$$A \cdot (B + C) = A \cdot B + A \cdot C$$

$$\varphi_A(B) = A \cdot B$$

$$F : L(V, U) \times V \to U$$

$$\varphi \in L(V,U), \quad x \in V$$

$$F(\varphi, x) = \varphi(x)$$

$$V \to K$$

$$\mathcal{L}(V, K) = V^*$$

$$[(x)_\alpha^T A](y)_\beta \qquad \alpha, \beta$$

$$F : U \times V \to K$$

$$[F]_{\alpha,\beta} = (F(u_i, v_j)) \in K^{m \times n}$$

$$x = \sum_{i=1}^{m} x_i u_i \,, \quad y = \sum_{j=1}^{m} y_j v_j$$

$$F(x,y) = F\left(\sum_{i=1}^{m} x_i u_i, y\right) =$$

$$= \sum_{i=1}^{m} x_i F\left(u_i, \sum_{j=1}^{m} y_j v_j\right) = \qquad a_{ij}$$

$$= \sum_{i=1}^{m} x_i \sum_{j=1}^{m} y_j F(u_i, v_j) = \sum_{i,j} x_i y_j F(u_i, v_j)$$

$$\Big(\underbrace{\begin{pmatrix} x_1 & \dots & x_m \end{pmatrix}}_{{}^t(x)_\beta} \big(F(u_i, v_j) \big)\Big) \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} = (y)_\beta$$

$$=$$

$$= \Big(\sum x_i F(u_i, v_1), \dots\dots, \sum x_i F(u_i, v_m) \Big) \begin{pmatrix} y_1 \\ \vdots \\ \vdots \\ y_m \end{pmatrix}$$

$$= \sum_j \Big(\sum_i x_i F(u_i, v_j) \Big) y_j$$

$$F(x,y) = (x)_\alpha^T B \cdot (y)_\beta$$

$i, j \qquad x = u_i, \quad y = v_j$

$$F(u_i, v_j) = (0, \dots, \underset{i}{1}, \dots 0)\, B \cdot \begin{array}{c} \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \end{array} \begin{array}{l} \\ \\ - j \\ \\ \\ \end{array}$$

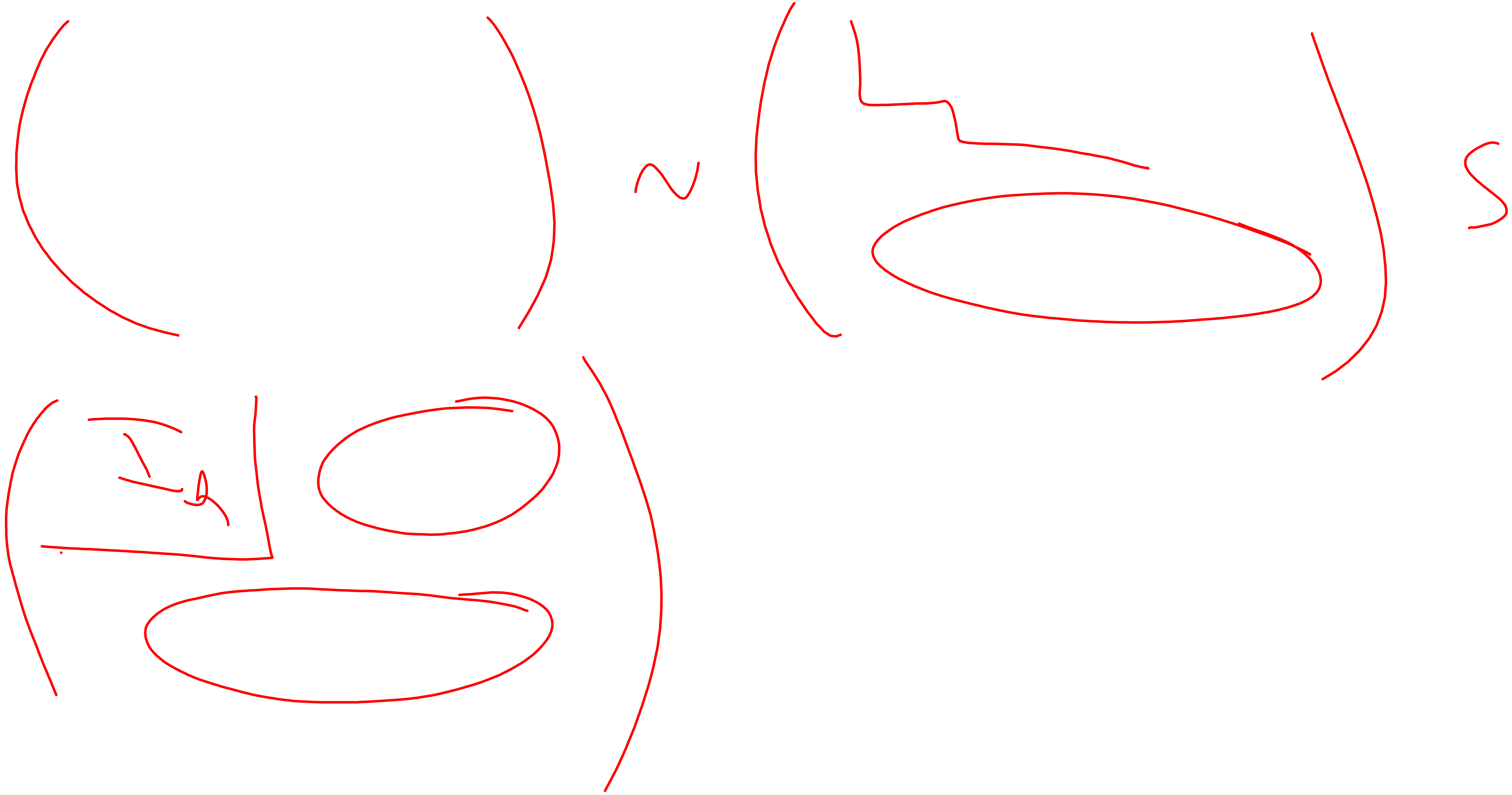
$$= r_i(B) \cdot \begin{pmatrix} 0 \\ \vdots \\ 1 \\ 0 \end{pmatrix} = b_{ij}$$

$$\underbrace{(x)^T_{\beta_1}\left(P_{\alpha_1,\beta_1}\right)^T}_{(x)^T_{\alpha_1}} \cdot [F]_{\alpha_1,\alpha_2} \cdot \underbrace{P_{\alpha_2,\beta_2}(y)_{\beta_2}}_{\overset{\parallel}{(y)_{\alpha_2}}}$$

$$= (x)^T_{\alpha_1}\,[F]_{\alpha_1,\alpha_2}\,(y)_{\alpha_2} = F(x,y)$$

$$A = [F]_{\alpha_1,\alpha_2}$$

$$B = \underset{\substack{\| \\ (P_{\alpha_1,\beta_1})^T}}{P^T} \, A \, \underset{\substack{\| \\ P_{\alpha_2,\beta_2}}}{Q}$$



$$F : U \times V \to K$$

$x \in U \qquad \varphi_x : V \to K \quad L2$

$$\varphi_x(y) = F(x, y)$$

$$F^* : U \to V^* = L(V, K)$$

$$F(x, y) = (x)_2^T (y)_\beta =$$

$$= \sum_{i,j} x_i y_j \delta_{ij} = \sum_{i=1}^{m} x_i y_i$$

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n_1, n_2 $\quad q$ $\quad$,, $n_1(1)\, q'(2) + n_2(1)\, q'(2)$

$$f(n_1 + n_2, q) = (n_1 + n_2)(1)\, q'(2)$$

$$F_0 = \frac{F(x,y)+F(y,x)}{2} \quad \boxed{F_0(x,y) = F_0(y,x)}$$

Sym

$$F_1(x,y) = \frac{F(x,y) - F(y,x)}{2}$$

$$F_1(y,x) = \frac{F(y,x) - F(x,y)}{2}$$

$$F_1(x,y) = -F(y,x)$$

$F = G + H$, G is sym.

H anti.

$\Rightarrow G = F_0$, $H = F_1$

$H(x, x) = -H(x, x)$

$2H(x, x) = 0$

$\boxed{H(x, x) = 0}$

$$0 = H(x+y,\, x+y) = \overbrace{H(x,x)}^{=0} + H(x,y) + H(y,x) + \underbrace{H(y,y)}_{=0}$$

$$F(x,x) = G(x,x) + \overbrace{H(x,x)}^{0\,=}$$

$$F(x,x) = G(x,x)$$

$$F(x+y,\, x+y) = G(x+y,\, x+y)$$

$$= \cancel{F(x,x)} + F(x,y) + F(y,x) + \cancel{F(y,y)}$$

$$= \cancel{G(x,x)} + G(x,y) + G(y,x) + \cancel{G(y,y)}$$

$$F(x,y) + F(y,x) = 2\, G(x,y)$$

$q: V \to K$ kv. f $\quad 2G(x,y) = 2F(x,y)$

$\exists$ sym. bil. F: $q(x) = F(x,x)$

$\exists\, G$ sym. bil. lad, že $\quad q(x) = G(x,x)$

$$q(x+y) = G(x+y, x+y) =$$
$$= G(x,x) + G(x,y) + G(y,x) + G(y,y)$$

$$\begin{pmatrix} \ddots & & a_{ji} \neq 0 \\ & 0 & \\ a_{ij} & & \ddots \end{pmatrix} \qquad 2a_{ij} \neq 0$$