$$\varphi \mapsto (\lambda_1, \dots, \lambda_r)$$
$$m_1 + \quad + m_r$$

$\varphi(v) = \lambda v, \quad v \neq 0$

$(\varphi - \lambda id)(v) = 0 \quad v \in \mathrm{Ker}(\varphi - \lambda id)$

$\dim \mathrm{Ker}(\varphi - \lambda id) = $ geom.más.

$v \in \mathrm{Ker}(\varphi - \lambda id) \Rightarrow$
$\Downarrow$
$\varphi(v) = \lambda v$

$S \subseteq V \quad \varphi(S) \subseteq S$

$u_1, \dots, u_r$ bázis S

bázis V

$(u_1, \dots, u_r, u_{r+1}, \dots u_n) = \alpha$

$$(\varphi)_\alpha = \left(\begin{array}{c|c} A_1 & B \\ \hline 0 & \boxed{A_2} \end{array}\right)$$

$$A = \begin{pmatrix} A_1 & B \\ O & A_2 \end{pmatrix} \qquad \varphi_1 \quad |A_1 - \lambda I_2|$$

$$\varphi \quad |A - \lambda I_n|$$

$$|A - \lambda I_n| = \begin{vmatrix} A_1 - \lambda I_2 & B \\ O & A_2 - \lambda I_{n-2} \end{vmatrix}$$

$$\Rightarrow |A_1 - \lambda I_2| \text{ divi } |A - \lambda I_n|$$

$$\lambda \qquad \varphi \qquad \dim \underbrace{\operatorname{Ker}(\varphi - \lambda\, id)}_{S}$$

$$\varphi_{|S} = \varphi_1$$

$$\varphi_1 : S \longrightarrow S$$

$$\begin{pmatrix} A_1 & B \\ O & A_2 \end{pmatrix}^{2 \times 2} \qquad \dim S \leq \text{alg.mól}$$

OG OP.

2.1. $u = v$ $\sqrt{\langle \varphi(u), \varphi(u) \rangle} = \sqrt{\langle u, u \rangle}$

2) $u \perp v$

$0 = \langle u, v \rangle = \langle \varphi(u), \varphi(v) \rangle$

$\Rightarrow \varphi(u) \perp \varphi(v)$

$\varphi(u) = \varphi(v) \Rightarrow u = v$

$\varphi(u) = 0 \Rightarrow u \overset{?}{=} 0$

$0 = \langle \varphi(u), \varphi(u) \rangle = \langle u, u \rangle$

4) $u_1, \ldots, u_k$ ON

$i \neq j \Rightarrow u_i \perp u_j \Rightarrow \varphi(u_i) \perp \varphi(u_j)$

$\|u_i\| = 1 \Rightarrow \|\varphi(u_i)\| = 1$

(5) $\dfrac{\langle u, v\rangle}{\|u\|\cdot\|v\|} = \dfrac{\langle \varphi(u), \varphi(v)\rangle}{\|\varphi(u)\|\cdot\|\varphi(v)\|}$

2.3. (1) ⟹ (2): ZŘEJMÉ.

(2) ⟹ (3)

$$\bar{A}^T = A$$

$$A = (\varphi)_\alpha \qquad s_j(A) = (\varphi(u_j))_\alpha$$

$$s_k(A)^T s_j(A) = \delta_{kj}$$

$$\langle \varphi(u_k), \varphi(u_j)\rangle = \delta_{kj}$$

$$\langle x, y\rangle_u = \sum_{i=1}^{n} x_i y_i \quad \begin{pmatrix} x_1 \\ x_n \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = (x)_\alpha, \quad \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = (y)_\alpha$$

$$A \quad B$$

$$B = P^{-1} A \cdot P$$

$$|A - \lambda I_n| = |B - \lambda I_n|$$

$$R(A - \lambda I_n) \sim R(B - \lambda I_n)$$

$$\begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$$

$$n = m_1 + \quad\quad + m_2$$

$$\vee$$

$$l_1 + \quad\quad + l_2$$

$$\lambda_1$$

$$\begin{pmatrix} \boxed{x_1 \ldots x_1}_{\,l_1} & & 0 \\ & \boxed{}_{\,l_2} & \\ 0 & & \boxed{x_2}_{\,l_2} \ldots\ldots \end{pmatrix} \qquad l_1 + \quad + l_2 = n$$

2.4 nd.

$$|\overline{A}^T| = \overline{|A|}$$

$$1 = |\overline{A}^T| \cdot |A| = \overline{|A|} \cdot |A|$$

$$\Rightarrow |A| = 1$$

(2.3) (3) $\Rightarrow$ (1)

u, v $\quad \langle \varphi(u), \varphi(v) \rangle \overset{?}{=} \langle u, v \rangle$

$$\langle \varphi(u), \varphi(v) \rangle = \overline{(\varphi(u))_d}^T \cdot (\varphi(v))_d =$$

$$\overline{(u)_d}^T \underbrace{\overline{A}^T A}_{I_n} \cdot (v)_d =$$

$$\overline{(u)_d}^T \cdot (v)_d = \langle u, v \rangle$$

2.4. $\quad \overline{A}^T \cdot A = \overline{I_n}$

Cauchy $\Rightarrow \det(\overline{A}^T) \cdot \det A = 1$

$$\overline{\sum_{\sigma \in S_n} (-1)^{|\sigma|} a_{1\sigma(1)} \cdots a_{n\sigma(n)}} =$$

$$= \sum_{\sigma \in S_n} (-1)^{|\sigma|} \overline{a_{1\sigma(1)}} \cdots \overline{a_{n\sigma(n)}}$$

$\boxed{V.2.6}$ $\langle u, v \rangle = \langle \varphi(u), \varphi(v) \rangle$

$\exists\, u_1 \neq 0,\ \lambda_1$

$\varphi(u_1) = \lambda u_1$

$\varphi(\langle u_1 \rangle) \subseteq \langle u_1 \rangle$

$x \in \varphi(\langle u_1 \rangle) \ni \varphi(c u_1) = c\lambda u_1 \in \langle u_1 \rangle$

x

$\varphi(\langle u_1 \rangle^{\perp}) \overset{?}{\subseteq} \langle u_1 \rangle^{\perp}$

$v \in \langle u_1 \rangle^{\perp} \Rightarrow \langle v, u_1 \rangle = 0$

$\langle \varphi(v), \varphi(u_1) \rangle = 0$

$\underbrace{\langle \varphi(v), u_1 \rangle}_{=0} \cdot \underbrace{\bar{\lambda}_1}_{\neq 0}$

$$\underbrace{\langle u_1\rangle}_{1} \oplus \underbrace{\langle u_1\rangle^{\perp}}_{n-1} = U$$

$$\varphi|_{\langle u_1\rangle^{\perp}}$$

indukc!

$$\begin{pmatrix} \lambda_1 & 0 \\ 0 & \boxed{\begin{matrix} \lambda_2 & 0 \\ 0 & \ddots\ \lambda_n \end{matrix}} \end{pmatrix}$$

2.5. $\varphi(v) = \lambda v, \quad v \neq 0$

$$\langle \varphi(v), \varphi(v) \rangle = \langle \lambda v, \lambda v \rangle$$

$$= \lambda \cdot \bar{\lambda} \langle v, v \rangle$$

$$\langle v, v \rangle \Rightarrow \lambda \cdot \bar{\lambda} = 1$$

$u, v \neq 0, \quad \lambda \neq \mu$

$\varphi(u) = \lambda u, \quad \varphi(v) = \mu v$

$$\langle \varphi(u), \varphi(v) \rangle = \langle \lambda u, \mu v \rangle =$$

$$= \lambda \bar{\mu} \langle u, v \rangle = \langle u, v \rangle^{\neq 0}$$

$$\Rightarrow \lambda \bar{\mu} = 1 \quad \Big| \, \mu \qquad \bar{\mu} \cdot \mu = 1$$

$$\lambda \underbrace{\bar{\mu} \mu}_{=1} = \mu$$

$$\Rightarrow \lambda = \mu \quad \text{SPOR}$$