant	6.67×10^{-11} N m^{2} kg^{-2}
h	Planck’s constant	6.62×10^{-34} J s
k	Boltzmann’s constant	1.38×10^{-23} J K^{-1}
m_e	mass of electron	9.11×10^{-31} kg
m_H	mass of hydrogen atom	1.67×10^{-27} kg
N_A	Avogardo’s number	6.02×10^{23} mol^{-1}
σ	Stefan Boltzmann constant	5.67×10^{-8} W m^{-2} K^{-4} ($\sigma = ac/4$)
R	gas constant (k/m_H)	8.26×10^{3} J K^{-1} kg^{-1}
e	charge of electron	1.60×10^{-19} C
$L_\odot$	luminosity of Sun	3.86×10^{26} W
$M_\odot$	mass of Sun	1.99×10^{30} kg
$T_{\mathrm{eff},\odot}$	effective temperature of Sun	5780 K
$R_\odot$	radius of Sun	6.96×10^{8} m
Parsec	unit of distance	3.09×10^{16} m

Classical Stellar Evolution



Star clusters

NGC 3766, ESO

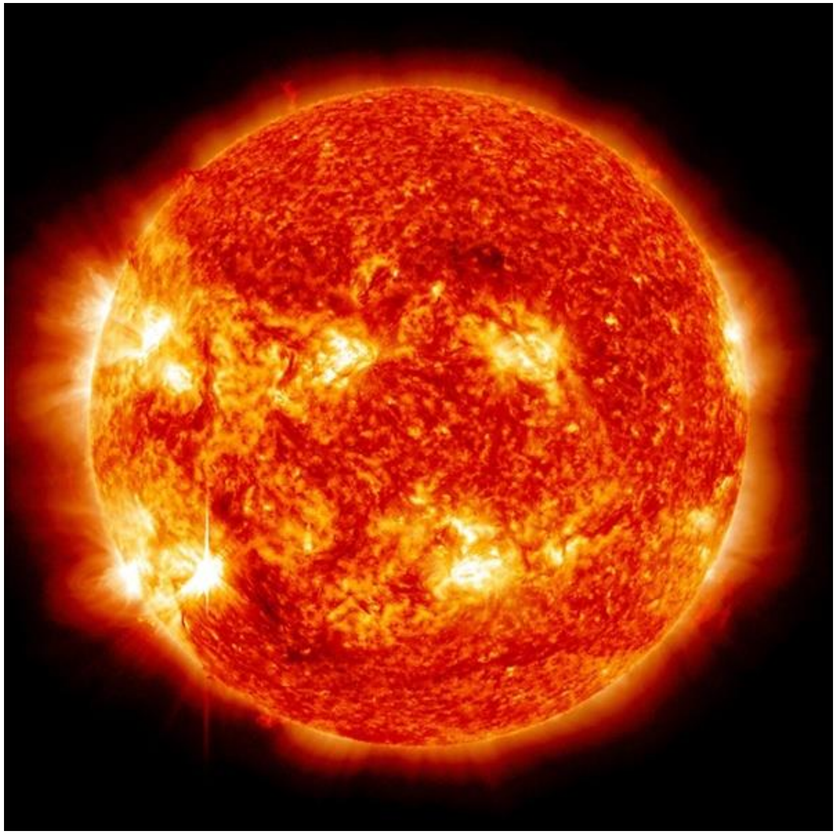
We observe star clusters

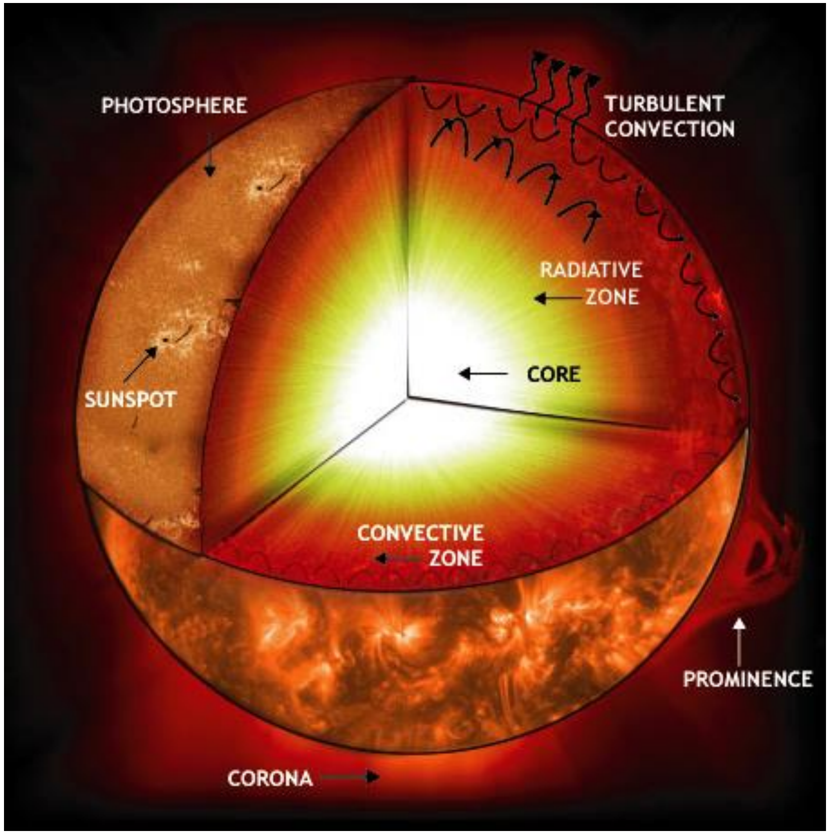
- Stars all at same distance
- Dynamically bound
- Same age
- Same chemical composition

Can contain 10^3 – 10^6 stars

Goal of this course is to understand the stellar content of such clusters

The Sun – best studied example





Stellar interiors not directly observable

Neutrinos emitted at core and detectable

Helioseismology - vibrations of solar surface can be used to probe density structure

Must construct models of stellar interiors – predictions of these models are tested by comparison with observed properties of individual stars

Observable properties of stars

Basic parameters to compare theory and observations:

- Mass (M)
- Luminosity (L)
  - The total energy radiated per second i.e. power (in W)

$$L = \int_0^\infty L_\lambda \, d\lambda$$

- Radius (R)
- Effective temperature (T_{eff})
  - The temperature of a black body of the same radius as the star that would radiate the same amount of energy

    $L = 4\pi R^2 \sigma T_{\text{eff}}$

$\Rightarrow$ 3 independent quantities

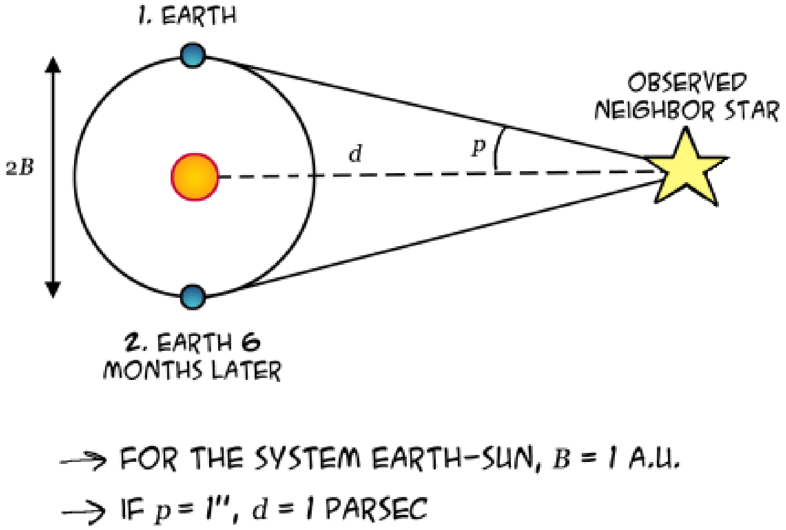
Basic definitions

Measured energy flux depends on distance to star (inverse square law)

$$F = \frac{L}{4\pi d^2}$$

Hence if *d* is known then *L* determined

We can determine distance if we measure *parallax*



Classical astrometric approach

Now: Gaia

Stellar radii

Angular diameter of sun at distance of 10 pc:

$\theta = 2R_{\odot}/10\ \text{pc} = 5\times 10^{-9}$ radians = 10^{-3} arcsec

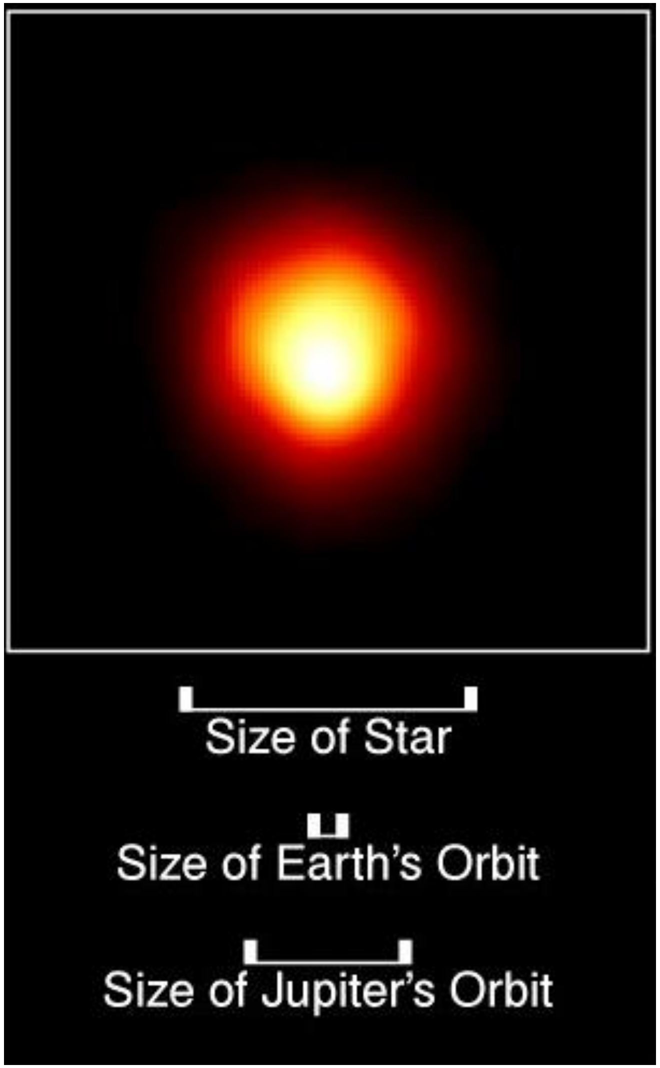
Compare with Hubble resolution of ~0.05 arcsec

$\Rightarrow$ very difficult to measure R directly

Radii of stars measured with techniques such as interferometry and eclipsing binaries

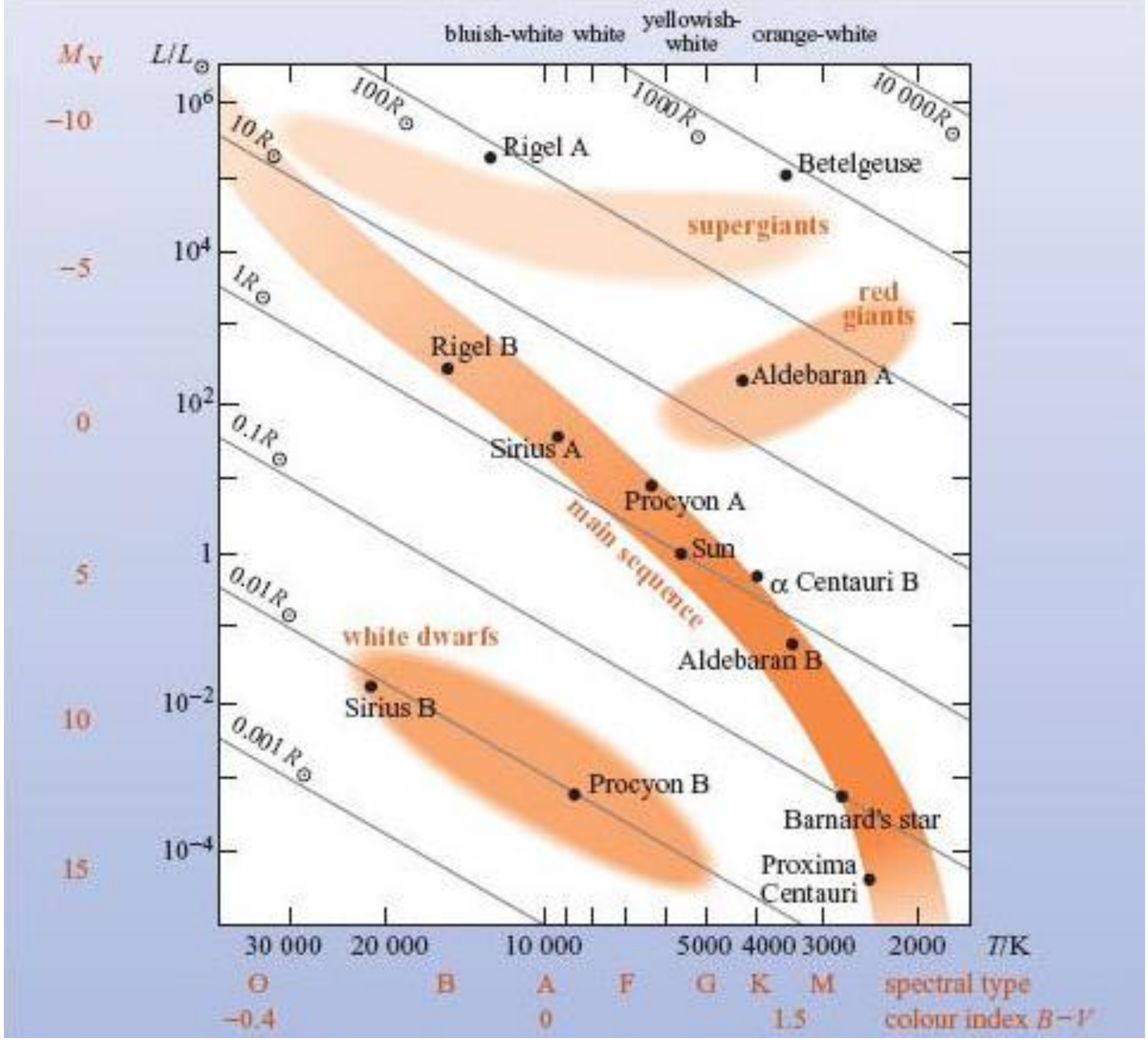
JMMC Stellar Diameters Catalogue - JSDC. Version 2: about 470 000 stars, median error of the diameters is around 1.5%

Stellar radii



Atmosphere of Betelgeuse

PRC96-04 · ST ScI OPO · January 15, 1995 · A. Dupree (CfA), NASA



The Hertzsprung - Russell diagram

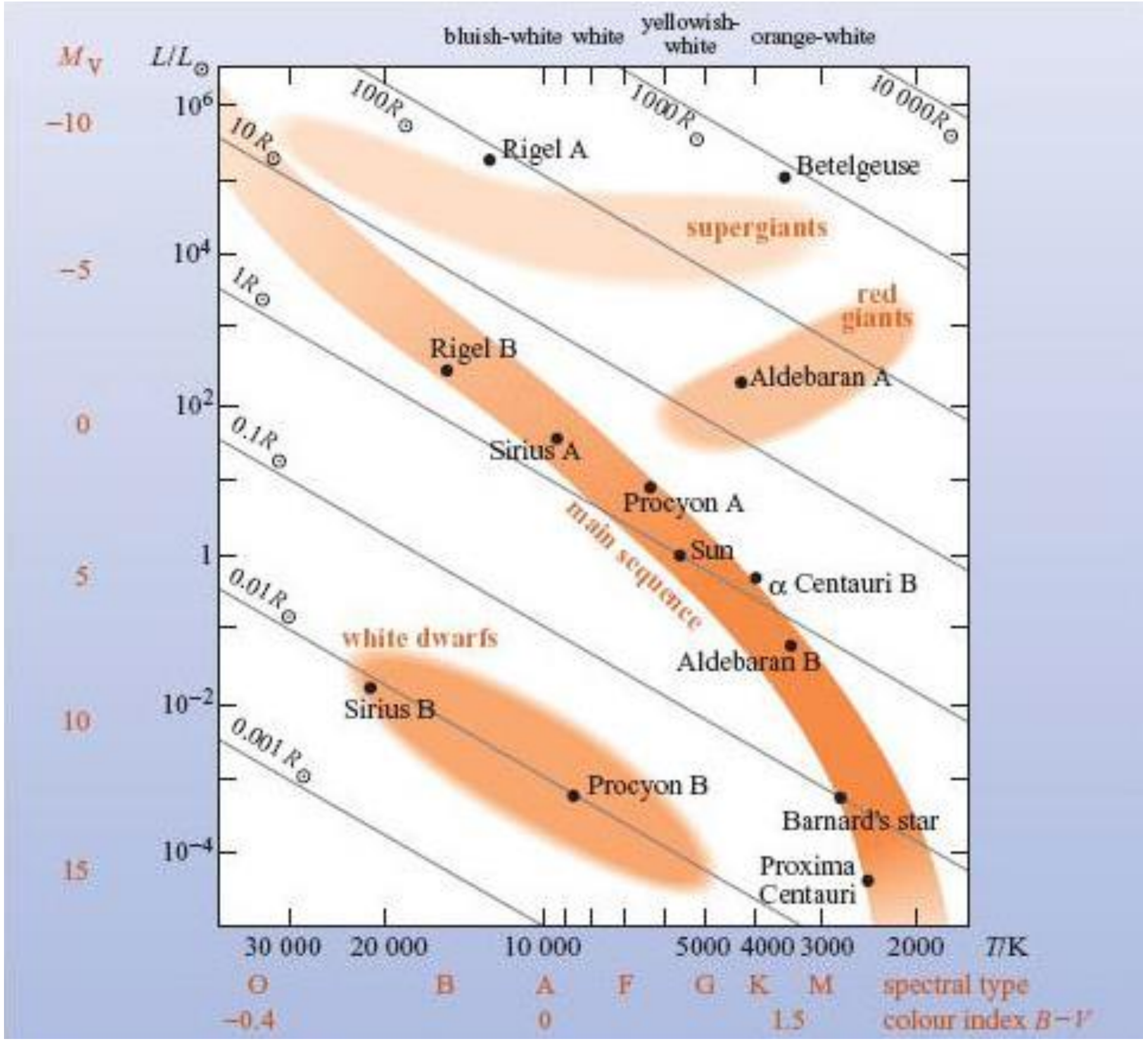
M, *R*, *L* and T_{eff} do not vary independently

Two major relationships

– *L* with T_{eff}

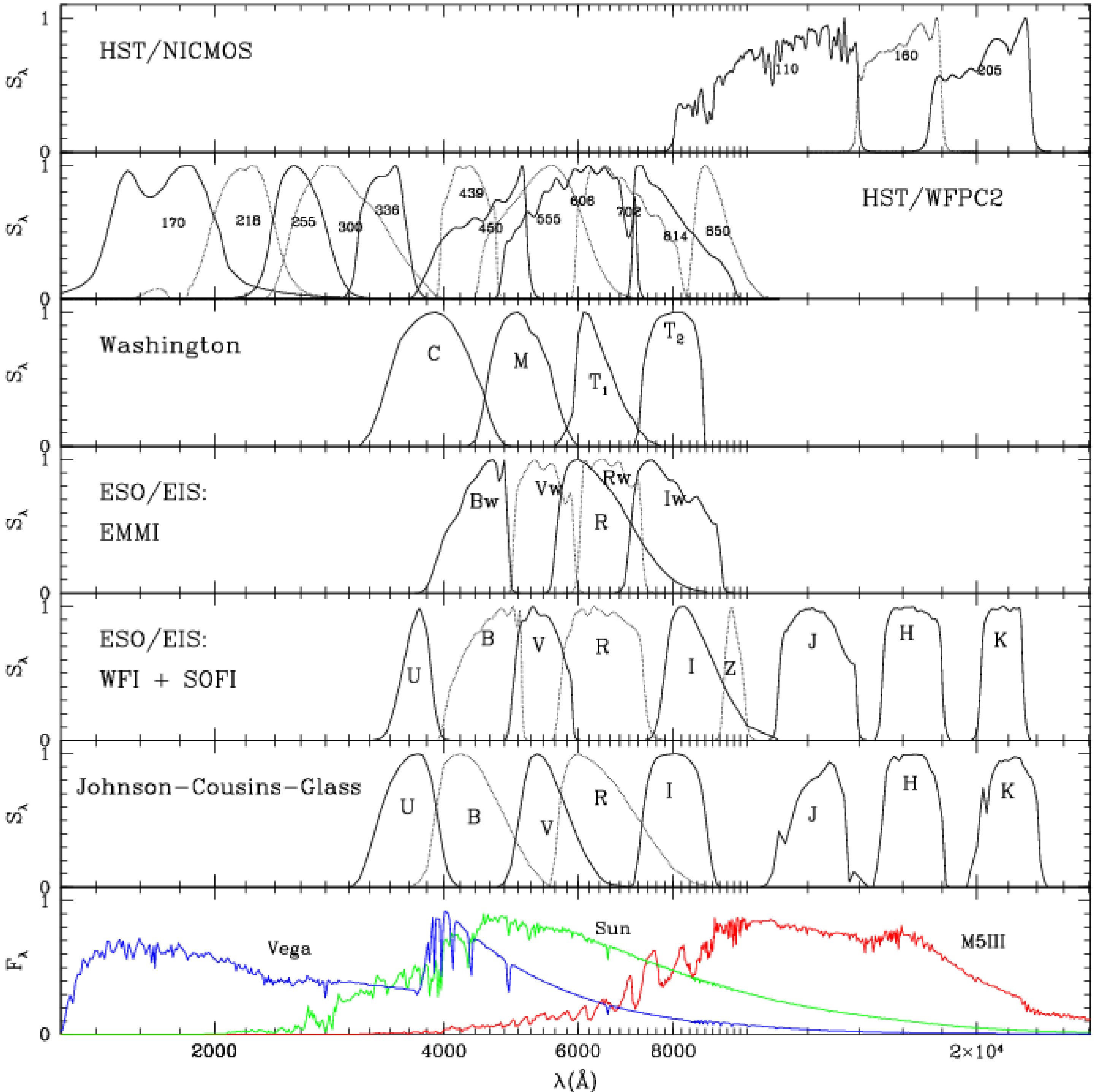
– *L* with *M*

The first is known as the *Hertzsprung-Russell* (HR) diagram or the *colour-magnitude* diagram

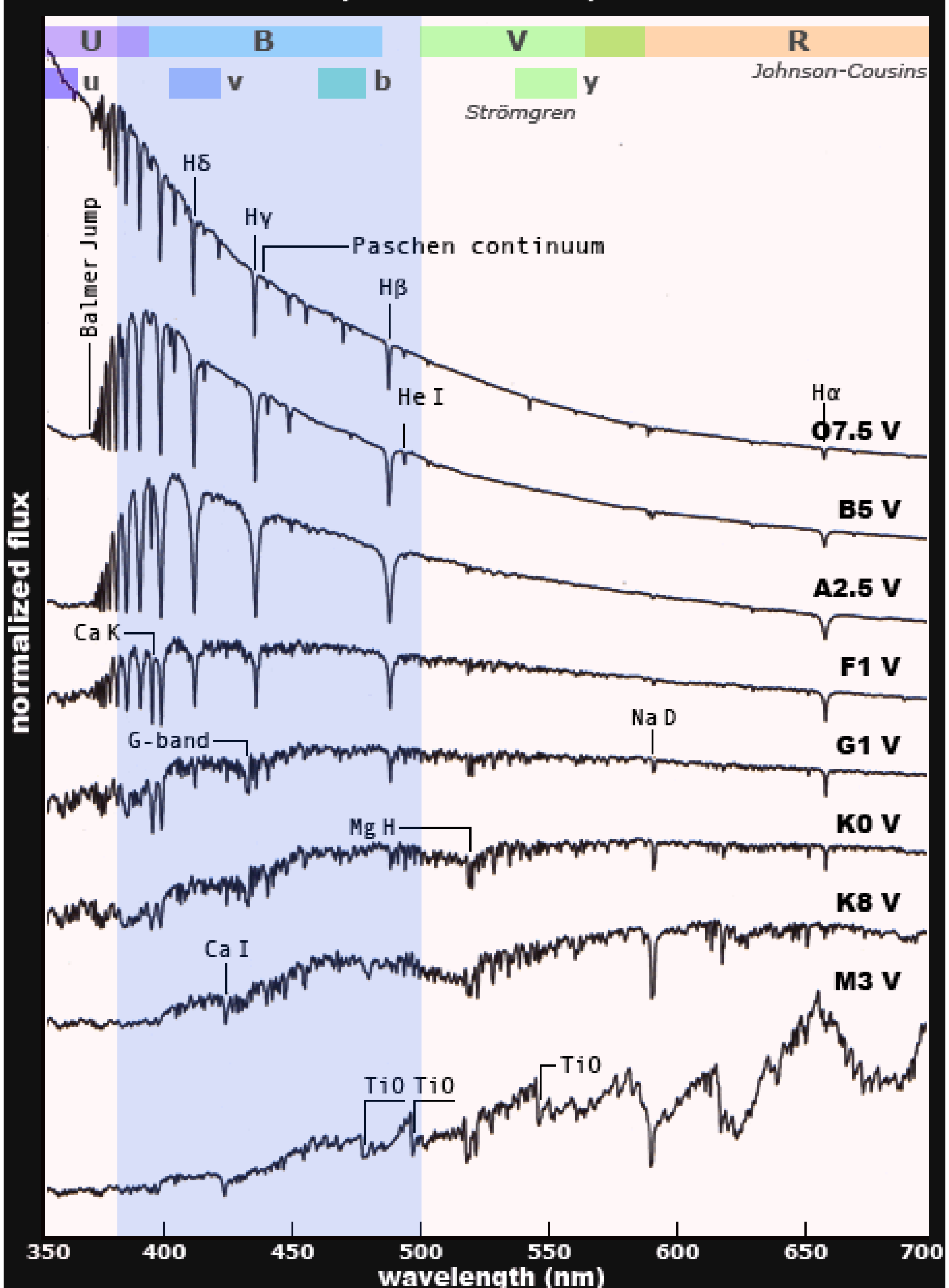


Colour and T_{eff}

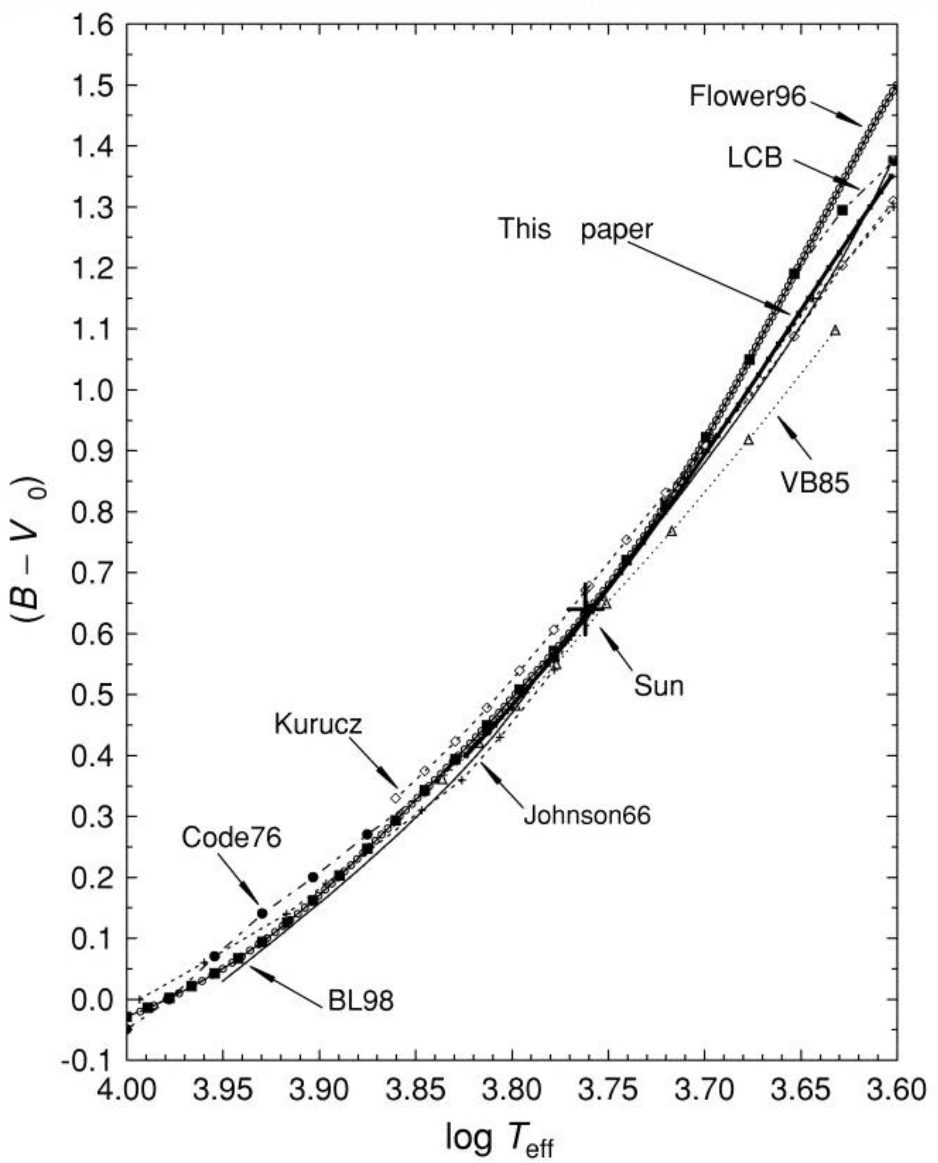
- Measuring accurate T_{eff} for stars is an intensive task – spectra needed and model atmospheres
- Magnitudes of stars are measured at different wavelengths
- Colours => Calibration => T_{eff}
- The Asiago Database on Photometric Systems (ADPS) lists about 200 different systems


HST/NICMOS
110
160
205
HST/WFPC2
170
218
255
300
336
439
450
555
606
702
814
850
Washington
C
M
T_1
T_2
ESO/EIS:
EMMI
Bw
Vw
Rw
R
Iw
ESO/EIS:
WFI + SOFI
U
B
V
R
I
Z
J
H
K
Johnson−Cousins−Glass
U
B
V
R
I
J
H
K
Vega
Sun
M5III
S_λ
F_λ
0
1
2000
4000
6000
8000
2×10⁴
λ(Å)

a sequence of stellar flux profiles



Colour and T_{eff}



Various calibrations can be used to provide the colour relation:

$(B - V) = f(T_{\mathrm{eff}})$

Remember that observed $(B - V)$ must be corrected for interstellar extinction to $(B - V)_0$

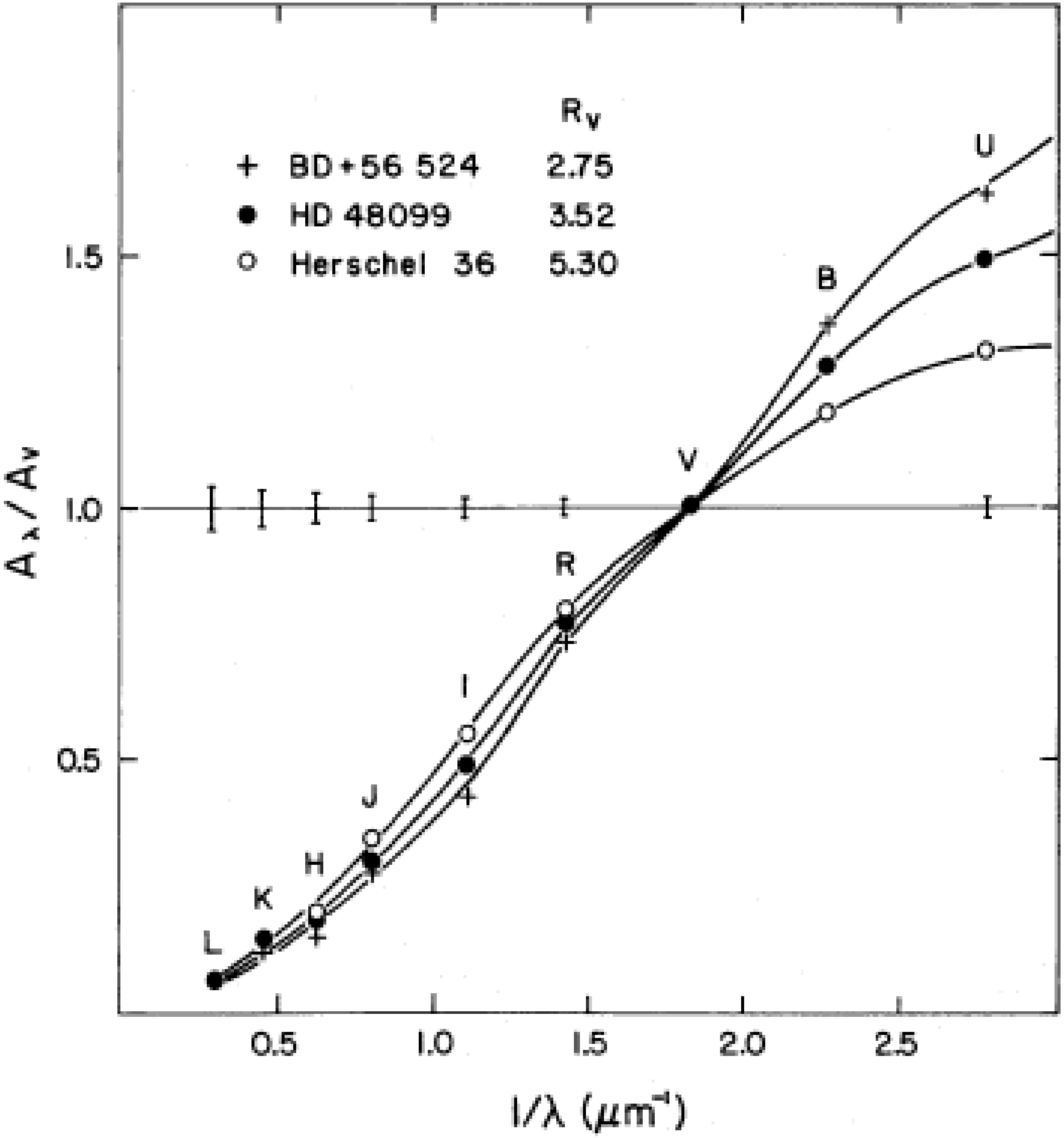
Absorption = Extinction = Reddening

- $A_V = k_1\ E(B\text{-}V) = k_2\ E(V\text{-}R) = \ldots$
- *General extinction* because of the ISM characteristics between the observer and the object
- *Differential extinction* within one star cluster because of local environment
- Both types are, in general *wavelength dependent*

Reasons for the interstellar extinction

- Light scatter at the interstellar dust
- Light absorption => Heating of the ISM
- Depending on the composition and density of the ISM
- Main contribution due to dust
- Simulations and calculations in Cardelli et al. (1989, ApJ, 345, 245)

Cardelli et al., 1989, ApJ, 345, 245



Important parameter:

$R_V = A_V/E(B - V)$

Normalization factor

Standard value used is 3.1

Be careful, different values used!

Depending on the line of sight

Absolute magnitude and bolometric magnitude

- **Absolute Magnitude** *M* defined as apparent magnitude of a star if it were placed at a distance of 10 pc

$$m - M = 5 \log(d/10) - 5$$

where d is in pc

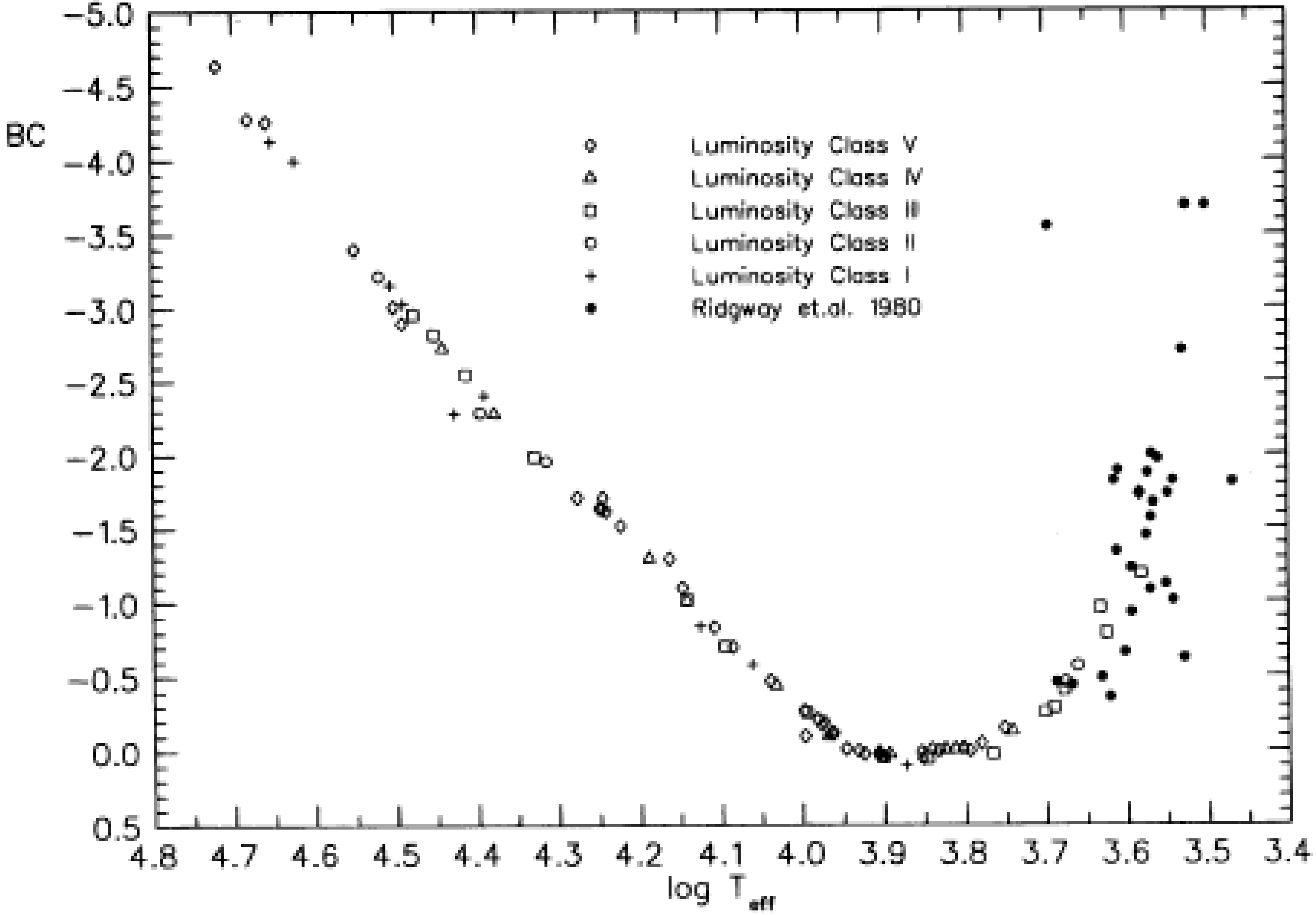
- Magnitudes are measured in some wavelength. To compare with theory it is more useful to determine **bolometric magnitude M_{bol}** – defined as absolute magnitude that would be measured by a bolometer sensitive to all wavelengths. We define the bolometric correction to be

$$BC = M_{\mathrm{bol}} - M_{\mathrm{V}}$$

Bolometric luminosity is then

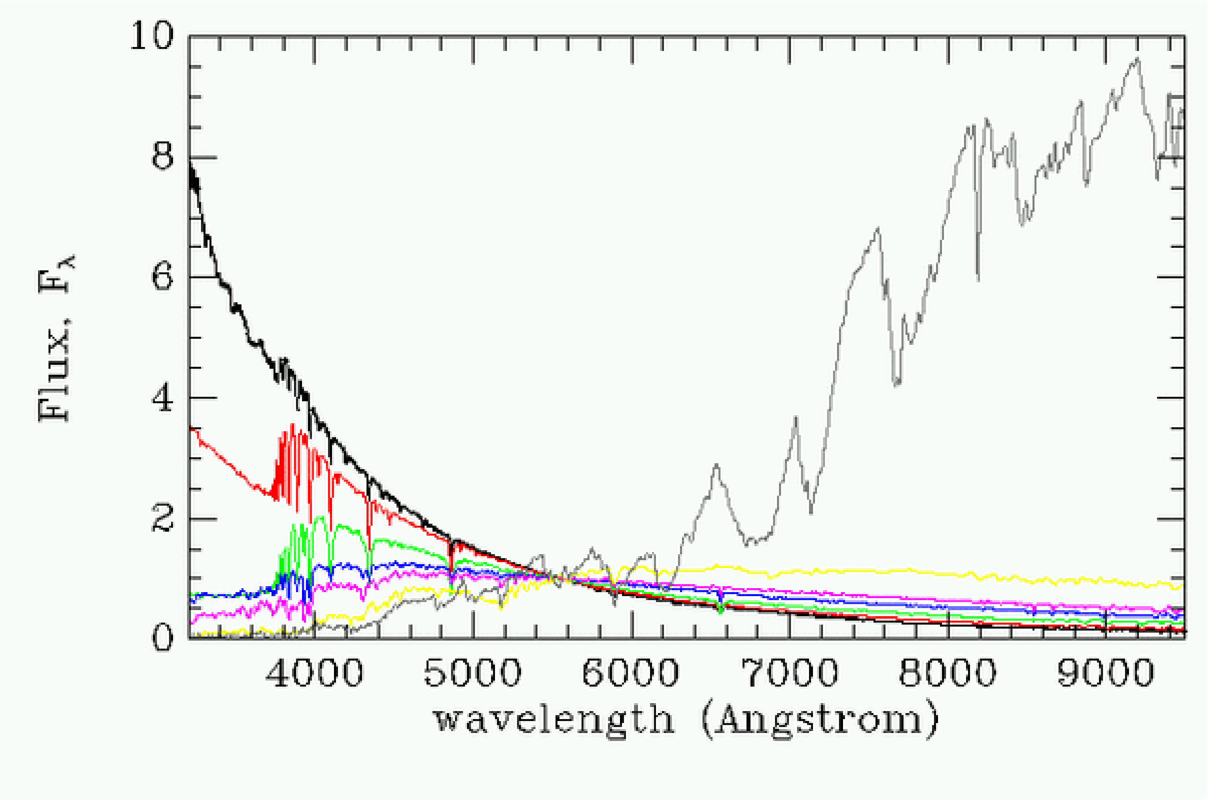
$$M_{\mathrm{bol}} - M_{\mathrm{bol},\odot} = -2.5 \log L/L_{\odot}$$

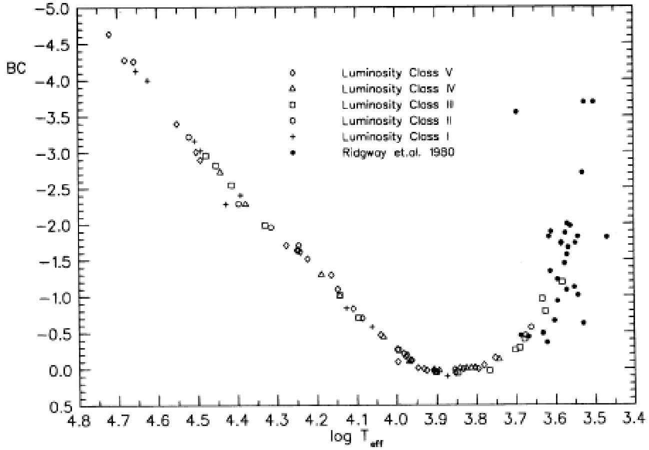
Bolometric Correction



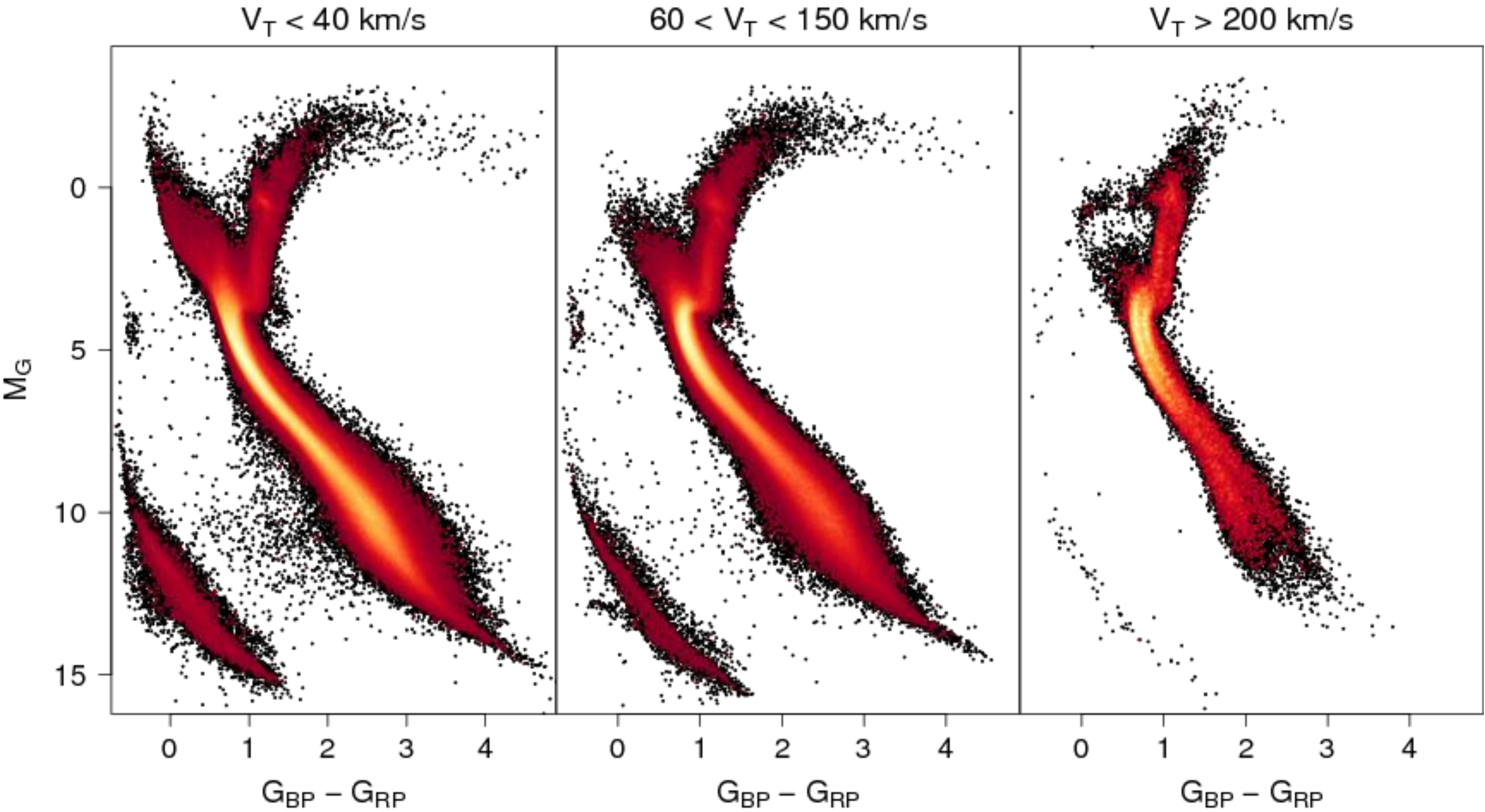
BC from Flower, 1996, ApJ, 469, 355

Bolometric Correction

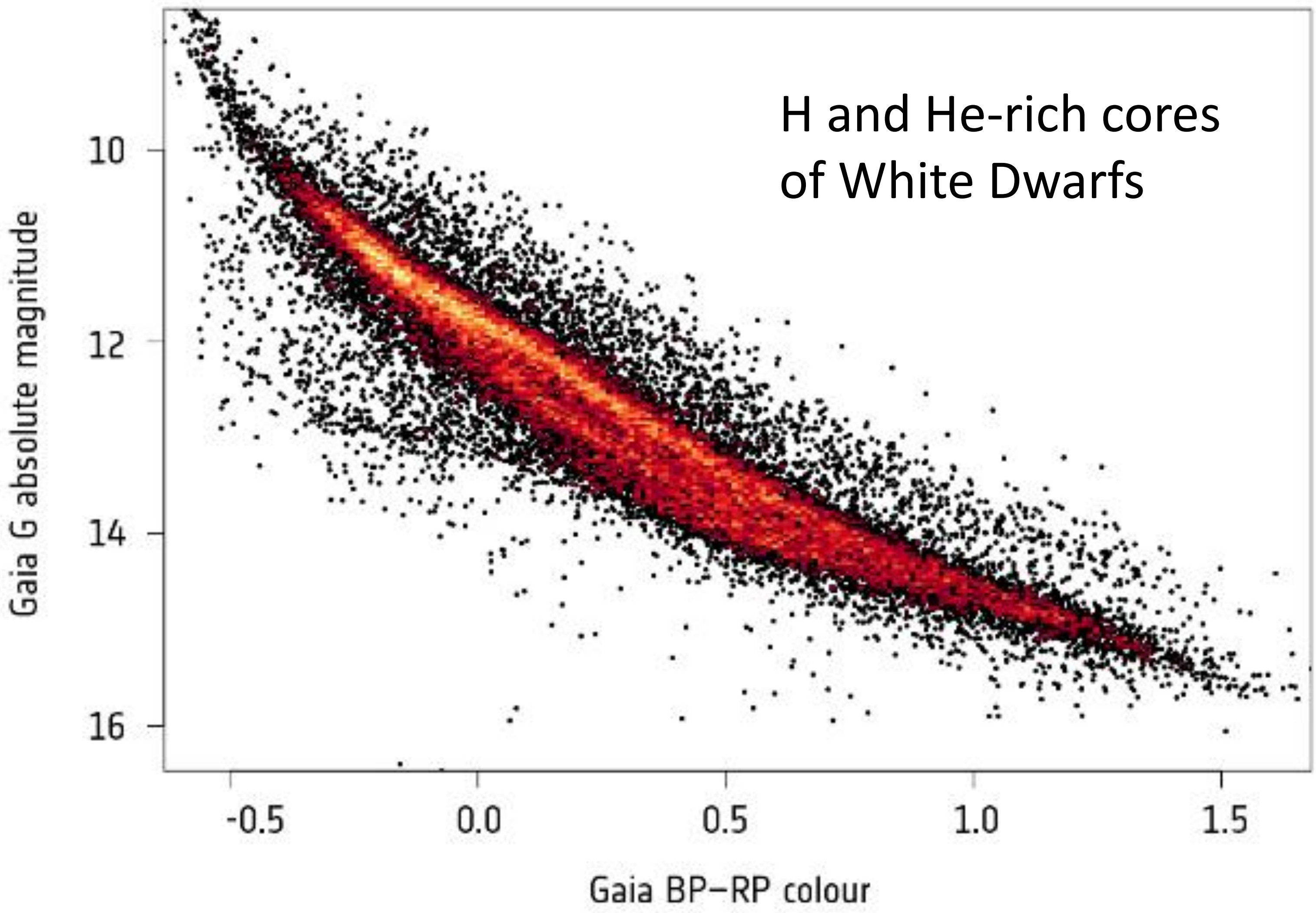




The Hertzsprung - Russell diagram - Gaia



The Hertzsprung - Russell diagram - Gaia



H and He-rich cores of White Dwarfs

Mass – Luminosity Relation

Masses measured in binary systems

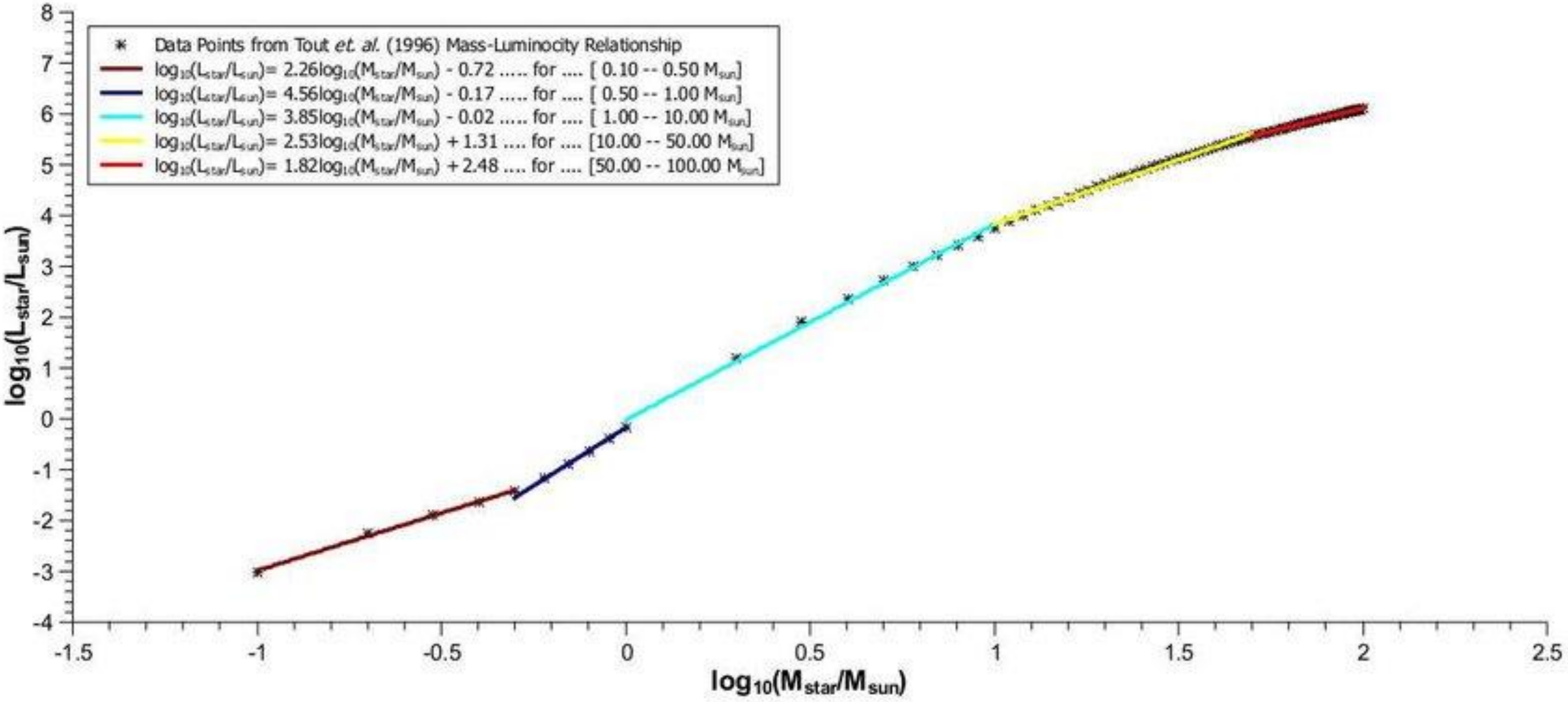
Heuristic mass-luminosity relation

$$L \propto M^{\alpha}$$

Where $\alpha = 2 - 5$; slope less steep for low and high mass stars

This implies that the main-sequence (MS) on the HRD is a function of mass, i.e. from bottom to top of MS, stars increase in mass

Mass – Luminosity Relation



We must understand the *M* - *L* relation and

L - T_{eff} relation theoretically

Models must reproduce observations

Metallicity - Basics

- Metallicity as [X:Y:Z]
- X = Hydrogen
- Y = Helium
- Z = „the rest“

$$X \equiv \frac{m_H}{M} \qquad Y \equiv \frac{m_{He}}{M} \qquad Z = \sum_{i>He} \frac{m_i}{M} = 1 - X - Y$$

Metallicity - designations

- In the literature you will find
  - [Z]
  - [Fe/H]
  - [M/H]
  - [Element 1 / Element 2]
- Relations for the transformation are necessary

$$[\mathrm{Fe/H}] = \log_{10}\left(\frac{N_{\mathrm{Fe}}}{N_{\mathrm{H}}}\right)_{\mathrm{star}} - \log_{10}\left(\frac{N_{\mathrm{Fe}}}{N_{\mathrm{H}}}\right)_{\mathrm{sun}}$$

$$[\mathrm{O/Fe}] = \log_{10}\left(\frac{N_{\mathrm{O}}}{N_{\mathrm{Fe}}}\right)_{\mathrm{star}} - \log_{10}\left(\frac{N_{\mathrm{O}}}{N_{\mathrm{Fe}}}\right)_{\mathrm{sun}}$$

$$= \left[\log_{10}\left(\frac{N_{\mathrm{O}}}{N_{\mathrm{H}}}\right)_{\mathrm{star}} - \log_{10}\left(\frac{N_{\mathrm{O}}}{N_{\mathrm{H}}}\right)_{\mathrm{sun}}\right] - \left[\log_{10}\left(\frac{N_{\mathrm{Fe}}}{N_{\mathrm{H}}}\right)_{\mathrm{star}} - \log_{10}\left(\frac{N_{\mathrm{Fe}}}{N_{\mathrm{H}}}\right)_{\mathrm{sun}}\right]$$

Metallicity - designations

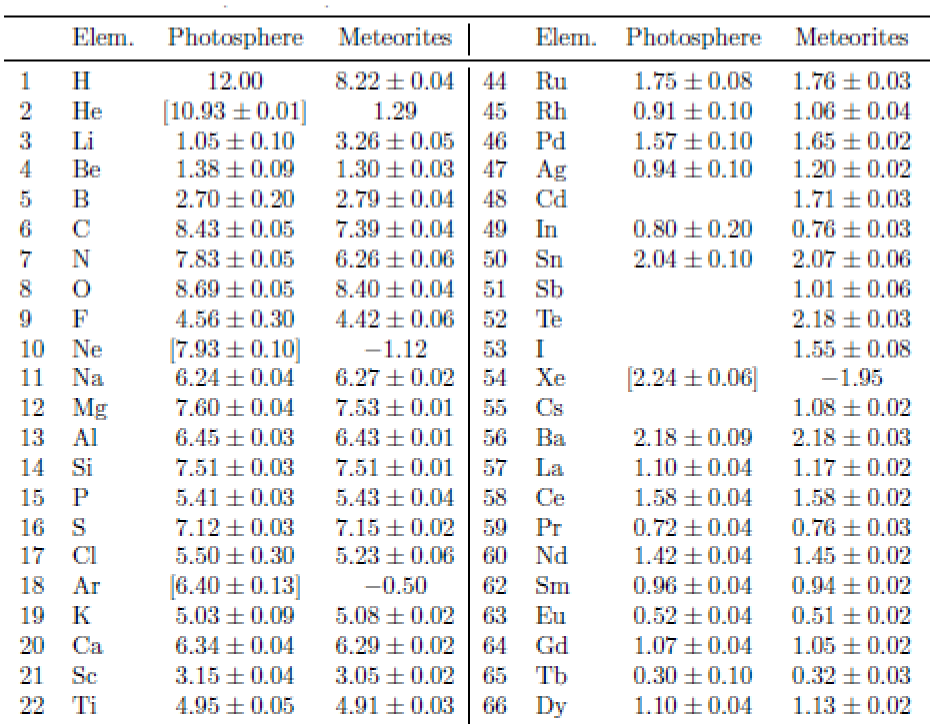
- [dex], e.g. [Fe/H] = -0,5 dex

dex	factor	dex	factor
-2	0,01	0,1	1,26
-1,5	0,03	0,2	1,58
-1	0,10	0,3	2,00
-0,9	0,13	0,4	2,51
-0,8	0,16	0,5	3,16
-0,7	0,20	0,6	3,98
-0,6	0,25	0,7	5,01
-0,5	0,32	0,8	6,31
-0,4	0,40	0,9	7,94
-0,3	0,50	1	10,00
-0,2	0,63	1,5	31,62
-0,1	0,79	2	100,00

Abundance - Sun

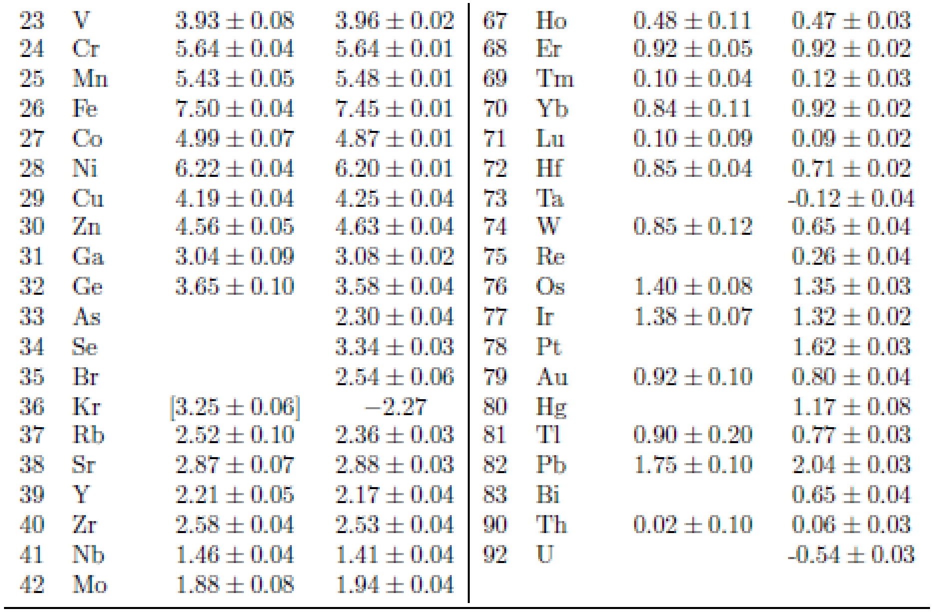
- Problems with
  - Hydrogen
  - Helium
  - Elements with only a few lines
  - Elements with only weak lines
- LTE versus NLTE (Local Thermodynamic Equilibrium)

Abundance - Sun



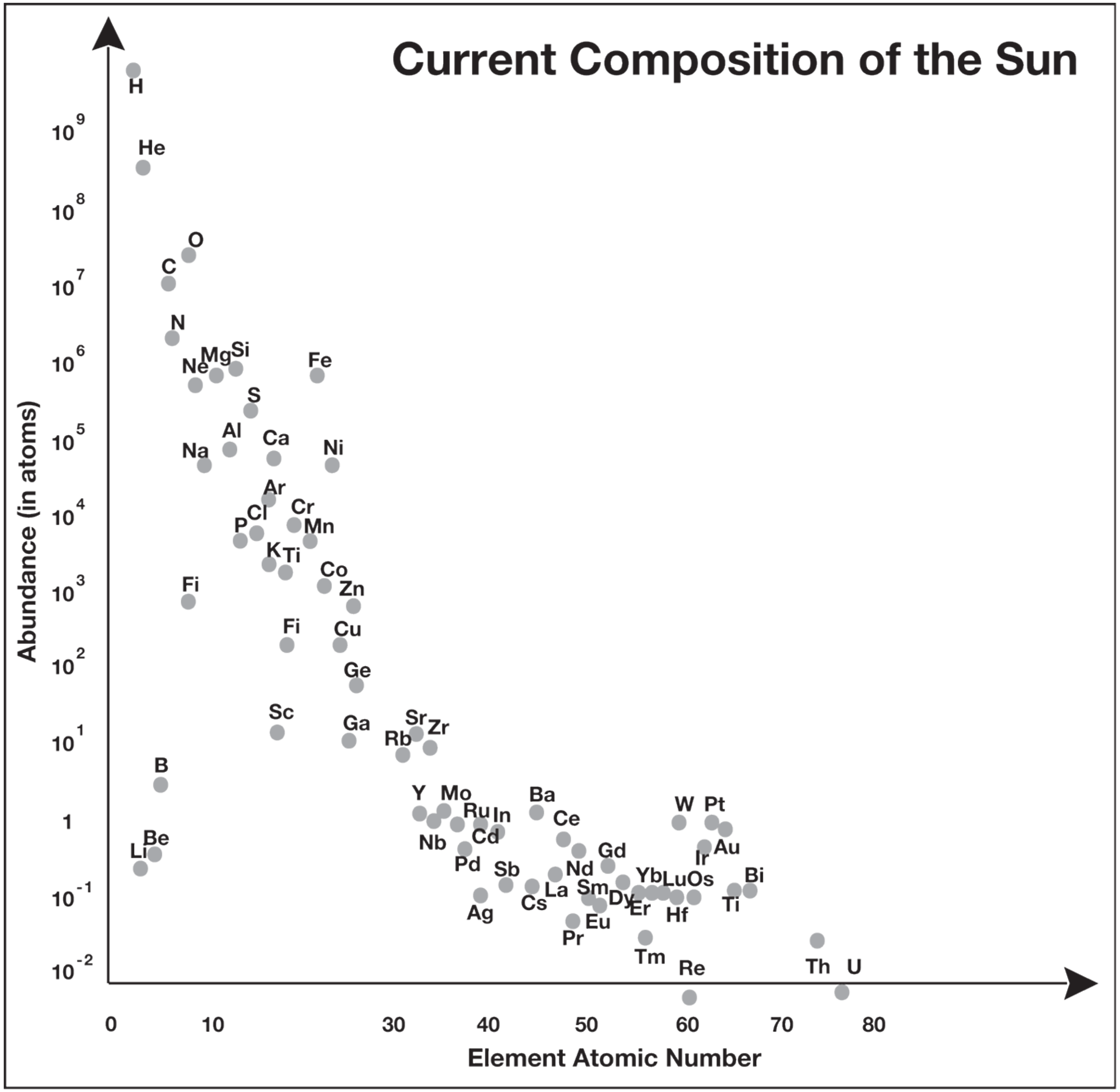
	Elem.	Photosphere	Meteorites		Elem.	Photosphere	Meteorites
1	H	12.00	8.22 ± 0.04	44	Ru	1.75 ± 0.08	1.76 ± 0.03
2	He	[10.93 ± 0.01]	1.29	45	Rh	0.91 ± 0.10	1.06 ± 0.04
3	Li	1.05 ± 0.10	3.26 ± 0.05	46	Pd	1.57 ± 0.10	1.65 ± 0.02
4	Be	1.38 ± 0.09	1.30 ± 0.03	47	Ag	0.94 ± 0.10	1.20 ± 0.02
5	B	2.70 ± 0.20	2.79 ± 0.04	48	Cd		1.71 ± 0.03
6	C	8.43 ± 0.05	7.39 ± 0.04	49	In	0.80 ± 0.20	0.76 ± 0.03
7	N	7.83 ± 0.05	6.26 ± 0.06	50	Sn	2.04 ± 0.10	2.07 ± 0.06
8	O	8.69 ± 0.05	8.40 ± 0.04	51	Sb		1.01 ± 0.06
9	F	4.56 ± 0.30	4.42 ± 0.06	52	Te		2.18 ± 0.03
10	Ne	[7.93 ± 0.10]	−1.12	53	I		1.55 ± 0.08
11	Na	6.24 ± 0.04	6.27 ± 0.02	54	Xe	[2.24 ± 0.06]	−1.95
12	Mg	7.60 ± 0.04	7.53 ± 0.01	55	Cs		1.08 ± 0.02
13	Al	6.45 ± 0.03	6.43 ± 0.01	56	Ba	2.18 ± 0.09	2.18 ± 0.03
14	Si	7.51 ± 0.03	7.51 ± 0.01	57	La	1.10 ± 0.04	1.17 ± 0.02
15	P	5.41 ± 0.03	5.43 ± 0.04	58	Ce	1.58 ± 0.04	1.58 ± 0.02
16	S	7.12 ± 0.03	7.15 ± 0.02	59	Pr	0.72 ± 0.04	0.76 ± 0.03
17	Cl	5.50 ± 0.30	5.23 ± 0.06	60	Nd	1.42 ± 0.04	1.45 ± 0.02
18	Ar	[6.40 ± 0.13]	−0.50	62	Sm	0.96 ± 0.04	0.94 ± 0.02
19	K	5.03 ± 0.09	5.08 ± 0.02	63	Eu	0.52 ± 0.04	0.51 ± 0.02
20	Ca	6.34 ± 0.04	6.29 ± 0.02	64	Gd	1.07 ± 0.04	1.05 ± 0.02
21	Sc	3.15 ± 0.04	3.05 ± 0.02	65	Tb	0.30 ± 0.10	0.32 ± 0.03
22	Ti	4.95 ± 0.05	4.91 ± 0.03	66	Dy	1.10 ± 0.04	1.13 ± 0.02

Asplund et al., 2009, Annual Review of Astronomy & Astrophysics, 47, 481



23	V	3.93 ± 0.08	3.96 ± 0.02	67	Ho	0.48 ± 0.11	0.47 ± 0.03
24	Cr	5.64 ± 0.04	5.64 ± 0.01	68	Er	0.92 ± 0.05	0.92 ± 0.02
25	Mn	5.43 ± 0.05	5.48 ± 0.01	69	Tm	0.10 ± 0.04	0.12 ± 0.03
26	Fe	7.50 ± 0.04	7.45 ± 0.01	70	Yb	0.84 ± 0.11	0.92 ± 0.02
27	Co	4.99 ± 0.07	4.87 ± 0.01	71	Lu	0.10 ± 0.09	0.09 ± 0.02
28	Ni	6.22 ± 0.04	6.20 ± 0.01	72	Hf	0.85 ± 0.04	0.71 ± 0.02
29	Cu	4.19 ± 0.04	4.25 ± 0.04	73	Ta		-0.12 ± 0.04
30	Zn	4.56 ± 0.05	4.63 ± 0.04	74	W	0.85 ± 0.12	0.65 ± 0.04
31	Ga	3.04 ± 0.09	3.08 ± 0.02	75	Re		0.26 ± 0.04
32	Ge	3.65 ± 0.10	3.58 ± 0.04	76	Os	1.40 ± 0.08	1.35 ± 0.03
33	As		2.30 ± 0.04	77	Ir	1.38 ± 0.07	1.32 ± 0.02
34	Se		3.34 ± 0.03	78	Pt		1.62 ± 0.03
35	Br		2.54 ± 0.06	79	Au	0.92 ± 0.10	0.80 ± 0.04
36	Kr	[3.25 ± 0.06]	−2.27	80	Hg		1.17 ± 0.08
37	Rb	2.52 ± 0.10	2.36 ± 0.03	81	Tl	0.90 ± 0.20	0.77 ± 0.03
38	Sr	2.87 ± 0.07	2.88 ± 0.03	82	Pb	1.75 ± 0.10	2.04 ± 0.03
39	Y	2.21 ± 0.05	2.17 ± 0.04	83	Bi		0.65 ± 0.04
40	Zr	2.58 ± 0.04	2.53 ± 0.04	90	Th	0.02 ± 0.10	0.06 ± 0.03
41	Nb	1.46 ± 0.04	1.41 ± 0.04	92	U		-0.54 ± 0.03
42	Mo	1.88 ± 0.08	1.94 ± 0.04				

Abundance - Sun



Asplund et al. (2009)

Abundance - Sun

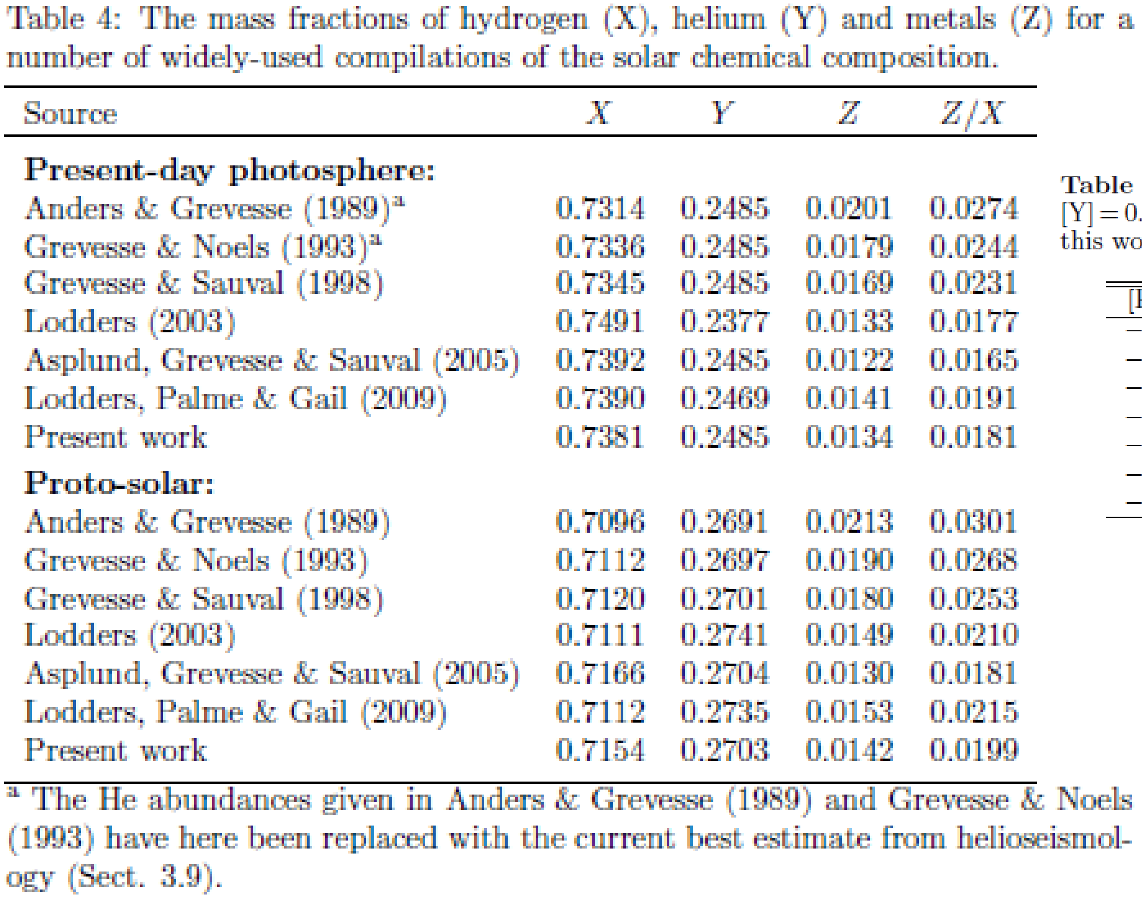


Table 4: The mass fractions of hydrogen (X), helium (Y) and metals (Z) for a number of widely-used compilations of the solar chemical composition.

Source	X	Y	Z	Z/X
Present-day photosphere:				
Anders & Grevesse (1989)[a]	0.7314	0.2485	0.0201	0.0274
Grevesse & Noels (1993)[a]	0.7336	0.2485	0.0179	0.0244
Grevesse & Sauval (1998)	0.7345	0.2485	0.0169	0.0231
Lodders (2003)	0.7491	0.2377	0.0133	0.0177
Asplund, Grevesse & Sauval (2005)	0.7392	0.2485	0.0122	0.0165
Lodders, Palme & Gail (2009)	0.7390	0.2469	0.0141	0.0191
Present work	0.7381	0.2485	0.0134	0.0181
Proto-solar:				
Anders & Grevesse (1989)	0.7096	0.2691	0.0213	0.0301
Grevesse & Noels (1993)	0.7112	0.2697	0.0190	0.0268
Grevesse & Sauval (1998)	0.7120	0.2701	0.0180	0.0253
Lodders (2003)	0.7111	0.2741	0.0149	0.0210
Asplund, Grevesse & Sauval (2005)	0.7166	0.2704	0.0130	0.0181
Lodders, Palme & Gail (2009)	0.7112	0.2735	0.0153	0.0215
Present work	0.7154	0.2703	0.0142	0.0199

[a] The He abundances given in Anders & Grevesse (1989) and Grevesse & Noels (1993) have here been replaced with the current best estimate from helioseismology (Sect. 3.9).

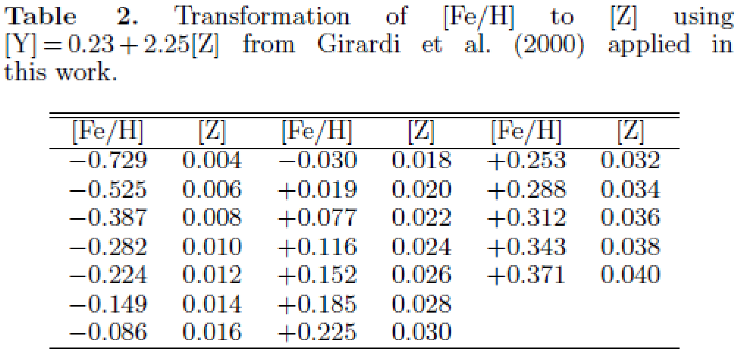
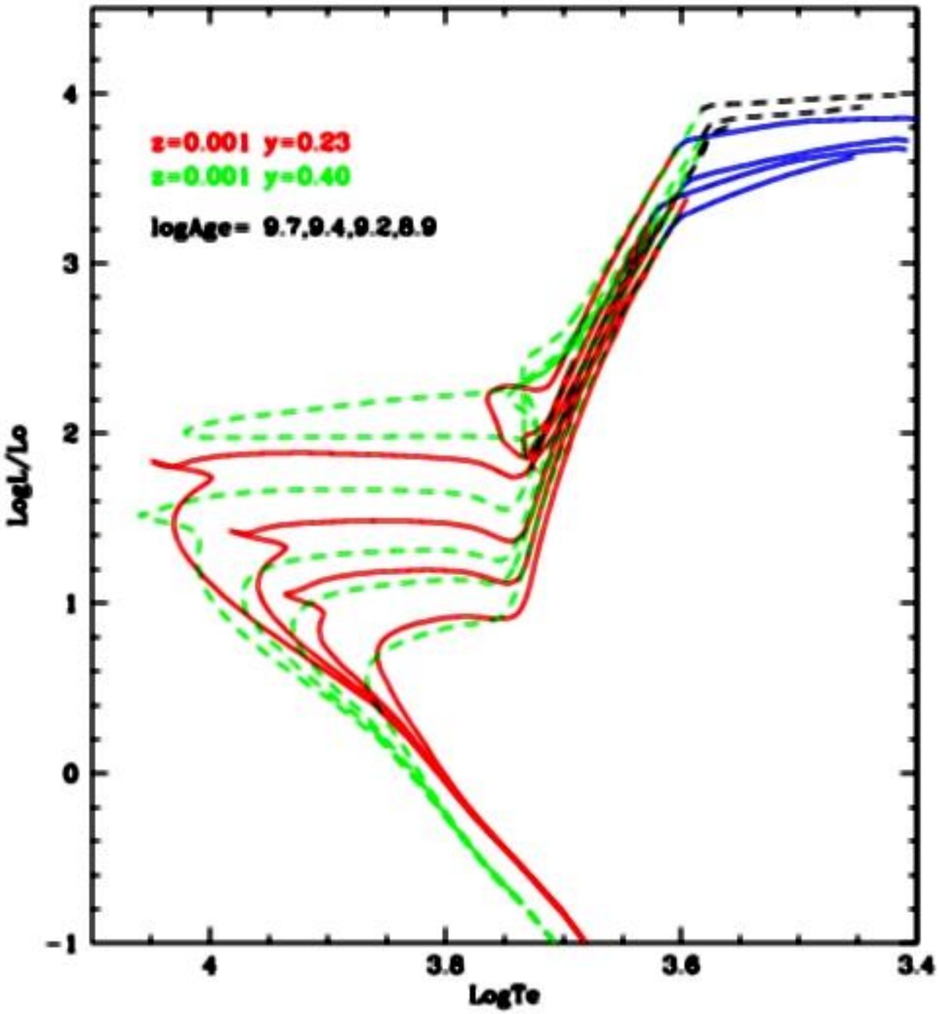


Table 2. Transformation of [Fe/H] to [Z] using [Y] = 0.23 + 2.25[Z] from Girardi et al. (2000) applied in this work.

[Fe/H]	[Z]	[Fe/H]	[Z]	[Fe/H]	[Z]
−0.729	0.004	−0.030	0.018	+0.253	0.032
−0.525	0.006	+0.019	0.020	+0.288	0.034
−0.387	0.008	+0.077	0.022	+0.312	0.036
−0.282	0.010	+0.116	0.024	+0.343	0.038
−0.224	0.012	+0.152	0.026	+0.371	0.040
−0.149	0.014	+0.185	0.028		
−0.086	0.016	+0.225	0.030		

Asplund et al. (2009)

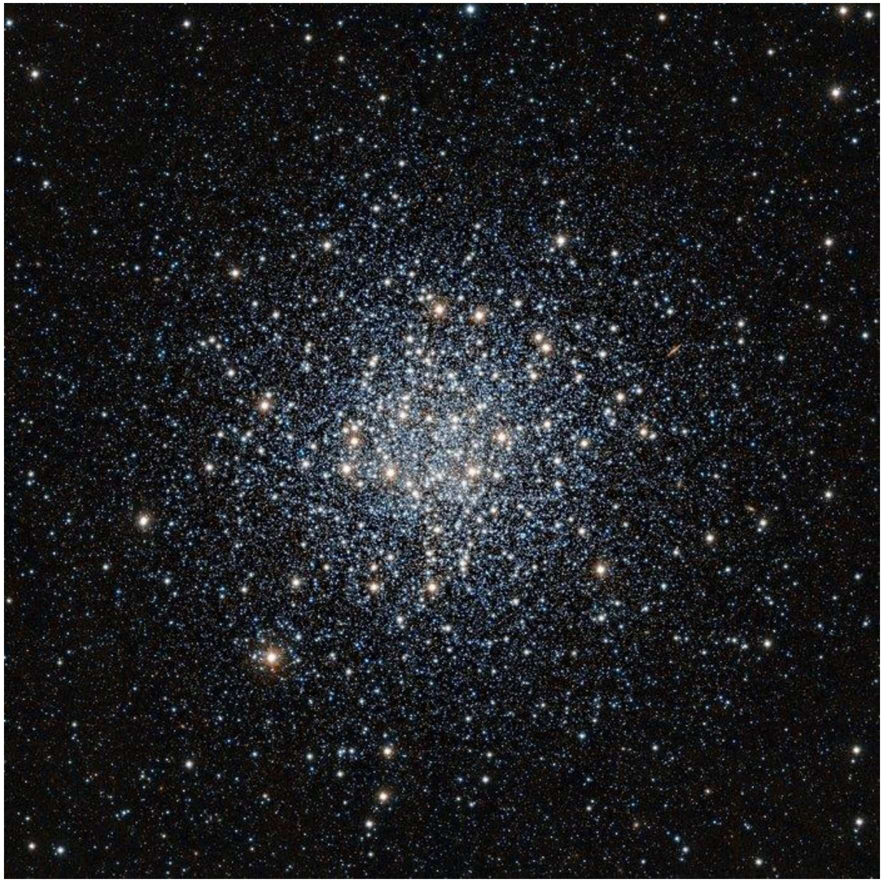
Isochrones - Metallicity



Different He abundances – [Z] constant

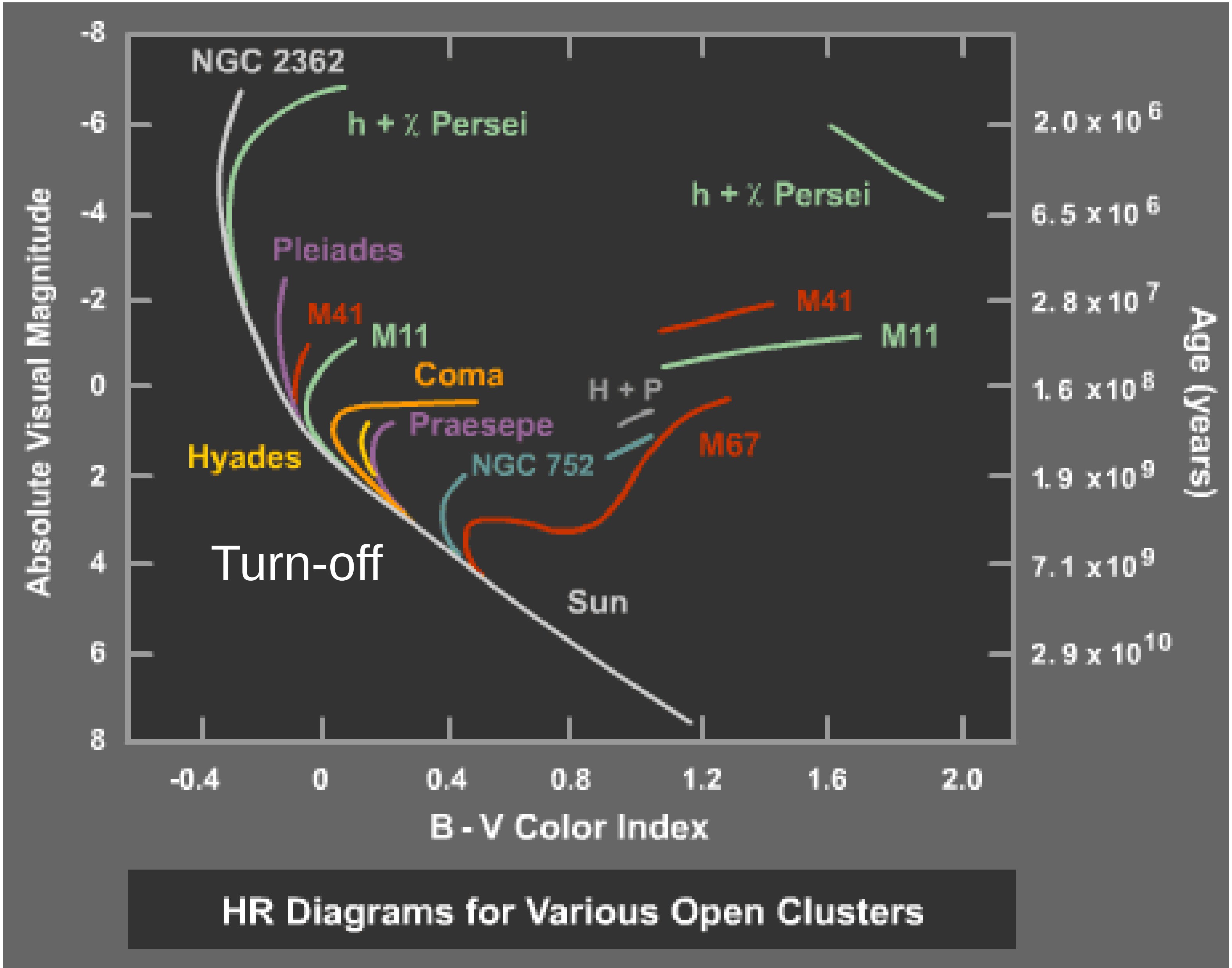
Star clusters

NGC 3766 – Open Cluster



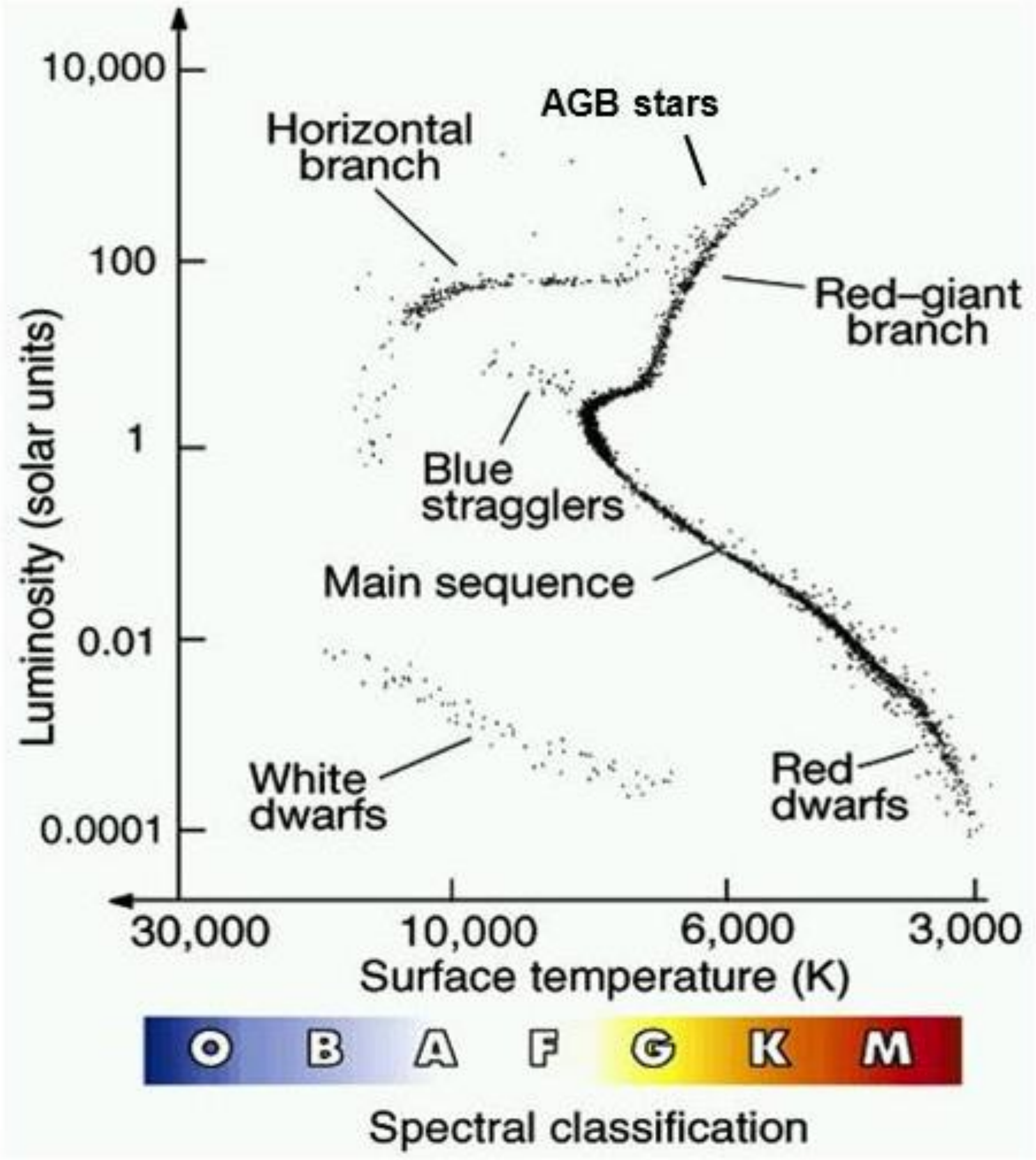
M55 – Globular Cluster

HRD- Open Clusters



HR Diagrams for Various Open Clusters

HRD – Globular Clusters



Star Clusters as Laboratories

- In clusters, age and metallicity must be same for all stars
- Hence differences must be due to masses
- Stellar evolution assumes that the differences in cluster stars are due only (or mainly) to initial masses (IMF)
- Cluster HR (or colour-magnitude) diagrams are quite similar – age determines overall appearance

Globular vs. Open clusters

Globular	Open
• MS turn-off points in similar position. Giant branch joining MS • Horizontal branch from giant branch to above the MS turn-off point • Horizontal branch often populated only with variable RR Lyrae stars	• MS turn off point varies massively, faintest is consistent with globulars • Maximum luminosity of stars can get to $M_\mathrm{v} \approx -10$ mag • Very massive stars found in these clusters

The differences are interpreted due to age – open clusters lie in the disk of the Milky Way and have large range of ages. The Globulars the oldest objects known tracing the earliest stages of the formation of Milky Way (~12×10^9 yrs)