A_n jel neg.

$$\begin{pmatrix} a_{11} & & \\ & \ddots & \\ & & a_{nn} \end{pmatrix}$$

$$A \overset{(1+)}{\sim} D$$

$$P = \begin{pmatrix} 1 & & \Delta \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$$

$\det P = 1$

$$D = P^T A P$$

$$D = P^{-1} A P$$

$$P_1^{-1} \begin{pmatrix} \boxed{A_1} & \\ & \end{pmatrix} P_1 = \begin{pmatrix} \boxed{D_1} & \neq 0 \\ & \neq 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & \boxed{} \\ & \end{pmatrix}$$

(ii) $\Rightarrow$ (iii) zřejmé, (iv) $\Rightarrow$ (i)

(iii) $\Rightarrow$ (iv) $\qquad |A_1| \neq 0 \neq a_{11}$

$\neq 0$

A is pr. def $\iff |A_k| > 0 \ \forall k$

$\iff$ JACOBI $\left(|A_1|, \frac{|A_2|}{|A_1|}, \dots, \frac{|A_n|}{|A_{n-1}|} \right)$

$\sum_{i=1}^{n} c_i x_i^2$, $c_i > 0$

$|A_1| > 0$, $\frac{|A_2|}{|A_1|} > 0$, ..., $\frac{|A_n|}{|A_{n-1}|} > 0$

$c_1 > 0$ .... $c_n > 0$

A is pos. def. $\qquad A = D = I_n$

$$D = P^T \cdot A \cdot P \qquad g(x) > 0$$

$\det P \cdot \det A \, \det P$ ($\det A > 0$) $\qquad x \neq 0$

$$\Rightarrow |A_1| \neq 0 \qquad S = [e_1, \dots, e_k] \to x$$

$$x^T A x = \bar{x}^T A_k \bar{x} \qquad \bar{x} = (x_1, \dots, x_k)$$

(b) $\quad x^T A x < 0 \quad \forall x \neq 0$

$x^T(-A)x > 0 \qquad |(-1)$

$\Leftrightarrow \; |-A_k| > 0 \; \Leftrightarrow \; (-1)^k |A_k| > 0$

$$\boxed{\sqrt{x_1^2 + x_2^2}}$$

$$\int_{\bar a}^{\bar b} f^2\,dx \le \int_a^b f^2\,dx$$

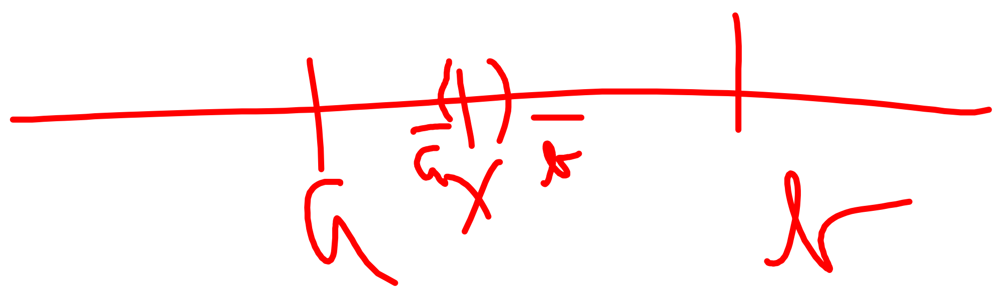
$$\vee|$$

$$\int_{\bar a}^{\bar b} \min = \min(\bar b - \bar a)\, b > 0$$

$$f \neq 0 \quad f \in C[a,b] \overset{?0}{\Rightarrow} \int_a^b f^2(x)\,dx > 0$$

$$\exists x \in (a,b) \quad f(x) \neq 0$$

$$f^2(x) \neq 0$$



$$\begin{pmatrix} 3 & 1 & 2 \\ 1 & 1 & 0 \\ 2 & 0 & 7 \end{pmatrix}$$

$$7 \cdot 1 + 2(-2) = 3 > 0$$

$\partial = (u_1, \dots, u_n)$ $\quad f$

$$A = (f(u_i, u_j))$$

$\mathbb{R}: \quad A = \overline{A^T} \qquad \mathbb{C}: \quad A = \overline{A^T}$

$$\langle x, x\rangle = \overline{\langle x, x\rangle} \in \mathbb{R}$$

$$f(u_i, u_i) > 0$$

$$\langle x, cy\rangle = \langle cy, x\rangle =$$

$$= \overline{c\langle y, x\rangle} = \bar{c}\,\overline{\langle y, x\rangle} = \bar{c}\langle x, y\rangle$$

$$z = a + ib$$

$$\bar{z} = a - ib$$

$\langle 0, 0 \rangle = 0 \cdot \langle a, b \rangle = 0$

$x = (x_1, \dots, x_k)$

$a_{ij} = \langle u_i, u_j \rangle = \langle u_j, u_i \rangle = a_{ji}$

$\forall x \in \mathbb{R}^k$

$$\boxed{x^T G(\alpha)\, x \geq 0}$$

$$0 \leq \left\langle \sum_{i=1}^{k} x_i u_i, \sum_{j=1}^{k} x_j u_j \right\rangle = \sum_i \sum_j x_i x_j \langle u_i, u_j \rangle$$

$$= x^T G(\alpha)\, x$$

$u_1 \dots u_k$ LN $\overset{?}{\Rightarrow}$ $G(\alpha)$ por. <u>def</u>

$$\forall x \in \mathbb{R}^k,\ x \neq 0 \qquad x^T G(\alpha) x \overset{?}{>} 0$$

$$0 = x^T G(\alpha) x = \left\langle \sum_{i=1}^k x_i u_i ,\ \sum_{i=1}^k x_i u_i \right\rangle \geq 0$$

<u>vždy</u>

$$\sum_{i=1}^k x_i u_i = 0 \ \overset{LN}{\Downarrow}\ x_1 = x_2 = \dots = x_n = 0$$

$\Longleftarrow$: $G(\alpha)$ is pos. def $\overset{?}{\Longrightarrow}$ α is $\underline{\text{LN}}$

$$x = (x_1, \dots, x_k), \quad \sum_{i=1}^{k} x_i u_i = 0$$

$$0 = \langle 0, 0 \rangle = \left\langle \sum x_i u_i, \sum x_j u_j \right\rangle =$$

$$= x^T \cdot G(\alpha) x \Rightarrow \underline{\underline{x = 0}}$$

$$P^T A P = D \qquad \begin{pmatrix} >0 & & \\ & >0 & \\ & & \end{pmatrix}$$

$$\det D > 0$$

$$\det P^T \, \det A \, \det P = \det D$$

$u, v \in V$

$$|G(u,v)| = \begin{vmatrix} \langle u,u\rangle & \langle u,v\rangle \\ \langle v,u\rangle & \langle v,v\rangle \end{vmatrix}$$

$$= \langle u,u\rangle \cdot \langle v,v\rangle - \langle u,v\rangle^2 \geq 0$$

$$|\langle u,v\rangle| \leq \underbrace{\sqrt{\langle u,u\rangle}}_{\|u\|} \cdot \underbrace{\sqrt{\langle v,v\rangle}}_{\|v\|}$$

$\|u\| \neq 0, \quad \|v\| \neq 0$

$$\frac{|\langle u, v \rangle|}{\|u\| \cdot \|v\|} \leq 1$$

$$-1 \leq \underbrace{\frac{\langle u, v \rangle}{\|u\| \, \|v\|}}_{} \leq 1$$

$= \cos\alpha$

$$\|x+y\| \le \|x\| + \|y\|$$

$$\langle 0, 0 \rangle = 0$$

$$\Downarrow$$

$$\|0\| = 0$$

$$\|cx\| = |c|\,\|x\|$$

$$\langle x, x \rangle = 0$$

$$\Downarrow$$

$$x = 0$$

$$x = 0 \Leftrightarrow \|x\| = 0$$

$$\sqrt{\langle cx, cx \rangle} = \sqrt{c^2 \langle x, x \rangle} = \|c\|\,\|x\|$$

$$\|$$

$$\|cx\|$$