$$\varphi(x) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

nemá reálne vl. čísla

$\mathbb{R}^2 \to \mathbb{R}^2$

$\mathbb{C}^2 \to \mathbb{C}^2$

$$\det \begin{pmatrix} \cos\alpha - \lambda & -\sin\alpha \\ \sin\alpha & \cos\alpha - \lambda \end{pmatrix} = (\cos\alpha - \lambda)^2 + \sin^2\alpha$$

$$= \lambda^2 - 2\lambda\cos\alpha + \cos^2\alpha + \sin^2\alpha = \lambda^2 - 2\lambda\cos\alpha + 1$$

$$D = 4\cos^2\alpha - 4 = 4(\cos^2\alpha - 1) \qquad \alpha \neq k\pi \Rightarrow D < 0$$

$$\lambda_1 = \cos\alpha + i\sin\alpha$$
$$\lambda_2 = \cos\alpha - i\sin\alpha$$

$$\lambda_{1,2} = \frac{2\cos\alpha \pm \sqrt{4\cos^2\alpha - \cancel{4}}}{2} = \cos\alpha \pm \sqrt{-\sin^2\alpha} = \cos\alpha \pm i\sin\alpha$$

ORTOGONÁLNÍ OPERÁTORY V DIMENZI 2

$\varphi : \mathbb{R}^2 \to \mathbb{R}^2 \quad \varphi(x) = Ax$, A má sloupce navzájem kolmé velikosti 1

Dvě možnosti:

$$A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \qquad \det A = a^2 + b^2 = 1$$

$$a^2 + b^2 = \underline{1}$$

Lze najít $\alpha \in \mathbb{R}$ $\quad a = \cos\alpha$, $b = \sin\alpha$

$$A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$ .... otočení o úhel α

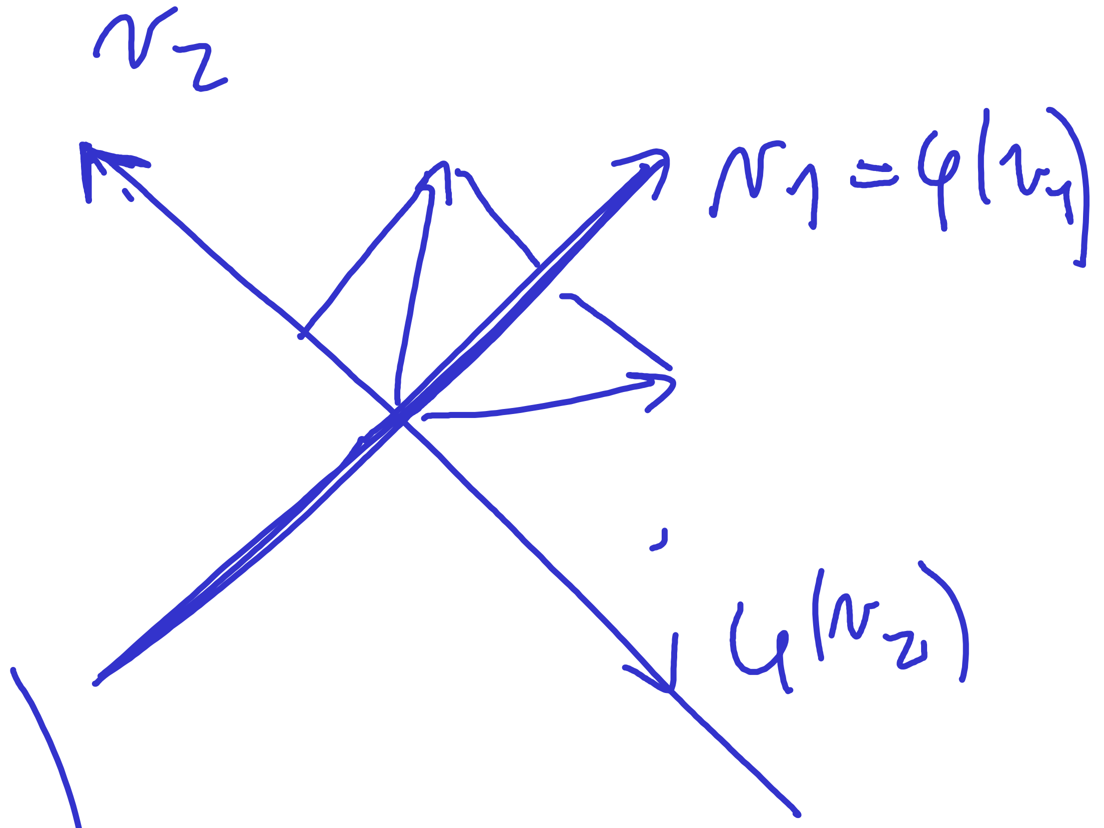
(2) $A = \begin{pmatrix} a & b \\ b & -a \end{pmatrix}$ $\quad a^2 + b^2 = 1$ $\quad \det A = -a^2 - b^2 = -1$

$$\det(A - \lambda) = \begin{pmatrix} a-\lambda & b \\ b & -a-\lambda \end{pmatrix} = (\lambda - a)(\lambda + a) - b^2 = \lambda^2 - a^2 - b^2 = \lambda^2 - 1 =$$
$$= (\lambda - 1)(\lambda + 1).$$

Vlastní vektory

$$\begin{pmatrix} a-1 & b \\ b & -a-1 \end{pmatrix} \sim \begin{pmatrix} a-1 & b \\ 0 & 0 \end{pmatrix} \qquad \begin{aligned} x_1 &= b \\ x_2 &= 1-a \end{aligned} \qquad v_1 = \begin{pmatrix} b \\ 1-a \end{pmatrix}$$

$$\begin{pmatrix} a+1 & b \\ b & -a+1 \end{pmatrix} \sim \begin{pmatrix} a+1 & b \\ 0 & 0 \end{pmatrix} \qquad \begin{aligned} x_1 &= -b \\ x_2 &= +a+1 \end{aligned} \qquad v_2 = \begin{pmatrix} -b \\ a+1 \end{pmatrix}$$



$$\langle v_1, v_2 \rangle = -b^2 + 1 - a^2 = 1 - a^2 - b^2 = 0$$

Symetrie podle přímky určené vl. vektorem k vl. číslu 1.