$$\mathrm{INV}\,[u_2, \dots, u_1] = U$$

$$(\varphi - \lambda \, id_U)(u_2) = u_1$$

$$\varphi(U) \subseteq U$$

u_1 jest LN

Zionym

$$\sum_{j=1}^{k} c_j u_j = 0 \overset{?}{\Longrightarrow} c_1 = \dots = c_k = 0$$

$$(\varphi - \lambda\, id_V)\left(\sum_{j=1}^{k} c_j u_j\right) =$$

$$= \sum_{j=1}^{k} c_j (\varphi - \lambda\, id_V)(u_j) = \sum_{j=2}^{k} c_j u_{j-1} \quad \text{ind. pod}$$

$$\Rightarrow c_1 u_1 = 0 \Rightarrow c_1 = 0 \Rightarrow c_2 = \quad = c_k = 0$$

$A \sim B$

$\wr$

J

$$A \sim J_A = J_B \sim B$$

$$\Downarrow$$

$$A \sim B$$

$$u_1,\ T(u_1),\ \dots\dots,\ T^{a_1-1}(u_1),$$

$$u_2,\ T(u_2),\ \dots\dots,\ T^{a_2-1}(u_2),$$

$$\dots\dots\dots$$

$$u_s,\ T(u_s),\ \dots\dots,\ T^{a_s-1}(u_s)$$

$$T = \varphi - \lambda\, \mathrm{id}_V$$

DR. WILDON

$T = 0$ Zvolíme lib. bázi V

$u_1, \ldots, u_n$

$1 = \dim \operatorname{Im} T + \dim \operatorname{Ker} T$

$T \neq 0$

$\boxed{\dim V = 1}$ $\boxed{T^2 = 0} \Rightarrow T = 0$

$\dim V = n \qquad \forall k < n$ to platí!

$T : V \to V$

$\dim V = \dim \operatorname{Im} T + \dim \operatorname{Ker} T$

a) $\dim \operatorname{Ker} T = 0 \Rightarrow T$ bijekce.

b) $\dim \operatorname{Ker} T = \dim V \Rightarrow T = 0$ není možné.

c) $0 < \dim \operatorname{Im} T < \dim W$

$\Rightarrow \quad T|_{\operatorname{Im} T} : \operatorname{Im} T \to \operatorname{Im} T$

$\Downarrow$

$w_1, \dots, w_k \;,\; \exists\, a_1, \dots, a_k$

$(w_1, \dots, T^{a_1 - 1}(w_1), \dots\dots\dots, T^{a_k - 1}(w_k))$

bázis $\operatorname{Im} T$.

$u_1 \in \operatorname{Im} T, \ldots \ldots, u_k \in \operatorname{Im} T$

$\exists\, w_1 \in V, T(w_1) = u_1, \ldots \ldots$

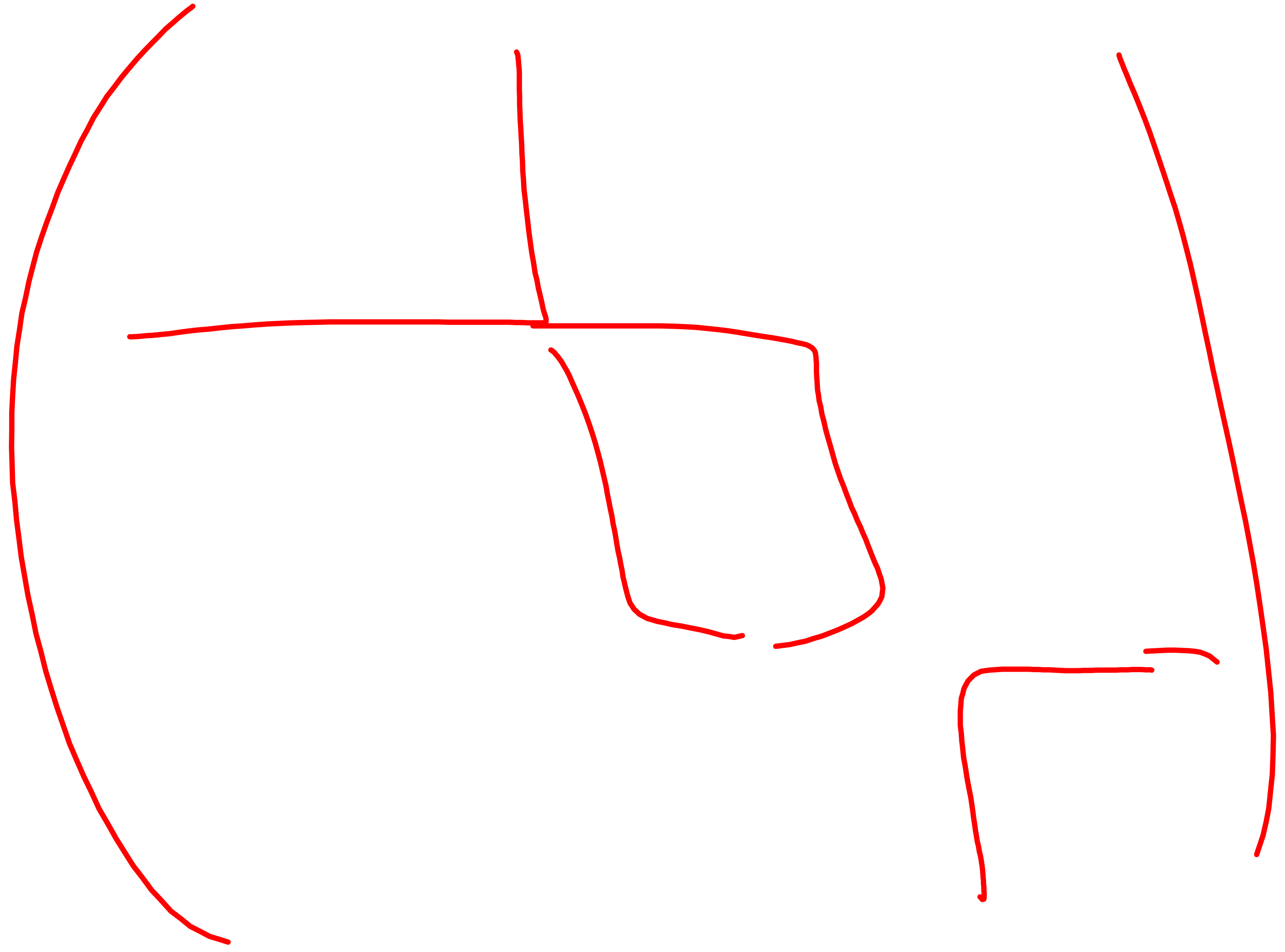
$\ldots \ldots w_k \in V, T(w_k) = u_k$

$w_1, \underset{\substack{\| \\ u_1}}{T(w_1)}, \ldots, T^{a_1}(w_1), \ldots \ldots, T^{a_k}(w_k)$

$\in \operatorname{Ker} T \qquad \in \operatorname{Ker} T$

$T|_{\mathrm{Ker}(\varphi-\chi_1)^{\rho}} : \mathrm{Ker}(\varphi-\chi_1\,\mathrm{id})^{\rho} \to \mathrm{Ker}(\varphi-\chi_1\,\mathrm{id})^{\rho}$

JBlock



L.2.6. λ EW. v. φ

$$\operatorname{Ker}(\varphi - \lambda\, id_V) \subseteq \operatorname{Ker}(\varphi - \lambda\, id_V)^2$$

$$\subseteq \ldots \subseteq \underline{\operatorname{Ker}(\varphi - \lambda\, id_V)^N}$$

③ $\underline{\mathrm{Ker}(\varphi-\lambda\,\mathrm{id}_V)^{\nu} \cap \mathrm{Im}(\varphi-\lambda\,\mathrm{id}_V)^{\nu} = \{0\}}$

$u \in$

$\exists\, v \in V,\quad (\varphi-\lambda\,\mathrm{id}_V)^{\nu}(v) = u$

$(\varphi-\lambda\,\mathrm{id}_V)^{\nu}(u) = 0 \qquad v \in \mathrm{Ker}(\varphi-\lambda\,\mathrm{id}_V)^{2\nu}$

$\Rightarrow (\varphi-\lambda\,\mathrm{id}_V)^{2\nu}(v) = 0 \qquad \mathrm{Ker}(\varphi-\lambda\,\mathrm{id}_V)^{2\nu} = \mathrm{Ker}(\varphi-\lambda\,\mathrm{id}_V)^{\nu}$

$\Rightarrow \underline{u = 0}$

$T^{a_1}(w_1), \ldots, T^{a_k}(w_k)$ je $\underline{\underline{LN}}$

a leží v jádře

Rozšíříme je na bázi jádra přidáním vektorů $R_1, \ldots, R_n$

$w_1, \dots, T^{a_1}(w_1), \dots, w_r, \dots, T^{a_r}(w_r),$

$R_1, \dots, R_n$ ZBAZE V

LN $\quad c^1_1 w_1 + \dots + \underline{c^1_{a_1} T^{a_1}(w_1)} + \dots$

$+ c^r_1 w_r + \dots + \underline{c^r_{a_r} T^{a_r}(w_r)} +$

$\underline{+ d_1 R_1 + \dots + d_n R_n} = 0$

Po aplikaci T dostaneme

$$c_1^1 u_1 + \dots + c_{a_1-1}^1 T^{a_1-1}(u_1) + \dots$$

$$+ c_1^k u_k + \dots c_{a_k-1}^k T^{a_k-1}(u_k) = 0$$

Z IND. PŘEDP. $c_1^1 = \dots = c_{a_1-1}^1 = 0 =$

$$= \dots = c_{a_k-1}^k$$

Z toho, že

$T^{a_1}(x_1), \dots, T^{a_k}(x_k), R_1, \dots, R_n$

je báze KerT máme i

$c^1_{a_1} = 0 = c^2_{a_2} = \dots = c^k_{a_k} = d_1 = \; = d_n.$

$\Rightarrow$ LN

$w_1, T(w_1), \ldots, T^{a_1}(w_1), \ldots$

$\ldots, w_2, T(w_2), \ldots, T^{a_2}(w_2),$

$R_1, \ldots, R_n$

$\dim \operatorname{Im} T = a_1 + \ldots + a_2$

$\dim V = \dim \operatorname{Im} T + \dim \operatorname{Ker} T$

$$\dim V = a_1 + \qquad + a_2 + \dim \ker T$$

$$= (a_1 + 1) + \qquad + (a_2 + 1) + n \geq k$$